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GETAL & RUIMTE

B

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9 Exponentiële en logaritmische functies

Voorkennis De logaritme

Bladzijde 9

1 **a** $^2\log(64) = {^2\log}(2^6) = 6$

b $^3\log(\frac{1}{27}) = {^3\log}(3^{-3}) = -3$

c $^6\log(6\sqrt{6}) = {^6\log}(6^{1\frac{1}{2}}) = 1\frac{1}{2}$

d $^{\frac{1}{3}}\log(1) = {^{\frac{1}{3}}\log}((\frac{1}{3})^0) = 0$

e $^{\frac{1}{2}}\log(\frac{1}{8}) = {^{\frac{1}{2}}\log}((\frac{1}{2})^3) = 3$

f $^3\log(\frac{1}{9}\sqrt{3}) = {^3\log}(3^{-2} \cdot 3^{\frac{1}{2}}) = {^3\log}(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$

2 **a** $^3\log(x) = 5$
$x = 3^5 = 243$

b $^2\log(3x + 7) = 4$
$3x + 7 = 2^4$
$3x + 7 = 16$
$3x = 9$
$x = 3$

c $5 + {^5\log}(x + 3) = 7$
$^5\log(x + 3) = 2$
$x + 3 = 5^2$
$x + 3 = 25$
$x = 22$

d $^{\frac{1}{2}}\log(3x + 4) = -2$
$3x + 4 = (\frac{1}{2})^{-2}$
$3x + 4 = 4$
$3x = 0$
$x = 0$

e $3 + {^2\log}(5 - 1\frac{1}{2}x) = 0$
$^2\log(5 - 1\frac{1}{2}x) = -3$
$5 - 1\frac{1}{2}x = 2^{-3}$
$5 - 1\frac{1}{2}x = \frac{1}{8}$
$40 - 12x = 1$
$-12x = -39$
$x = 3\frac{1}{4}$

f $4 - 2 \cdot {^3\log}(x) = 3$
$-2 \cdot {^3\log}(x) = -1$
$^3\log(x) = \frac{1}{2}$
$x = 3^{\frac{1}{2}} = \sqrt{3}$

9.1 Rekenregels voor logaritmen

Bladzijde 10

1 **a** $^2\log(a)$ is de exponent van een macht met grondtal 2 waarmee de macht gelijk is aan a, dus $2^{^2\log(a)} = a$. Omdat een macht met grondtal 2 altijd groter is dan 0, geldt dit alleen voor $a > 0$.

b $^g\log(x) = y$ oftewel $y = {^g\log}(x)$ substitueren in $x = g^y$ geeft $x = g^{^g\log(x)}$, dus $g^{^g\log(x)} = x$.

Bladzijde 11

2 $2 \cdot {^2\log}(800) = 2 \cdot {^2\log}(32 \cdot 25) = 2 \cdot ({^2\log}(32) + {^2\log}(25)) = 2 \cdot ({^2\log}(2^5) + {^2\log}(5^2)) =$
$2 \cdot (5 + 2 \cdot {^2\log}(5)) = 10 + 4 \cdot {^2\log}(5)$

3 **a** $^2\log(6) + {^2\log}(10) = {^2\log}(6 \cdot 10) = {^2\log}(60)$

b $^3\log(30) - {^3\log}(6) = {^3\log}(\frac{30}{6}) = {^3\log}(5)$

c $2 \cdot {^5\log}(3) + {^5\log}(\frac{1}{2}) = {^5\log}(3^2) + {^5\log}(\frac{1}{2}) = {^5\log}(9) + {^5\log}(\frac{1}{2}) = {^5\log}(9 \cdot \frac{1}{2}) = {^5\log}(4\frac{1}{2})$

d $^{\frac{1}{2}}\log(15) - 4 \cdot {^{\frac{1}{2}}\log}(3) = {^{\frac{1}{2}}\log}(15) - {^{\frac{1}{2}}\log}(3^4) = {^{\frac{1}{2}}\log}(15) - {^{\frac{1}{2}}\log}(81) = {^{\frac{1}{2}}\log}(\frac{15}{81}) = {^{\frac{1}{2}}\log}(\frac{5}{27})$

e $-2 \cdot {^4\log}(6) + {^4\log}(12) = {^4\log}(6^{-2}) + {^4\log}(12) = {^4\log}(\frac{1}{36}) + {^4\log}(12) = {^4\log}(\frac{1}{36} \cdot 12) = {^4\log}(\frac{1}{3})$

f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log(\frac{50}{25}) = \log(2)$

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4 **a** $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$

b $3 - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log(\frac{1}{8}) - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log\left(\frac{\frac{1}{8}}{10}\right) = {}^{\frac{1}{2}}\log(\frac{1}{80})$

c $2 - \log(5) = \log(10^2) - \log(5) = \log(100) - \log(5) = \log(\frac{100}{5}) = \log(20)$

d ${}^2\log(12) - {}^3\log(9) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(12) - {}^2\log(4) = {}^2\log(\frac{12}{4}) = {}^2\log(3)$

e $\frac{1}{2} \cdot {}^3\log(16) + {}^{\frac{1}{3}}\log(8) = {}^3\log(16^{\frac{1}{2}}) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log(\frac{4}{27})$

f $\log(500) - {}^5\log(125) = \log(500) - 3 = \log(500) - \log(10^3) = \log(500) - \log(1000) = \log(\frac{500}{1000}) = \log(\frac{1}{2})$

5 **a** ${}^3\log(6) + {}^3\log(1\frac{1}{2}) = {}^3\log(6 \cdot 1\frac{1}{2}) = {}^3\log(9) = 2$

b ${}^5\log(2) - {}^5\log(50) = {}^5\log(\frac{2}{50}) = {}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$

c ${}^2\log(27) + 3 \cdot {}^2\log(\frac{1}{6}) = {}^2\log(27) + {}^2\log((\frac{1}{6})^3) = {}^2\log(27) + {}^2\log(\frac{1}{216}) = {}^2\log(\frac{27}{216}) = {}^2\log(\frac{1}{8}) = {}^2\log(2^{-3}) = -3$

d $2 \cdot {}^4\log(6) - 2 \cdot {}^4\log(3) = {}^4\log(6^2) - {}^4\log(3^2) = {}^4\log(36) - {}^4\log(9) = {}^4\log(\frac{36}{9}) = {}^4\log(4) = 1$

6 **a** $g^{{}^g\log(a) - {}^g\log(b)} = \frac{g^{{}^g\log(a)}}{g^{{}^g\log(b)}} = \frac{a}{b} = g^{{}^g\log(\frac{a}{b})}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log\left(\frac{a}{b}\right)$.

b $g^{p \cdot {}^g\log(a)} = (g^{{}^g\log(a)})^p = a^p = g^{{}^g\log(a^p)}$, dus $p \cdot {}^g\log(a) = {}^g\log(a^p)$.

7 **a** ${}^2\log(a) + 3 \cdot {}^2\log(b) = {}^2\log(a) + {}^2\log(b^3) = {}^2\log(ab^3)$

b $5 \cdot {}^3\log(a) - 2 \cdot {}^3\log(b) = {}^3\log(a^5) - {}^3\log(b^2) = {}^3\log\left(\frac{a^5}{b^2}\right)$

c $2 + {}^5\log(a) = {}^5\log(5^2) + {}^5\log(a) = {}^5\log(25) + {}^5\log(a) = {}^5\log(25a)$

d $2 - {}^3\log(a) = {}^3\log(3^2) - {}^3\log(a) = {}^3\log(9) - {}^3\log(a) = {}^3\log\left(\frac{9}{a}\right)$

e ${}^6\log(a) - 1 = {}^6\log(a) - {}^6\log(6) = {}^6\log\left(\frac{a}{6}\right) = {}^6\log(\frac{1}{6}a)$

f $2 \cdot {}^5\log(b) + \frac{1}{2} \cdot {}^5\log(a) = {}^5\log(b^2) + {}^5\log(a^{\frac{1}{2}}) = {}^5\log(b^2) + {}^5\log(\sqrt{a}) = {}^5\log(b^2 \cdot \sqrt{a})$

Bladzijde 12

8 **a** $\log(600) = \log(100 \cdot 6) = \log(100) + \log(6) = 2 + \log(6)$

b ${}^2\log(24) = {}^2\log(8 \cdot 3) = {}^2\log(8) + {}^2\log(3) = 3 + {}^2\log(3)$

c $4 \cdot {}^3\log(54) = 4 \cdot {}^3\log(27 \cdot 2) = 4 \cdot ({}^3\log(27) + {}^3\log(2)) = 4 \cdot (3 + {}^3\log(2)) =$
$12 + 4 \cdot {}^3\log(2) = 12 + {}^3\log(2^4) = 12 + {}^3\log(16)$

d $2 \cdot {}^5\log(1250) = 2 \cdot {}^5\log(625 \cdot 2) = 2 \cdot ({}^5\log(625) + {}^5\log(2)) = 2 \cdot (4 + {}^5\log(2)) =$
$8 + 2 \cdot {}^5\log(2) = 8 + {}^5\log(2^2) = 8 + {}^5\log(4)$

e $\log(1600) - 1 = \log(100 \cdot 16) - 1 = \log(100) + \log(16) - 1 = 2 + \log(16) - 1 = 1 + \log(16)$

f $7 + {}^2\log(1600) = 7 + {}^2\log(64 \cdot 25) = 7 + {}^2\log(64) + {}^2\log(25) = 7 + 6 + {}^2\log(25) = 13 + {}^2\log(25)$

9 **a** $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } 81} y = {}^3\log(\frac{1}{81}x)$

Er geldt ${}^3\log(\frac{1}{81}x) = {}^3\log(\frac{1}{81}) + {}^3\log(x) = -4 + {}^3\log(x)$.
Dus $a = -4$.

b Je had de grafiek van g ook kunnen krijgen met de translatie (0, −4).

10 **a** $y = {}^2\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{32}} y = {}^2\log(32x)$
Er geldt ${}^2\log(32x) = {}^2\log(32) + {}^2\log(x) = 5 + {}^2\log(x)$.
Dus de translatie (0, 5) levert dezelfde beeldgrafiek op.

b $y = {}^2\log(x) \xrightarrow{\text{translatie } (0,\ 3)} y = {}^2\log(x) + 3$
Er geldt ${}^2\log(x) + 3 = {}^2\log(x) + {}^2\log(2^3) = {}^2\log(x) + {}^2\log(8) = {}^2\log(8x)$
Dus de vermenigvuldiging ten opzichte van de y-as met $\frac{1}{8}$ levert dezelfde beeldgrafiek op.

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11 $y = {}^2\log(x) \xrightarrow{\text{translatie }(3,\,0)} y = {}^2\log(x-3) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{4}} y = {}^2\log(4x-3)$

Er geldt ${}^2\log(4x-3) = {}^2\log(4(x-\frac{3}{4})) = {}^2\log(4) + {}^2\log(x-\frac{3}{4}) = 2 + {}^2\log(x-\frac{3}{4})$.

Dus $p = 2$ en $q = -\frac{3}{4}$.

12 $y = 3^x \xrightarrow{\text{translatie }(a,\,0)} y = 3^{x-a} \xrightarrow{\text{verm. } y\text{-as, } b} y = 3^{\frac{1}{b}x - a}$

$g(x) = 81 \cdot 27^x = 3^4 \cdot (3^3)^x = 3^4 \cdot 3^{3x} = 3^{3x+4}$

Dus $b = \frac{1}{3}$ en $a = -4$.

$y = {}^3\log(x) \xrightarrow{\text{translatie }(-4,\,0)} y = {}^3\log(x+4) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = {}^3\log(3x+4)$

Er geldt ${}^3\log(3x+4) = {}^3\log(3(x+1\frac{1}{3})) = {}^3\log(3) + {}^3\log(x+1\frac{1}{3}) = 1 + {}^3\log(x+1\frac{1}{3})$.

Dus $p = 1$ en $q = 1\frac{1}{3}$.

13 **a** $3 + {}^2\log(3) = {}^2\log(2^3) + {}^2\log(3) = {}^2\log(8) + {}^2\log(3) = {}^2\log(8 \cdot 3) = {}^2\log(24)$

b ${}^2\log(x+1) = 3 + {}^2\log(3)$
${}^2\log(x+1) = {}^2\log(24)$
$x + 1 = 24$
$x = 23$

Bladzijde 14

14 **a** ${}^5\log(x) = 3 \cdot {}^5\log(2) - 2 \cdot {}^5\log(3)$
${}^5\log(x) = {}^5\log(2^3) - {}^5\log(3^2)$
${}^5\log(x) = {}^5\log(8) - {}^5\log(9)$
${}^5\log(x) = {}^5\log(\frac{8}{9})$
$x = \frac{8}{9}$

b ${}^2\log(x) = 4 - {}^2\log(3)$
${}^2\log(x) = {}^2\log(2^4) - {}^2\log(3)$
${}^2\log(x) = {}^2\log(16) - {}^2\log(3)$
${}^2\log(x) = {}^2\log(\frac{16}{3})$
$x = 5\frac{1}{3}$

c ${}^2\log(x+3) = 3 + {}^2\log(x)$
${}^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$
${}^2\log(x+3) = {}^2\log(8) + {}^2\log(x)$
${}^2\log(x+3) = {}^2\log(8x)$
$x + 3 = 8x$
$-7x = -3$
$x = \frac{3}{7}$

d ${}^3\log(2x) = 1 + {}^3\log(x+1)$
${}^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$
${}^3\log(2x) = {}^3\log(3(x+1))$
${}^3\log(2x) = {}^3\log(3x+3)$
$2x = 3x + 3$
$-x = 3$
$x = -3$
vold. niet

15 **a** $5 \cdot \log(x) = 5 - \log(3125)$
$5 \cdot \log(x) = 5 - \log(5^5)$
$5 \cdot \log(x) = 5 - 5 \cdot \log(5)$
$\log(x) = 1 - \log(5)$
$\log(x) = \log(10) - \log(5)$
$\log(x) = \log(\frac{10}{5})$
$x = 2$

b ${}^{\frac{1}{2}}\log(2x-1) = 2 + {}^{\frac{1}{2}}\log(x+2)$
${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(x+2)$
${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}) + {}^{\frac{1}{2}}\log(x+2)$
${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}(x+2))$
${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}x + \frac{1}{2})$
$2x - 1 = \frac{1}{4}x + \frac{1}{2}$
$1\frac{3}{4}x = 1\frac{1}{2}$
$x = \frac{6}{7}$

c ${}^3\log(x+2) = 1 - {}^3\log(x)$
${}^3\log(x+2) = {}^3\log(3) - {}^3\log(x)$
${}^3\log(x+2) = {}^3\log\left(\frac{3}{x}\right)$
$x + 2 = \frac{3}{x}$
$x^2 + 2x = 3$
$x^2 + 2x - 3 = 0$
$(x-1)(x+3) = 0$
$x = 1 \vee x = -3$
vold. niet

d $2 \cdot {}^3\log(x) + 1 = {}^3\log(5x-2)$
${}^3\log(x^2) + {}^3\log(3) = {}^3\log(5x-2)$
${}^3\log(3x^2) = {}^3\log(5x-2)$
$3x^2 = 5x - 2$
$3x^2 - 5x + 2 = 0$
$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$
$x = \frac{5+1}{6} = 1 \vee x = \frac{5-1}{6} = \frac{2}{3}$

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16 **a** $^5\log(x) = 2 + \frac{1}{2} \cdot {^5\log(3)}$
$^5\log(x) = {^5\log(5^2)} + {^5\log(3^{\frac{1}{2}})}$
$^5\log(x) = {^5\log(25)} + {^5\log(\sqrt{3})}$
$^5\log(x) = {^5\log(25\sqrt{3})}$
$x = 25\sqrt{3}$

b $^3\log(x + 4) + 1 = 2 \cdot {^3\log(x - 2)}$
$^3\log(x + 4) + {^3\log(3)} = {^3\log((x - 2)^2)}$
$^3\log(3(x + 4)) = {^3\log(x^2 - 4x + 4)}$
$^3\log(3x + 12) = {^3\log(x^2 - 4x + 4)}$
$3x + 12 = x^2 - 4x + 4$
$x^2 - 7x - 8 = 0$
$(x + 1)(x - 8) = 0$
$x = -1 \quad \vee \quad x = 8$
vold. niet

c $^2\log(2x) - {^2\log(x + 3)} = {^2\log(x)} - 2$
$^2\log\left(\frac{2x}{x + 3}\right) = {^2\log(x)} - {^2\log(2^2)}$
$^2\log\left(\frac{2x}{x + 3}\right) = {^2\log\left(\frac{x}{4}\right)}$
$\frac{2x}{x + 3} = \frac{x}{4}$
$x^2 + 3x = 8x$
$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0 \quad \vee \quad x = 5$
vold. niet

d $^3\log(x) = 2 - {^3\log(x - 1)}$
$^3\log(x) = {^3\log(3^2)} - {^3\log(x - 1)}$
$^3\log(x) = {^3\log\left(\frac{9}{x - 1}\right)}$
$x = \frac{9}{x - 1}$
$x^2 - x = 9$
$x^2 - x - 9 = 0$
$D = (-1)^2 - 4 \cdot 1 \cdot -9 = 37$
$x = \frac{1 + \sqrt{37}}{2} \vee x = \frac{1 - \sqrt{37}}{2}$
$x = \frac{1}{2} + \frac{1}{2}\sqrt{37} \vee x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$
vold. niet

Bladzijde 15

17 $y = {^2\log(x)} \xrightarrow{\text{verm. } x\text{-as, } a} y = a \cdot {^2\log(x)} \xrightarrow{\text{verm. } y\text{-as, } b} y = a \cdot {^2\log\left(\frac{1}{b}x\right)}$

Er geldt $a \cdot {^2\log\left(\frac{1}{b}x\right)} = a \cdot ({^2\log(x)} - {^2\log(b)}) = -a \cdot {^2\log(b)} + a \cdot {^2\log(x)}$.

Dus $a = 10$ en $-10 \cdot {^2\log(b)} = -5$
$^2\log(b) = \frac{1}{2}$
$b = 2^{\frac{1}{2}} = \sqrt{2}$

18 **a** $p^{{^p\log(g)}} = g$
$(p^{{^p\log(g)}})^{{^g\log(a)}} = g^{{^g\log(a)}}$
$p^{{^p\log(g)} \cdot {^g\log(a)}} = g^{{^g\log(a)}}$
$p^{{^p\log(g)} \cdot {^g\log(a)}} = a$

b $p^{{^p\log(g)} \cdot {^g\log(a)}} = p^{{^p\log(a)}}$
$^p\log(g) \cdot {^g\log(a)} = {^p\log(a)}$
$^g\log(a) = \frac{^p\log(a)}{^p\log(g)}$

Bladzijde 16

19 **a** Omdat is gegeven dat $^4\log(x) = {^2\log(x - 2)}$ zal gelden dat $a = b$ als $^4\log(x) = a$ en $^2\log(x - 2) = b$.
Substitutie van $^4\log(x) = a$ en $^2\log(x - 2) = b$ in $^4\log(x) = {^2\log(x - 2)}$ geeft $a = b$.
Uit $^g\log(x) = y$ geeft $x = g^y$ volgt $^4\log(x) = a$ geeft $x = 4^a$ en $^2\log(x - 2) = b$ geeft $x - 2 = 2^b$ oftewel $x = 2^b + 2$.

b Uit $x = 4^a$, $x = 2^b + 2$ en $a = b$ volgt $4^a = 2^a + 2$

$(2^2)^a = 2^a + 2$
$(2^a)^2 - 2^a - 2 = 0$
Stel $2^a = u$.
$u^2 - u - 2 = 0$
$(u + 1)(u - 2) = 0$
$u = -1 \vee u = 2$
$2^a = -1 \vee 2^a = 2$
$a = 1$

$a = 1$ geeft $x = 4$

Bladzijde 17

20 **a** ${}^{3}\log(3x - 5) + {}^{\frac{1}{3}}\log(x - 1) = 0$
${}^{3}\log(3x - 5) - {}^{3}\log(x - 1) = 0$
${}^{3}\log(3x - 5) = {}^{3}\log(x - 1)$
$3x - 5 = x - 1$
$2x = 4$
$x = 2$

b ${}^{5}\log(3x) + 2 \cdot {}^{\frac{1}{5}}\log(x) = 0$
${}^{5}\log(3x) + {}^{\frac{1}{5}}\log(x^2) = 0$
${}^{5}\log(3x) - {}^{5}\log(x^2) = 0$
${}^{5}\log(3x) = {}^{5}\log(x^2)$
$3x = x^2$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0 \quad \vee \quad x = 3$
vold. niet

c $2x \cdot {}^{\frac{1}{3}}\log(3x + 5) = {}^{\frac{1}{3}}\log(3x + 5)$
${}^{\frac{1}{3}}\log(3x + 5) = 0 \vee 2x = 1$
$3x + 5 = 1 \vee x = \frac{1}{2}$
$3x = -4 \vee x = \frac{1}{2}$
$x = -1\frac{1}{3} \vee x = \frac{1}{2}$

d ${}^{2}\log(x) = {}^{4}\log(x + 20)$
${}^{2}\log(x) = \dfrac{{}^{2}\log(x + 20)}{{}^{2}\log(4)}$
${}^{2}\log(x) = \dfrac{{}^{2}\log(x + 20)}{2}$
$2 \cdot {}^{2}\log(x) = {}^{2}\log(x + 20)$
${}^{2}\log(x^2) = {}^{2}\log(x + 20)$
$x^2 = x + 20$
$x^2 - x - 20 = 0$
$(x + 4)(x - 5) = 0$
$x = -4 \quad \vee \quad x = 5$
vold. niet

21 **a** $-2 \cdot {}^{\frac{1}{2}}\log(x) = 2 + {}^{2}\log(3 - x)$
$2 \cdot {}^{2}\log(x) = {}^{2}\log(2^2) + {}^{2}\log(3 - x)$
${}^{2}\log(x^2) = {}^{2}\log(4(3 - x))$
$x^2 = 12 - 4x$
$x^2 + 4x - 12 = 0$
$(x - 2)(x + 6) = 0$
$x = 2 \vee x = -6$
vold. niet

b ${}^{9}\log(2x) = {}^{3}\log(x - 4)$
$\dfrac{{}^{3}\log(2x)}{{}^{3}\log(9)} = {}^{3}\log(x - 4)$
$\dfrac{{}^{3}\log(2x)}{2} = {}^{3}\log(x - 4)$
${}^{3}\log(2x) = 2 \cdot {}^{3}\log(x - 4)$
${}^{3}\log(2x) = {}^{3}\log((x - 4)^2)$
$2x = (x - 4)^2$
$2x = x^2 - 8x + 16$
$x^2 - 10x + 16 = 0$
$(x - 2)(x - 8) = 0$
$x = 2 \quad \vee \quad x = 8$
vold. niet

c $4x \cdot {}^{4}\log(2x - 1) + 3 \cdot {}^{4}\log(2x - 1) = 0$
$(4x + 3) \cdot {}^{4}\log(2x - 1) = 0$
$4x + 3 = 0 \vee {}^{4}\log(2x - 1) = 0$
$4x = -3 \vee 2x - 1 = 1$
$x = -\frac{3}{4} \vee 2x = 2$
$x = -\frac{3}{4} \quad \vee \quad x = 1$
vold. niet

d $x^2 \cdot {}^{5}\log(2x + 1) + 9 \cdot {}^{\frac{1}{5}}\log(2x + 1) = 0$
$x^2 \cdot {}^{5}\log(2x + 1) - 9 \cdot {}^{5}\log(2x + 1) = 0$
$(x^2 - 9) \cdot {}^{5}\log(2x + 1) = 0$
$x^2 - 9 = 0 \vee {}^{5}\log(2x + 1) = 0$
$x^2 = 9 \vee 2x + 1 = 1$
$x = 3 \vee x = -3 \vee 2x = 0$
$x = 3 \vee x = -3 \quad \vee \quad x = 0$
vold. niet

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22 ${}^3\log^2(x) = 2 \cdot {}^3\log(x) + 15$
Stel ${}^3\log(x) = u$.
$u^2 = 2u + 15$
$u^2 - 2u - 15 = 0$
$(u + 3)(u - 5) = 0$
$u = -3 \vee u = 5$
${}^3\log(x) = -3 \vee {}^3\log(x) = 5$
$x = 3^{-3} = \frac{1}{27} \vee x = 3^5 = 243$

23 **a** ${}^2\log^2(x) = 2 \cdot {}^2\log(x) + 3$
Stel ${}^2\log(x) = u$.
$u^2 = 2u + 3$
$u^2 - 2u - 3 = 0$
$(u + 1)(u - 3) = 0$
$u = -1 \vee u = 3$
${}^2\log(x) = -1 \vee {}^2\log(x) = 3$
$x = \frac{1}{2} \vee x = 8$

b ${}^{\frac{1}{2}}\log^2(x + 2) + 3 \cdot {}^{\frac{1}{2}}\log(x + 2) = 0$
Stel ${}^{\frac{1}{2}}\log(x + 2) = u$.
$u^2 + 3u = 0$
$u(u + 3) = 0$
$u = 0 \vee u = -3$
${}^{\frac{1}{2}}\log(x + 2) = 0 \vee {}^{\frac{1}{2}}\log(x + 2) = -3$
$x + 2 = 1 \vee x + 2 = 8$
$x = -1 \vee x = 6$

c $2 \cdot {}^3\log^2(x) + 2 = 5 \cdot {}^3\log(x)$
Stel ${}^3\log(x) = u$.
$2u^2 + 2 = 5u$
$2u^2 - 5u + 2 = 0$
$D = (-5)^2 - 4 \cdot 2 \cdot 2 = 9$
$u = \dfrac{5 + 3}{4} = 2 \vee u = \dfrac{5 - 3}{4} = \frac{1}{2}$
${}^3\log(x) = 2 \vee {}^3\log(x) = \frac{1}{2}$
$x = 9 \vee x = \sqrt{3}$

d ${}^5\log^2(x) + 3 \cdot {}^{\frac{1}{5}}\log(x) + 2 = 0$
${}^5\log^2(x) - 3 \cdot {}^5\log(x) + 2 = 0$
Stel ${}^5\log(x) = u$.
$u^2 - 3u + 2 = 0$
$(u - 1)(u - 2) = 0$
$u = 1 \vee u = 2$
${}^5\log(x) = 1 \vee {}^5\log(x) = 2$
$x = 5 \vee x = 25$

9.2 Exponentiële en logaritmische formules

Bladzijde 19

24 **a** Jaarlijkse toename van 5%, dus de vermenigvuldigingsfactor is 1,05.
Het bedrag op 1 januari 2021 is $1000 \cdot 1{,}05 = 1050$ euro.
Het bedrag op 1 januari 2025 is $1000 \cdot 1{,}05^5 \approx 1276{,}28$ euro.
b $g = 1{,}05$
c procentuele toename $\approx \dfrac{1276{,}28 - 1000}{1000} \cdot 100\% \approx 27{,}6\%$

Bladzijde 21

25 **a** $g_{\text{jaar}} = 1{,}127$
b $g_{\text{maand}} = 0{,}932$
c Het groeipercentage per maand is 73,5%.
d De afname per dag is 15,5%.
e Het groeipercentage per jaar is 142%.
f $g_{\text{dag}} = 0{,}993$

26 **a** $g_{\text{kwartier}} = 1{,}12$
$g_{\text{uur}} = 1{,}12^4 \approx 1{,}574$
De toename is 57,4% per uur.
b $g_{\text{kwartier}} = 1{,}12$
$g_{\text{5 minuten}} = 1{,}12^{\frac{1}{3}} \approx 1{,}038$
De procentuele toename per vijf minuten is 3,8%.
c $g_{\text{kwartier}} = 1{,}12$
$g_{\text{5 uur}} = 1{,}12^{20} \approx 9{,}65$
De toename is 865% per vijf uur.

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27 **a** $g_{\text{dag}} = 1{,}3$

$g_{\text{week}} = 1{,}3^7 \approx 6{,}27$

Het groeipercentage per week is 527%.

b $g_{\text{dag}} = 1{,}3$

$g_{4\text{ uur}} = 1{,}3^{\frac{1}{6}} \approx 1{,}045$

Het groeipercentage per vier uur is 4,5%.

28 **a** $g_{\text{dag}} = 0{,}84$

$g_{\text{week}} = 0{,}84^7 \approx 0{,}295$

b $g_{\text{dag}} = 0{,}84$

$g_{\text{uur}} = 0{,}84^{\frac{1}{24}} \approx 0{,}993$

De hoeveelheid neemt met 0,7% per uur af.

c $g_{\text{dag}} = 0{,}84$

$g_{\text{kwartier}} = 0{,}84^{\frac{1}{24 \cdot 4}} \approx 0{,}9982$

Het groeipercentage per kwartier is −0,18%.

Bladzijde 22

29 **a** $g_{\text{uur}} = 0{,}805$

$g_{\text{kwartier}} = 0{,}805^{\frac{1}{4}} \approx 0{,}947$

De afname is 5,3% per kwartier.

b $g_{\text{jaar}} = 1{,}086$

$g_{25\text{ jaar}} = 1{,}086^{25} \approx 7{,}87$

De toename is 687% per 25 jaar.

c $g_{\text{week}} = 2{,}80$

$g_{\text{dag}} = 2{,}80^{\frac{1}{7}} \approx 1{,}158$

30 **a** De hoeveelheid N neemt in 10 − 3 = 7 uur toe van 1600 tot 4100, dus $g_{7\text{ uur}} = \frac{4100}{1600}$.

b $g_{\text{uur}} = \left(\frac{4100}{1600}\right)^{\frac{1}{7}} = 1{,}1438...$

c $\left.\begin{array}{l} N = b \cdot 1{,}1438...^t \\ t = 3 \text{ en } N = 1600 \end{array}\right\} b \cdot 1{,}1438...^3 = 1600$

$b = \frac{1600}{1{,}1438...^3} \approx 1070$

Dus op $t = 0$ is $N \approx 1070$.

De formule van N is $N = 1070 \cdot 1{,}144^t$.

31 **a** Stel $N = b \cdot g^t$.

$g_{6\text{ uur}} = \frac{1250}{150} = 8{,}333...$

$g_{\text{uur}} = 8{,}333...^{\frac{1}{6}} = 1{,}4238...$

$\left.\begin{array}{l} N = b \cdot 1{,}4238...^t \\ t = 2 \text{ en } N = 150 \end{array}\right\} b \cdot 1{,}4238...^2 = 150$

$b = \frac{150}{1{,}4238...^2} \approx 74$

Dus $N = 74 \cdot 1{,}424^t$.

b Stel $H = b \cdot g^t$.

$g_{3\text{ dagen}} = \frac{0{,}47}{0{,}60} = 0{,}7833...$

$g_{\text{dag}} = 0{,}7833...^{\frac{1}{3}} = 0{,}9218...$

$\left.\begin{array}{l} H = b \cdot 0{,}9218...^t \\ t = 5 \text{ en } H = 0{,}60 \end{array}\right\} b \cdot 0{,}9218...^5 = 0{,}60$

$b = \frac{0{,}60}{0{,}9218...^5} = 0{,}901...$

Dus $H = 0{,}90 \cdot 0{,}922^t$.

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32 Stel $A = b \cdot g^t$.

$g_{4\text{ dagen}} = \frac{11}{31} = 0{,}3548...$

$g_{\text{dag}} = 0{,}3548...^{\frac{1}{4}} = 0{,}7718...$

$\left.\begin{array}{l} A = b \cdot 0{,}7718...^t \\ t = 3 \text{ en } A = 31 \end{array}\right\} b \cdot 0{,}7718...^3 = 31$

$b = \frac{31}{0{,}7718...^3} = 67{,}4...$

60 uur $= \frac{60}{24}$ dag $= 2{,}5$ dag

$t = 2{,}5$ geeft $A = 67{,}4... \cdot 0{,}7718...^{2{,}5} \approx 35$

Na 60 uur was de oppervlakte van de wond ongeveer 35 mm^2.

33 **a** Voer in $y_1 = 17{,}0 \cdot 1{,}031^x$ en $y_2 = 34{,}0$.
De optie snijpunt geeft $x = 22{,}70...$
Na 22,7 jaar is het aantal verdubbeld.

b Voer in $y_1 = 12{,}9 \cdot 1{,}031^x$ en $y_2 = 25{,}8$.
De optie snijpunt geeft $x = 22{,}70...$
Na 22,7 jaar is het aantal verdubbeld.

Bladzijde 23

34 **a** $g_{\text{jaar}} = 1{,}131$
$1{,}131^T = 2$
$T = {}^{1{,}131}\log(2) = 5{,}630...$
De verdubbelingstijd is 5 jaar en $0{,}630... \cdot 12 \approx 8$ maanden.

b $g_{\text{week}} = 0{,}915$
$0{,}915^T = \frac{1}{2}$
$T = {}^{0{,}915}\log(\frac{1}{2}) = 7{,}802...$
De halveringstijd is 7 weken en $0{,}802... \cdot 7 \approx 6$ dagen.

Bladzijde 24

35 **a** $g_{\text{jaar}} = 1{,}0325$
$1{,}0325^T = 2$
$T = {}^{1{,}0325}\log(2) = 21{,}6...$
De verdubbelingstijd is 22 jaar.

b $g_{10\text{ jaar}} = 1{,}081$
$1{,}081^T = 2$
$T = {}^{1{,}081}\log(2) = 8{,}89...$
De verdubbelingstijd is $8{,}89... \cdot 10$ jaar ≈ 89 jaar.

36 **a** $g_{25\text{ jaar}} = 2$
$g_{\text{jaar}} = 2^{\frac{1}{25}} \approx 1{,}028$
Het groeipercentage per jaar is 2,8%.

b $g_{28\text{ jaar}} = \frac{1}{2}$
$g_{\text{jaar}} = (\frac{1}{2})^{\frac{1}{28}} \approx 0{,}976$
De hoeveelheid radioactieve stof neemt per jaar met 2,4% af.

37 **a** $g_{\text{dag}} = 0{,}917$
$0{,}917^T = \frac{1}{2}$
$T = {}^{0{,}917}\log(\frac{1}{2}) = 7{,}9...$
De halveringstijd is 8 dagen.

b $g_{\text{dag}} = 0{,}917$
$0{,}917^T = 0{,}1$
$T = {}^{0{,}917}\log(0{,}1) = 26{,}5...$
Na 27 dagen is nog 10% van de beginhoeveelheid over.

38 **a** $g_{dag} = 0{,}81$
$g_{week} = 0{,}81^7 = 0{,}2287...$
De afname per week is 77,1%.
b $g_{week} = 0{,}38$
$g_{dag} = 0{,}38^{\frac{1}{7}} = 0{,}8709...$
De afname per dag is 12,9%.
c De groeifactor per dag is 0,845.
Dus $BZV = 300 \cdot 0{,}845^t$.
d $0{,}845^T = \frac{1}{2}$
$T = {}^{0{,}845}\log(\frac{1}{2}) = 4{,}115...$
De halveringstijd is 4 dagen en $0{,}115... \cdot 24 \approx 3$ uur.
e $300 \cdot 0{,}845^t = 10$
$0{,}845^t = \frac{10}{300} = \frac{1}{30}$
$t = {}^{0{,}845}\log(\frac{1}{30}) = 20{,}1...$
Dus na 20 dagen zuiveren is het BZV afgenomen tot 10 mg/liter.
f $T = 10$ geeft $h = 7{,}6 \cdot 0{,}96^{10} = 5{,}0527...$
Bij 10 °C is de groeifactor per dag 0,8709... (zie vraag b).
$0{,}8709...^{5{,}0527...} \approx 0{,}5$, dus de formule klopt voor $T = 10$.
$T = 20$ geeft $h = 7{,}6 \cdot 0{,}96^{20} = 3{,}3592...$
Bij 20 °C is de groeifactor per dag 0,81 (zie vraag a).
$0{,}81^{3{,}3592...} \approx 0{,}5$, dus de formule klopt voor $T = 20$.

Bladzijde 25

39 $y = 2^{x-1}$
$2^{x-1} = y$
$x - 1 = {}^2\log(y)$
$x = 1 + {}^2\log(y)$

Bladzijde 26

40 **a** $t = 1 + {}^2\log(0{,}2N)$
$t = {}^2\log(2) + {}^2\log(0{,}2N)$
$t = {}^2\log(0{,}4N)$
$t = \frac{\log(0{,}4N)}{\log(2)}$
$t \approx 3{,}32 \cdot \log(0{,}4N)$
Dus $t = 3{,}32 \cdot \log(0{,}4N)$.
b $t = 1 + {}^2\log(0{,}2N)$
$t = 1 + {}^2\log(0{,}2) + {}^2\log(N)$
$t = 1 + {}^2\log(0{,}2) + \frac{{}^3\log(N)}{{}^3\log(2)}$
$t \approx -1{,}32 + 1{,}58 \cdot {}^3\log(N)$
Dus $t = -1{,}32 + 1{,}58 \cdot {}^3\log(N)$.
c $x = 64 \cdot 8^y$
$x = 64 \cdot (3^{{}^3\log(8)})^y$
$x \approx 64 \cdot (3^{1{,}893})^y$
$x \approx 64 \cdot 3^{1{,}893y}$
Dus $x = 64 \cdot 3^{1{,}893y}$.

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41 **a** $N = 34 \cdot 3^{2t+3}$
$34 \cdot 3^{2t+3} = N$
$3^{2t+3} = \frac{1}{34}N$
$2t + 3 = {}^3\log(\frac{1}{34}N)$
$2t = -3 + {}^3\log(\frac{1}{34}) + {}^3\log(N)$
$t = -1\frac{1}{2} + \frac{1}{2} \cdot {}^3\log(\frac{1}{34}) + \frac{1}{2} \cdot {}^3\log(N)$
$t \approx -3{,}10 + 0{,}5 \cdot {}^3\log(N)$
Dus $t = -3{,}10 + 0{,}5 \cdot {}^3\log(N)$.

b $y = 13 \cdot 2^{3x-1}$
$13 \cdot 2^{3x-1} = y$
$2^{3x-1} = \frac{1}{13}y$
$3x - 1 = {}^2\log(\frac{1}{13}y)$
$3x = 1 + {}^2\log(\frac{1}{13}y)$
$3x = {}^2\log(2) + {}^2\log(\frac{1}{13}y)$
$3x = {}^2\log(\frac{2}{13}y)$
$x = \frac{1}{3} \cdot {}^2\log(\frac{2}{13}y)$
$x \approx 0{,}33 \cdot {}^2\log(0{,}15y)$
Dus $x = 0{,}33 \cdot {}^2\log(0{,}15y)$.

c $y = 3 \cdot \log(x) - 5$
$3 \cdot \log(x) = y + 5$
$\log(x) = \frac{1}{3}y + \frac{5}{3}$
$x = 10^{\frac{1}{3}y + \frac{5}{3}} = 10^{\frac{1}{3}y} \cdot 10^{\frac{5}{3}} \approx 46{,}42 \cdot 10^{0{,}33y}$
Dus $x = 46{,}42 \cdot 10^{0{,}33y}$.

d $K = 1 - \frac{1}{2} \cdot {}^3\log(2v)$
$\frac{1}{2} \cdot {}^3\log(2v) = 1 - K$
${}^3\log(2v) = 2 - 2K$
$2v = 3^{2-2K}$
$v = \frac{1}{2} \cdot 3^2 \cdot 3^{-2K}$
$v = \frac{1}{2} \cdot 9 \cdot (3^{-2})^K$
$v = 4\frac{1}{2} \cdot (\frac{1}{9})^K$
Dus $v = 4\frac{1}{2} \cdot (\frac{1}{9})^K$.

42 **a** ${}^2\log(4t - 1) = 5N + 3$
$4t - 1 = 2^{5N+3}$
$4t = 1 + (2^5)^N \cdot 2^3$
$4t = 1 + 8 \cdot 32^N$
$4t = 1 + 8 \cdot (10^{\log(32)})^N$
$4t \approx 1 + 8 \cdot (10^{1{,}51})^N$
$t \approx \frac{1}{4} + 2 \cdot 10^{1{,}51N}$
Dus $t = \frac{1}{4} + 2 \cdot 10^{1{,}51N}$.

b $M = 14 \cdot 1{,}3^{4N-3}$
$14 \cdot 1{,}3^{4N-3} = M$
$1{,}3^{4N-3} = \frac{1}{14}M$
$4N - 3 = {}^{1,3}\log(\frac{1}{14}M)$
$4N = 3 + {}^{1,3}\log(\frac{1}{14}) + {}^{1,3}\log(M)$
$4N = 3 + {}^{1,3}\log(\frac{1}{14}) + \dfrac{\log(M)}{\log(1{,}3)}$
$N = \dfrac{3 + {}^{1,3}\log(\frac{1}{14})}{4} + \dfrac{\log(M)}{4 \cdot \log(1{,}3)}$
$N \approx -1{,}76 + 2{,}19 \cdot \log(M)$
Dus $N = -1{,}76 + 2{,}19 \cdot \log(M)$.

c $w = 125 \cdot \log(3T + 15) - 8$
$125 \cdot \log(3T + 15) = w + 8$
$\log(3T + 15) = 0{,}008w + 0{,}064$
$3T + 15 = 10^{0{,}008w + 0{,}064}$
$3T = -15 + (10^{0{,}008})^w \cdot 10^{0{,}064}$
$T \approx -5 + 0{,}386 \cdot 1{,}019^w$
Dus $T = -5 + 0{,}386 \cdot 1{,}019^w$.

d $p = 10 \cdot 5^{0{,}1q-3}$
$10 \cdot 5^{0{,}1q-3} = p$
$5^{0{,}1q-3} = 0{,}1p$
$0{,}1q - 3 = {}^5\log(0{,}1p)$
$0{,}1q = 3 + {}^5\log(0{,}1p)$
$0{,}1q = {}^5\log(125) + {}^5\log(0{,}1p)$
$0{,}1q = {}^5\log(12{,}5p)$
$0{,}1q = \dfrac{\log(12{,}5p)}{\log(5)}$
$q = \dfrac{10 \cdot \log(12{,}5p)}{\log(5)}$
$q \approx 14{,}31 \cdot \log(12{,}5p)$
Dus $q = 14{,}31 \cdot \log(12{,}5p)$.

43 **a** $f(x) = g_a(x)$ geeft ${}^2\log(x) = {}^2\log(x\sqrt{x}) + a$
${}^2\log(x) = {}^2\log(x^{1\frac{1}{2}}) + a$
${}^2\log(x) = 1\frac{1}{2} \cdot {}^2\log(x) + a$
$\frac{1}{2} \cdot {}^2\log(x) = -a$
${}^2\log(x) = -2a$
$x = 2^{-2a} = (2^{-2})^a = (\frac{1}{4})^a$

$\left.\begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -2a \end{array}\right\} y = -2a$

Dus $S_a((\frac{1}{4})^a, -2a)$.

b Uit $A_aB_a = 4$ en $y_{A_a} = 0$ volgt $y_{B_a} = 4 \vee y_{B_a} = -4$.

$g_a(x) = 0$ geeft ${}^2\log(x\sqrt{x}) + a = 0$

$${}^2\log(x^{1\frac{1}{2}}) = -a$$

$$1\tfrac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -\tfrac{2}{3}a$$

$$\left.\begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -\tfrac{2}{3}a \end{array}\right\} y_{B_a} = -\tfrac{2}{3}a$$

Dus $-\frac{2}{3}a = 4 \vee -\frac{2}{3}a = -4$ en dit geeft $a = -6 \vee a = 6$.

c De grafiek van g_a^{inv} snijdt de y-as in (0, 64), dus de grafiek van g_a snijdt de x-as in (64, 0).

$g_a(64) = 0$ geeft ${}^2\log(64\sqrt{64}) + a = 0$

$$a = -{}^2\log(64 \cdot 8)$$

$$a = -{}^2\log(512)$$

$$a = -{}^2\log(2^9)$$

$$a = -9$$

Dus $g_{-9}(x) = {}^2\log(x\sqrt{x}) - 9$.

Voor g_{-9} geldt $y = {}^2\log(x\sqrt{x}) - 9$, dus voor g_{-9}^{inv} geldt $x = {}^2\log(y\sqrt{y}) - 9$.

$x = {}^2\log(y\sqrt{y}) - 9$ geeft ${}^2\log(y\sqrt{y}) = x + 9$

$${}^2\log(y^{1\frac{1}{2}}) = x + 9$$

$$1\tfrac{1}{2} \cdot {}^2\log(y) = x + 9$$

$${}^2\log(y) = \tfrac{2}{3}x + 6$$

$$y = 2^{\frac{2}{3}x + 6} = 2^{\frac{2}{3}x} \cdot 2^6 = 64 \cdot 2^{\frac{2}{3}x}$$

Dus $g_{-9}^{\text{inv}}(x) = 64 \cdot 2^{\frac{2}{3}x}$.

44 ${}^2\log(N) = -1{,}9 + 0{,}3 \cdot {}^3\log(t)$

$0{,}3 \cdot {}^3\log(t) = 1{,}9 + {}^2\log(N)$

$${}^3\log(t) = \frac{1{,}9}{0{,}3} + \frac{1}{0{,}3} \cdot {}^2\log(N)$$

$$t = 3^{\frac{1{,}9}{0{,}3} + \frac{1}{0{,}3} \cdot {}^2\log(N)}$$

$$t = 3^{\frac{1{,}9}{0{,}3}} \cdot 3^{\frac{1}{0{,}3} \cdot \frac{{}^3\log(N)}{{}^3\log(2)}}$$

$$t = 3^{\frac{1{,}9}{0{,}3}} \cdot \left(3^{{}^3\log(N)}\right)^{\frac{1}{0{,}3 \cdot {}^3\log(2)}}$$

$$t = 3^{\frac{1{,}9}{0{,}3}} \cdot N^{\frac{1}{0{,}3 \cdot {}^3\log(2)}}$$

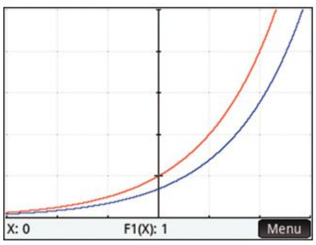
$t \approx 1051N^{5{,}28}$

Dus $t = 1051N^{5{,}28}$.

9.3 Het grondtal e

Bladzijde 28

45 **a**



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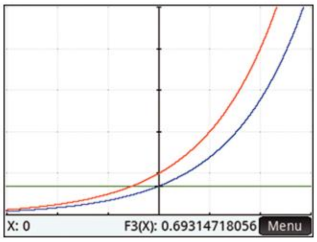
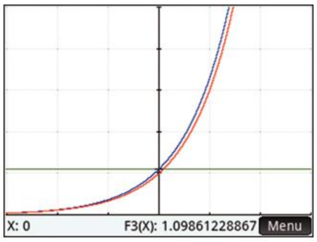
b



$c \approx 0{,}6931$

c



Dus ook hier is $\frac{y_2}{y_1}$ constant en wel ongeveer 1,0986.

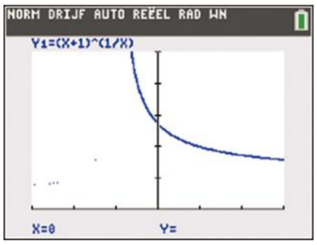
46 **a** $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \frac{2^{x+h} - 2^x}{h} = \lim\limits_{h \to 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim\limits_{h \to 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$

b $f'(0) = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^0 = \lim\limits_{h \to 0} \frac{2^h - 1}{h}$

c $f'(x) = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x = f'(0) \cdot 2^x$

Bladzijde 29

47 **a**



b Omdat voor $x = 0$ de exponent $\frac{1}{x}$ niet bestaat.

c

x	y_1
0,01	2,7048
0,001	2,7169
0,0001	2,7181
0,00001	2,7183

d $g \approx 2{,}718$

Bladzijde 30

48 **a** Dit mag omdat $e^x \neq 0$ voor elke x, dus $e^x = 0$ geeft geen oplossing.

b $e^x = 1$ geeft $x = 0$ en $e^x = -3$ heeft geen oplossing.

Bladzijde 31

49 a $2e^2 - e^2 = e^2$
b $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
c $5e^2 \cdot 3e^3 = 15e^5$
d $\frac{12e^6}{4e^2} = 3e^4$
e $e^{5x} \cdot e^x = e^{6x}$
f $e^x \cdot e^2 = e^{x+2}$
g $5e^x - 3e^x = 2e^x$
h $e^x(e^2 + 1) = e^{x+2} + e^x$
i $e^x(e^x + 1) = e^{2x} + e^x$
j $(e^x + 1)^2 = (e^x)^2 + 2 \cdot e^x \cdot 1 + 1 = e^{2x} + 2e^x + 1$
k $(e^{3x} + 3)^2 = (e^{3x})^2 + 2 \cdot e^{3x} \cdot 3 + 9 = e^{6x} + 6e^{3x} + 9$
l $\frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$

50 a $(2 + 3e^{\frac{1}{2}x})^2 = 4 + 2 \cdot 2 \cdot 3e^{\frac{1}{2}x} + (3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$
b $(e^x + e^{-x})^2 = (e^x)^2 + 2 \cdot e^x \cdot e^{-x} + (e^{-x})^2 = e^{2x} + 2 + e^{-2x}$
c $\frac{e^{2x} - 4}{e^x + 2} = \frac{(e^x + 2)(e^x - 2)}{e^x + 2} = e^x - 2$

51 a $(2x + 4)e^x = 0$
$2x + 4 = 0$
$2x = -4$
$x = -2$
b $x^2 e^x = 3x e^x$
$x^2 = 3x$
$x = 0 \vee x = 3$
c $x^2 e^x = e^x$
$x^2 = 1$
$x = 1 \vee x = -1$
d $e^{3x} - e^x = 0$
$e^{3x} = e^x$
$3x = x$
$x = 0$
e $e^{4x} - 1 = 0$
$e^{4x} = 1$
$4x = 0$
$x = 0$
f $e^x \cdot e^x = e^6$
$e^{2x} = e^6$
$2x = 6$
$x = 3$

52 a $e^x + e^x = 2e^6$
$2e^x = 2e^6$
$e^x = e^6$
$x = 6$
b $\frac{e^{5x}}{e^x} = e$
$e^{4x} = e^1$
$4x = 1$
$x = \frac{1}{4}$
c $2x e^x + e^x = 0$
$e^x(2x + 1) = 0$
$2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$
d $e^{x+2} - \sqrt{e} = 0$
$e^{x+2} = \sqrt{e}$
$e^{x+2} = e^{\frac{1}{2}}$
$x + 2 = \frac{1}{2}$
$x = -1\frac{1}{2}$
e $e^{2x} + e^x = 2$
$(e^x)^2 + e^x = 2$
Stel $e^x = u$.
$u^2 + u = 2$
$u^2 + u - 2 = 0$
$(u - 1)(u + 2) = 0$
$u = 1 \vee u = -2$
$e^x = 1 \vee e^x = -2$
$x = 0$
f $e^{6x} + 1 = 2e^{3x}$
$(e^{3x})^2 + 1 = 2e^{3x}$
Stel $e^{3x} = u$.
$u^2 + 1 = 2u$
$u^2 - 2u + 1 = 0$
$(u - 1)^2 = 0$
$u = 1$
$e^{3x} = 1$
$3x = 0$
$x = 0$

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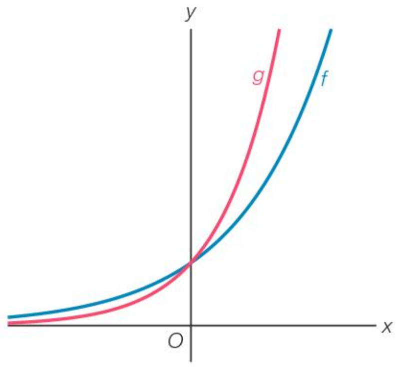
53 $e^{2x} + e^3 = (e + 1)\,e^{x+1}$
$e^{2x} + e^3 = (e + 1)\,e^x \cdot e$
$e^{2x} + e^3 = (e^2 + e)\,e^x$
$(e^x)^2 - (e^2 + e)\,e^x + e^3 = 0$
Stel $e^x = u$.
$u^2 - (e^2 + e)u + e^3 = 0$
$(u - e^2)(u - e) = 0$
$u = e^2 \vee u = e$
$e^x = e^2 \vee e^x = e$
$x = 2 \vee x = 1$

54 **a**



b $2 < e < 3$, dus voor $x > 0$ is $2^x < e^x < 3^x$ en voor $x < 0$ is $3^x < e^x < 2^x$.
Voor $x = 0$ is $2^x = e^x = 3^x = 1$.
Dus de grafiek van $h(x) = e^x$ ligt voor $x \neq 0$ tussen de grafieken van $f(x) = 2^x$ en $g(x) = 3^x$.

Bladzijde 32

55 De productregel is gebruikt in de onderdelen b en c.
De quotiëntregel is gebruikt in het onderdeel d.
De kettingregel is gebruikt in de onderdelen c en d.

56 **a** $f(x) = e^x + 2$ geeft $f'(x) = e^x$

b $f(x) = 2\,e^x + \dfrac{1}{x} = 2\,e^x + x^{-1}$ geeft $f'(x) = 2\,e^x - x^{-2} = 2\,e^x - \dfrac{1}{x^2}$

c $f(x) = x\,e^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (x + 1)\,e^x$

d $f(x) = \dfrac{x}{e^x}$ geeft $f'(x) = \dfrac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \dfrac{(1 - x)\,e^x}{(e^x)^2} = \dfrac{1 - x}{e^x}$

e $f(x) = \dfrac{2\,e^x}{x - 1}$ geeft $f'(x) = \dfrac{(x - 1) \cdot 2\,e^x - 2\,e^x \cdot 1}{(x - 1)^2} = \dfrac{2x\,e^x - 2\,e^x - 2\,e^x}{(x - 1)^2} = \dfrac{(2x - 4)\,e^x}{(x - 1)^2}$

f $f(x) = (2x - 4)\,e^x$ geeft $f'(x) = 2 \cdot e^x + (2x - 4) \cdot e^x = (2x - 2)\,e^x$

Bladzijde 33

57 **a** $f(x) = e^{x^2 + x}$ geeft $f'(x) = e^{x^2 + x} \cdot (2x + 1) = (2x + 1)\,e^{x^2 + x}$

b $f(x) = x^2 + 2\,e^{3x}$ geeft $f'(x) = 2x + 2 \cdot e^{3x} \cdot 3 = 2x + 6\,e^{3x}$

c $f(x) = x\,e^{x^2}$ geeft $f'(x) = 1 \cdot e^{x^2} + x \cdot e^{x^2} \cdot 2x = (2x^2 + 1)\,e^{x^2}$

d $f(x) = \dfrac{2\,e^{-x-1}}{x^2}$ geeft

$$f'(x) = \frac{x^2 \cdot 2\,e^{-x-1} \cdot -1 - 2\,e^{-x-1} \cdot 2x}{x^4} = \frac{-2x^2\,e^{-x-1} - 4x\,e^{-x-1}}{x^4} = \frac{-2x(x + 2)\,e^{-x-1}}{x^4} = \frac{-2(x + 2)\,e^{-x-1}}{x^3}$$

e $f(x) = 3x\,e^{2x-1}$ geeft $f'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{2x-1} \cdot 2 = (6x + 3)\,e^{2x-1}$

f $f(x) = \dfrac{e^{2x}}{e^{2x} + 1}$ geeft $f'(x) = \dfrac{(e^{2x} + 1) \cdot e^{2x} \cdot 2 - e^{2x} \cdot e^{2x} \cdot 2}{(e^{2x} + 1)^2} = \dfrac{2\,e^{4x} + 2\,e^{2x} - 2\,e^{4x}}{(e^{2x} + 1)^2} = \dfrac{2\,e^{2x}}{(e^{2x} + 1)^2}$

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58 **a** $-\frac{1}{e^2} \approx -0{,}135$

b $\frac{3e}{(e+2)^2} \approx 0{,}366$

c $1\frac{1}{3}e^3 \approx 26{,}781$

d $\frac{e^2}{2e-3} \approx 3{,}033$

59 **a** $f(x) = x\,e^x + 2$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)\,e^x$

b $f'(x) = 0$ geeft $(1+x)\,e^x = 0$

$1 + x = 0$

$x = -1$

min. is $f(-1) = -e^{-1} + 2 = 2 - \frac{1}{e}$

c $f'(x) = (1+x)\,e^x$ geeft $f''(x) = 1 \cdot e^x + (1+x) \cdot e^x = (2+x)\,e^x$

d $f''(x) = 0$ geeft $(2+x)\,e^x = 0$

$2 + x = 0$

$x = -2$

$f(-2) = -2e^{-2} + 2 = 2 - \frac{2}{e^2}$

Dus de coördinaten van het buigpunt zijn $\left(-2, 2 - \frac{2}{e^2}\right)$.

e Stel k: $y = ax + b$ met $a = f'(-2) = (1-2)e^{-2} = -\frac{1}{e^2}$.

$$\left.\begin{array}{l} y = -\frac{1}{e^2}x + b \\ \text{door } \left(-2, 2 - \frac{2}{e^2}\right) \end{array}\right\} -\frac{1}{e^2} \cdot -2 + b = 2 - \frac{2}{e^2}$$

$\frac{2}{e^2} + b = 2 - \frac{2}{e^2}$

$b = 2 - \frac{4}{e^2}$

Dus k: $y = -\frac{1}{e^2}x + 2 - \frac{4}{e^2}$.

60 **a** $f(x) = -x\,e^x$ geeft $f'(x) = -1 \cdot e^x + -x \cdot e^x = (-x-1)\,e^x$

$f'(x) = 0$ geeft $(-x-1)\,e^x = 0$

$-x - 1 = 0$

$-x = 1$

$x = -1$

max. is $f(-1) = e^{-1} = \frac{1}{e}$

b $f'(x) = (-x-1)\,e^x$ geeft $f''(x) = -1 \cdot e^x + (-x-1) \cdot e^x = (-x-2)\,e^x$

$f''(x) = 0$ geeft $(-x-2)\,e^x = 0$

$-x - 2 = 0$

$-x = 2$

$x = -2$

Stel k: $y = ax + b$ met $a = f'(-2) = (2-1)e^{-2} = \frac{1}{e^2}$.

$$\left.\begin{array}{l} y = \frac{1}{e^2}x + b \\ f(-2) = 2e^{-2} = \frac{2}{e^2} \end{array}\right\} \frac{1}{e^2} \cdot -2 + b = \frac{2}{e^2}$$

$\frac{-2}{e^2} + b = \frac{2}{e^2}$

$b = \frac{4}{e^2}$

Dus k: $y = \frac{1}{e^2}x + \frac{4}{e^2}$.

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61 **a** $f(x) = 0$ geeft $(x^2 - 3)\,e^x = 0$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \sqrt{3} \vee x = -\sqrt{3}$

De nulpunten zijn $\sqrt{3}$ en $-\sqrt{3}$.

b $f(x) = (x^2 - 3)\,e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3)\,e^x$

$f'(x) = 0$ geeft $(x^2 + 2x - 3)\,e^x = 0$

$x^2 + 2x - 3 = 0$

$(x - 1)(x + 3) = 0$

$x = 1 \vee x = -3$

max. is $f(-3) = (9 - 3)e^{-3} = \dfrac{6}{e^3}$ en min. is $f(1) = (1 - 3)e^1 = -2e$.

c $f'(x) = (x^2 + 2x - 3)\,e^x$ geeft $f''(x) = (2x + 2) \cdot e^x + (x^2 + 2x - 3) \cdot e^x = (x^2 + 4x - 1)\,e^x$

$f''(x) = 0$ geeft $(x^2 + 4x - 1)\,e^x = 0$

$x^2 + 4x - 1 = 0$

$D = 4^2 - 4 \cdot 1 \cdot -1 = 20$, dus $\sqrt{D} = \sqrt{20} = 2\sqrt{5}$

$x = \dfrac{-4 + 2\sqrt{5}}{2} \vee x = \dfrac{-4 - 2\sqrt{5}}{2}$

$x = -2 + \sqrt{5} \vee x = -2 - \sqrt{5}$

De x-coördinaten van de buigpunten zijn $-2 + \sqrt{5}$ en $-2 - \sqrt{5}$.

Bladzijde 34

62 **a** $f(x) = \frac{1}{2}e^{2x}$ geeft $f'(x) = \frac{1}{2} \cdot e^{2x} \cdot 2 = e^{2x}$

Stel k: $y = ax + b$ met $a = f'(-1) = e^{-2} = \dfrac{1}{e^2}$.

$$\left.\begin{array}{l} y = \dfrac{1}{e^2}x + b \\ f(-1) = \dfrac{1}{2e^2}, \text{dus } A\left(-1, \dfrac{1}{2e^2}\right) \end{array}\right\} \begin{array}{l} \dfrac{1}{e^2} \cdot -1 + b = \dfrac{1}{2e^2} \\ -\dfrac{2}{2e^2} + b = \dfrac{1}{2e^2} \\ b = \dfrac{3}{2e^2} \end{array}$$

Dus k: $y = \dfrac{1}{e^2}x + \dfrac{3}{2e^2}$.

$g(x) = \dfrac{1}{e^{x+3}} = e^{-x-3}$ geeft $g'(x) = e^{-x-3} \cdot -1 = -e^{-x-3}$

Stel l: $y = ax + b$ met $a = g'(-1) = -e^{1-3} = -e^{-2} = -\dfrac{1}{e^2}$.

$$\left.\begin{array}{l} y = -\dfrac{1}{e^2}x + b \\ g(-1) = e^{1-3} = \dfrac{1}{e^2}, \text{dus } B\left(-1, \dfrac{1}{e^2}\right) \end{array}\right\} \begin{array}{l} -\dfrac{1}{e^2} \cdot -1 + b = \dfrac{1}{e^2} \\ \dfrac{1}{e^2} + b = \dfrac{1}{e^2} \\ b = 0 \end{array}$$

Dus l: $y = -\dfrac{1}{e^2}x$.

k en l snijden geeft $\dfrac{1}{e^2}x + \dfrac{3}{2e^2} = -\dfrac{1}{e^2}x$

$\dfrac{2}{e^2}x = -\dfrac{3}{2e^2}$

$x = -\dfrac{3}{2e^2} \cdot \dfrac{e^2}{2}$

$x = -\frac{3}{4}$

b $h(x) = f(x) + g(x) = \frac{1}{2}e^{2x} + \frac{1}{e^{x+3}}$ geeft $h'(x) = e^{2x} - e^{-x-3}$

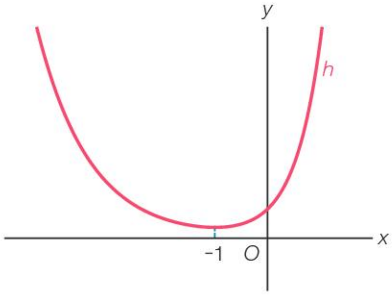
$h'(x) = 0$ geeft $e^{2x} - e^{-x-3} = 0$

$e^{2x} = e^{-x-3}$

$2x = -x - 3$

$3x = -3$

$x = -1$



min. is $h(-1) = \frac{1}{2}e^{-2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$

Het bereik is dus $B_h = \left[\frac{3}{2e^2}, \rightarrow\right\rangle$.

63 **a** $h(4) = 5$ geeft $2(e^{\frac{1}{4}\cdot 4} + e^{-\frac{1}{4}\cdot 4}) + c = 5$

$2e^1 + 2e^{-1} + c = 5$

$c = 5 - 2e - \frac{2}{e}$

b $h(x) = 5(e^{0,1x} + e^{-0,1x}) - 5,81$ geeft $h'(x) = 5(e^{0,1x} \cdot 0,1 + e^{-0,1x} \cdot -0,1) = 0,5(e^{0,1x} - e^{-0,1x})$

De helling in B is $h'(4) = 0,5(e^{0,1 \cdot 4} - e^{-0,1 \cdot 4}) = 0,5(e^{0,4} - e^{-0,4}) \approx 0,41$.

c $h_k(0) = 0$ geeft $\frac{1}{2k}(e^0 + e^0 - e^{4k} - e^{-4k}) + 5 = 0$ oftewel $\frac{1}{2k}(2 - e^{4k} - e^{-4k}) + 5 = 0$.

Voer in $y_1 = \frac{1}{2x}(2 - e^{4x} - e^{-4x}) + 5$.

De optie nulpunt geeft $x = 0,469...$

Dus $k \approx 0,47$.

Bladzijde 35

64 **a** $f(x) = 2x\,e^{-x^2}$ geeft $f'(x) = 2 \cdot e^{-x^2} + 2x \cdot e^{-x^2} \cdot -2x = (2 - 4x^2)\,e^{-x^2}$

$f'(x) = 0$ geeft $(2 - 4x^2)\,e^{-x^2} = 0$

$2 - 4x^2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \vee x = -\frac{1}{2}\sqrt{2}$

$y_A = 2 \cdot -\frac{1}{2}\sqrt{2} \cdot e^{-\frac{1}{2}} = -\frac{\sqrt{2}}{e^{\frac{1}{2}}} = -\frac{\sqrt{2}}{\sqrt{e}} = -\sqrt{\frac{2}{e}}$

$y_B = 2 \cdot \frac{1}{2}\sqrt{2} \cdot e^{\frac{1}{2}} = \sqrt{\frac{2}{e}}$

$y_B - y_A = \sqrt{\frac{2}{e}} - -\sqrt{\frac{2}{e}} = 2\sqrt{\frac{2}{e}}$

Dus $a = \frac{2}{e}$.

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b $f'(x) = (2 - 4x^2)\,e^{-x^2}$ geeft
$f''(x) = -8x \cdot e^{-x^2} + (2 - 4x^2) \cdot e^{-x^2} \cdot -2x = -8x \cdot e^{-x^2} + (8x^3 - 4x) \cdot e^{-x^2} = (8x^3 - 12x)\,e^{-x^2}$
$f''(x) = 0$ geeft $(8x^3 - 12x)\,e^{-x^2} = 0$
$8x^3 - 12x = 0$
$4x(2x^2 - 3) = 0$
$4x = 0 \vee 2x^2 = 3$
$x = 0 \vee x^2 = 1\frac{1}{2}$
$x = 0 \vee x = \sqrt{1\frac{1}{2}} = \frac{1}{2}\sqrt{6} \vee x = -\sqrt{1\frac{1}{2}} = -\frac{1}{2}\sqrt{6}$

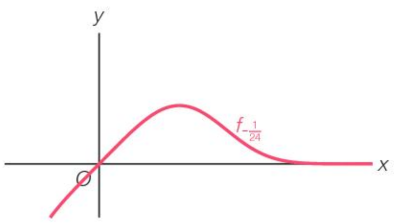
$f(0) = 0$

$f(\frac{1}{2}\sqrt{6}) = \sqrt{6} \cdot e^{-1\frac{1}{2}} = \dfrac{\sqrt{6}}{e\sqrt{e}}$

$f(-\frac{1}{2}\sqrt{6}) = -\sqrt{6} \cdot e^{-1\frac{1}{2}} = -\dfrac{\sqrt{6}}{e\sqrt{e}}$

De coördinaten van de buigpunten van de grafiek van f zijn (0, 0),
$\left(\frac{1}{2}\sqrt{6}, \dfrac{\sqrt{6}}{e\sqrt{e}}\right)$ en $\left(-\frac{1}{2}\sqrt{6}, -\dfrac{\sqrt{6}}{e\sqrt{e}}\right)$.

65 $f_a(x) = x\,e^{ax^3}$ geeft $f_a'(x) = 1 \cdot e^{ax^3} + x \cdot e^{ax^3} \cdot 3ax^2 = (1 + 3ax^3)\,e^{ax^3}$
$f_a'(2) = 0$ geeft $(1 + 3a \cdot 2^3)\,e^{a \cdot 2^3} = 0$
$(1 + 3a \cdot 8)\,e^{8a} = 0$
$(1 + 24a)\,e^{8a} = 0$
$1 + 24a = 0$
$24a = -1$
$a = -\frac{1}{24}$
$a = -\frac{1}{24}$ geeft $f_{-\frac{1}{24}}(x) = x\,e^{-\frac{1}{24}x^3}$



De extreme waarde voor $x = 2$ is een maximum.

9.4 De natuurlijke logaritme

Bladzijde 37

66 **a** $f(x) = 2^x$ geeft $f^{\text{inv}}(x) = {}^{2}\log(x)$
b $g(x) = e^x$ geeft $g^{\text{inv}}(x) = {}^{e}\log(x)$

Bladzijde 38

67 **a** $\ln(e) = 1$
b $\ln(e\sqrt{e}) = \ln(e \cdot e^{\frac{1}{2}}) = \ln(e^{1\frac{1}{2}}) = 1\frac{1}{2}$
c $\ln\left(\dfrac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = 0$
e $3\ln(e \cdot \sqrt[3]{e}) = 3\ln(e \cdot e^{\frac{1}{3}}) = 3\ln(e^{1\frac{1}{3}}) = 3 \cdot 1\frac{1}{3} = 4$
f $\ln^2(e^3) = 3^2 = 9$
g $\ln^3(e^2) = 2^3 = 8$
h $e^{\ln(7)} + e^{2\ln(7)} = 7 + e^{\ln(7^2)} = 7 + 7^2 = 56$
i $e^{\frac{1}{2}\ln(5)} = e^{\ln(5^{\frac{1}{2}})} = 5^{\frac{1}{2}} = \sqrt{5}$
j $e^{\ln(10)} \cdot e^{\ln(3)} = 10 \cdot 3 = 30$

68 **a** $e^{3x} = 12$
$3x = \ln(12)$
$x = \frac{1}{3}\ln(12) \approx 0{,}828$
b $5\,e^{2x} = 60$
$e^{2x} = 12$
$2x = \ln(12)$
$x = \frac{1}{2}\ln(12) \approx 1{,}242$
c $6 + e^{0{,}5x+2} = 10$
$e^{0{,}5x+2} = 4$
$0{,}5x + 2 = \ln(4)$
$x + 4 = 2\ln(4)$
$x = 2\ln(4) - 4 \approx -1{,}227$
d $\frac{3}{5\,e^{2x-1}} = 10$
$50\,e^{2x-1} = 3$
$e^{2x-1} = \frac{3}{50}$
$2x - 1 = \ln(\frac{3}{50})$
$2x = 1 + \ln(\frac{3}{50})$
$x = \frac{1}{2} + \frac{1}{2}\ln(\frac{3}{50}) \approx -0{,}907$

69 **a** $2\ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(9 \cdot 4) = \ln(36)$
b $\ln(20) - 3\ln(2) = \ln(20) - \ln(2^3) = \ln(20) - \ln(8) = \ln(\frac{20}{8}) = \ln(2\frac{1}{2})$
c $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
d $1 + \ln(10) = \ln(e) + \ln(10) = \ln(10e)$
e $\frac{1}{2} + 2\ln(6) = \ln(e^{\frac{1}{2}}) + \ln(6^2) = \ln(\sqrt{e}) + \ln(36) = \ln(36\sqrt{e})$
f $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

70 **a** $\ln(x) = -1$
$x = e^{-1} = \frac{1}{e}$
b $4\ln(x) = 2$
$\ln(x) = \frac{1}{2}$
$x = e^{\frac{1}{2}} = \sqrt{e}$
c $\ln(3x) = 3$
$3x = e^3$
$x = \frac{1}{3}e^3$
d $\ln(-x + 2) = -2$
$-x + 2 = e^{-2}$
$-x = -2 + \frac{1}{e^2}$
$x = 2 - \frac{1}{e^2}$
e $\ln^2(x) = \frac{1}{4}$
$\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$
$x = e^{\frac{1}{2}} = \sqrt{e} \vee x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$
f $\ln(x) = 1 + \ln(5)$
$\ln(x) = \ln(e) + \ln(5)$
$\ln(x) = \ln(5e)$
$x = 5e$

Bladzijde 39

71 **a** $3x\ln(x) = 2\ln(x)$
$\ln(x) = 0 \vee 3x = 2$
$x = 1 \vee x = \frac{2}{3}$
b $\ln^2(x) - \ln(x) = 0$
$\ln(x)(\ln(x) - 1) = 0$
$\ln(x) = 0 \vee \ln(x) = 1$
$x = 1 \vee x = e$
c $x^2\ln(x + 1) = 4\ln(x + 1)$
$\ln(x + 1) = 0 \vee x^2 = 4$
$x + 1 = 1 \vee x = 2 \vee x = -2$
$x = 0 \vee x = 2 \vee x = -2$
vold. niet
d $4\,e^{1-3x} = 20$
$e^{1-3x} = 5$
$1 - 3x = \ln(5)$
$-3x = -1 + \ln(5)$
$x = \frac{1}{3} - \frac{1}{3}\ln(5)$
e $\ln^2(x) - 2\ln(x) - 3 = 0$
Stel $\ln(x) = u$.
$u^2 - 2u - 3 = 0$
$(u + 1)(u - 3) = 0$
$u = -1 \vee u = 3$
$\ln(x) = -1 \vee \ln(x) = 3$
$x = e^{-1} = \frac{1}{e} \vee x = e^3$
f $\ln(x + 3) - \ln(x - 1) = \ln(2)$
$\ln\left(\frac{x+3}{x-1}\right) = \ln(2)$
$\frac{x+3}{x-1} = 2$
$2x - 2 = x + 3$
$x = 5$

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g $2\ln(x) = \ln(2) + \ln(x+4)$
$\ln(x^2) = \ln(2(x+4))$
$x^2 = 2x + 8$
$x^2 - 2x - 8 = 0$
$(x+2)(x-4) = 0$
$x = -2 \vee x = 4$
vold. niet

h $e^{x^2} = 100$
$x^2 = \ln(100)$
$x = \sqrt{\ln(100)} \vee x = -\sqrt{\ln(100)}$

72 **a** $G = 100$ geeft $F = 16(0{,}6 + \ln(100)) \approx 83$
De hartslagfrequentie is 83 slagen per minuut.

b $F = 78$ geeft $16(0{,}6 + \ln(G)) = 78$
$0{,}6 + \ln(G) = 4{,}875$
$\ln(G) = 4{,}275$
$G = e^{4{,}275} \approx 72$
Dus het hiermee corresponderende gewicht is 72 kg.

c $F = 16(0{,}6 + \ln(G))$
$16(0{,}6 + \ln(G)) = F$
$0{,}6 + \ln(G) = \frac{1}{16}F$
$\ln(G) = \frac{1}{16}F - 0{,}6$
$G = e^{\frac{1}{16}F - 0{,}6}$
$G = e^{\frac{1}{16}F} \cdot e^{-0{,}6}$
$G = e^{-0{,}6} \cdot (e^{\frac{1}{16}})^F$
$G \approx 0{,}549 \cdot 1{,}064^F$
Dus $G = 0{,}549 \cdot 1{,}064^F$.

73 **a** $2^x = (e^{\ln(2)})^x = e^{\ln(2) \cdot x}$

b $f(x) = 2^x = e^{\ln(2) \cdot x}$ geeft $f'(x) = e^{\ln(2) \cdot x} \cdot \ln(2) = 2^x \cdot \ln(2)$

74 **a** $e^{\ln(x)} = x$ geeft $[e^{\ln(x)}]' = x'$
$e^{\ln(x)} \cdot [\ln(x)]' = 1$
$e^{\ln(x)} \cdot f'(x) = 1$

b $e^{\ln(x)} \cdot f'(x) = 1$
$x \cdot f'(x) = 1$
$f'(x) = \frac{1}{x}$

c $h(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \cdot \ln(x)$ geeft $h'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \frac{1}{x\ln(2)}$

Bladzijde 40

75 **a** $f(x) = \ln(ax)$ geeft $f'(x) = \frac{1}{ax} \cdot a = \frac{1}{x}$

b $f(x) = \ln(2x)$ geeft $f'(x) = \frac{1}{x}$
$g(x) = \ln(x\sqrt{2})$ geeft $g'(x) = \frac{1}{x}$

c $f(x) = \ln(x^n) = n \cdot \ln(x)$ geeft $f'(x) = n \cdot \frac{1}{x} = \frac{n}{x}$

d $f(x) = \ln(x^2)$ geeft $f'(x) = \frac{2}{x}$
$g(x) = \ln\left(\frac{1}{x}\right)$ geeft $g'(x) = \frac{-1}{x}$
$h(x) = \ln(\sqrt{x})$ geeft $h'(x) = \frac{1}{2x}$

Bladzijde 41

76 **a** $f(x) = 3^{4x-2}$ geeft $f'(x) = 3^{4x-2} \cdot \ln(3) \cdot 4 = 4 \cdot 3^{4x-2} \cdot \ln(3)$

b $f(x) = \dfrac{1-\ln(x)}{x}$ geeft $f'(x) = \dfrac{x \cdot -\frac{1}{x} - (1-\ln(x)) \cdot 1}{x^2} = \dfrac{-1-1+\ln(x)}{x^2} = \dfrac{\ln(x)-2}{x^2}$

c $f(x) = {}^2\log(4x-1)$ geeft $f'(x) = \dfrac{1}{(4x-1)\ln(2)} \cdot 4 = \dfrac{4}{(4x-1)\ln(2)}$

d $f(x) = \dfrac{\ln(3x)}{x}$ geeft $f'(x) = \dfrac{x \cdot \frac{1}{x} - \ln(3x) \cdot 1}{x^2} = \dfrac{1-\ln(3x)}{x^2}$

e $f(x) = x\ln(x^3)$ geeft $f'(x) = 1 \cdot \ln(x^3) + x \cdot \dfrac{3}{x} = \ln(x^3) + 3$

f $f(x) = \ln(x^2+x)$ geeft $f'(x) = \dfrac{1}{x^2+x} \cdot (2x+1) = \dfrac{2x+1}{x^2+x}$

77 **a** $f(x) = \ln(2^x) = x\ln(2)$ geeft $f'(x) = \ln(2)$

b $f(x) = {}^2\log(x^2+1)$ geeft $f'(x) = \dfrac{1}{(x^2+1)\ln(2)} \cdot 2x = \dfrac{2x}{(x^2+1)\ln(2)}$

c $f(x) = x\ln^2(x)$ geeft $f'(x) = 1 \cdot \ln^2(x) + x \cdot 2\ln(x) \cdot \dfrac{1}{x} = \ln^2(x) + 2\ln(x)$

d $f(x) = x^2 \cdot {}^3\log(4x)$ geeft $f'(x) = 2x \cdot {}^3\log(4x) + x^2 \cdot \dfrac{1}{x\ln(3)} = 2x \cdot {}^3\log(4x) + \dfrac{x}{\ln(3)}$

e $f(x) = (2^x - 1) \cdot 2^x = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = 2^x \cdot \ln(2) \cdot (2 \cdot 2^x - 1)$

f $f(x) = \ln^2(4x)$ geeft $f'(x) = 2\ln(4x) \cdot \dfrac{1}{x} = \dfrac{2\ln(4x)}{x}$

78 **a** $x^n = (e^{\ln(x)})^n = e^{n\ln(x)}$

b $f(x) = e^{n\ln(x)}$ geeft $f'(x) = e^{n\ln(x)} \cdot \dfrac{n}{x}$

c $e^{n\ln(x)} \cdot \dfrac{n}{x} = x^n \cdot \dfrac{n}{x} = n \cdot \dfrac{x^n}{x} = nx^{n-1}$

Er is geen gebruikgemaakt van enige beperking van n, dus de regel geldt ook voor elke niet-gehele n van $\mathbb{R}$.

79 **a** $x - 2 > 0$ oftewel $x > 2$, dus $D_f = \langle 2, \rightarrow \rangle$.

$7 - x > 0$ oftewel $x < 7$, dus $D_g = \langle \leftarrow, 7 \rangle$.

$f(x) = g(x)$ geeft $\ln(x-2) = \ln(7-x)$

$x - 2 = 7 - x$

$2x = 9$

$x = 4\frac{1}{2}$

$f(x) \leq g(x)$ geeft $2 < x \leq 4\frac{1}{2}$

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b $s(x) = f(x) + g(x) = \ln(x-2) + \ln(7-x)$ geeft $s'(x) = \dfrac{1}{x-2} + \dfrac{1}{7-x} \cdot -1 = \dfrac{1}{x-2} - \dfrac{1}{7-x}$

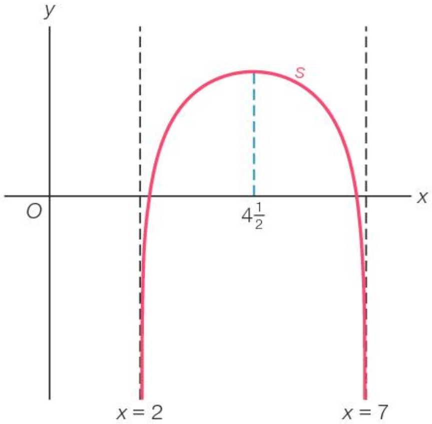
$s'(x) = 0$ geeft $\dfrac{1}{x-2} - \dfrac{1}{7-x} = 0$

$\dfrac{1}{x-2} = \dfrac{1}{7-x}$

$x - 2 = 7 - x$

$2x = 9$

$x = 4\frac{1}{2}$



max. is $s(4\frac{1}{2}) = \ln(2\frac{1}{2}) + \ln(2\frac{1}{2}) = 2\ln(2\frac{1}{2})$

80 **a** $f(x) = \dfrac{x}{\ln(x)}$ geeft $f'(x) = \dfrac{\ln(x) \cdot 1 - x \cdot \dfrac{1}{x}}{\ln^2(x)} = \dfrac{\ln(x) - 1}{\ln^2(x)}$

Stel $k{:}\, y = ax + b$ met $a = f'\left(\dfrac{1}{e}\right) = \dfrac{\ln\left(\dfrac{1}{e}\right) - 1}{\ln^2\left(\dfrac{1}{e}\right)} = \dfrac{-1-1}{(-1)^2} = -2.$

$$\left.\begin{array}{l} y = -2x + b \\ f\left(\dfrac{1}{e}\right) = \dfrac{\dfrac{1}{e}}{\ln\left(\dfrac{1}{e}\right)} = \dfrac{\dfrac{1}{e}}{-1} = -\dfrac{1}{e}, \text{dus } A\left(\dfrac{1}{e}, -\dfrac{1}{e}\right) \end{array}\right\} \begin{array}{l} -2 \cdot \dfrac{1}{e} + b = -\dfrac{1}{e} \\ b = \dfrac{1}{e} \end{array}$$

Dus $k{:}\, y = -2x + \dfrac{1}{e}.$

b $f'(x) = -6$ geeft $\dfrac{\ln(x) - 1}{\ln^2(x)} = -6$

$\ln(x) - 1 = -6\ln^2(x)$

Stel $\ln(x) = u$.

$u - 1 = -6u^2$

$6u^2 + u - 1 = 0$

$D = 1^2 - 4 \cdot 6 \cdot -1 = 25$

$u = \dfrac{-1+5}{12} = \frac{1}{3} \vee u = \dfrac{-1-5}{12} = -\frac{1}{2}$

$\ln(x) = \frac{1}{3} \vee \ln(x) = -\frac{1}{2}$

$x = e^{\frac{1}{3}} \vee x = e^{-\frac{1}{2}}$

$f(e^{\frac{1}{3}}) = \dfrac{e^{\frac{1}{3}}}{\ln(e^{\frac{1}{3}})} = \dfrac{e^{\frac{1}{3}}}{\frac{1}{3}} = 3e^{\frac{1}{3}}$, dus raakpunt $(e^{\frac{1}{3}}, 3e^{\frac{1}{3}}) = (\sqrt[3]{e}, 3 \cdot \sqrt[3]{e})$.

$f(e^{-\frac{1}{2}}) = \dfrac{e^{-\frac{1}{2}}}{\ln(e^{-\frac{1}{2}})} = \dfrac{e^{-\frac{1}{2}}}{-\frac{1}{2}} = -2e^{-\frac{1}{2}}$, dus raakpunt $(e^{-\frac{1}{2}}, -2e^{-\frac{1}{2}}) = \left(\dfrac{1}{\sqrt{e}}, -\dfrac{2}{\sqrt{e}}\right)$.

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Bladzijde 42

81 **a** $f(x) = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = (2^{2x} \cdot 2 - 2^x)\ln(2) = (2^{2x+1} - 2^x)\ln(2)$

$f'(x) = 0$ geeft $(2^{2x+1} - 2^x)\ln(2) = 0$

$2^{2x+1} - 2^x = 0$

$2^{2x+1} = 2^x$

$2x + 1 = x$

$x = -1$

min. is $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, dus $B_f = [-\frac{1}{4}, \rightarrow\rangle$.

b $f'(x) = (2^{2x+1} - 2^x)\ln(2)$ geeft $f''(x) = (2^{2x+1} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2))\ln(2) = (2^{2x+2} - 2^x)\ln^2(2)$

$f''(x) = 0$ geeft $(2^{2x+2} - 2^x)\ln^2(2) = 0$

$2^{2x+2} - 2^x = 0$

$2^{2x+2} = 2^x$

$2x + 2 = x$

$x = -2$

$f(-2) = 2^{-4} - 2^{-2} = \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}$, dus buigpunt $(-2, -\frac{3}{16})$.

82 **a** $f(x) = \sqrt{x}\ln(\sqrt{x})$ geeft $f'(x) = \dfrac{1}{2\sqrt{x}} \cdot \ln(\sqrt{x}) + \sqrt{x} \cdot \dfrac{1}{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$

$f'(x) = 0$ geeft $\dfrac{\ln(\sqrt{x}) + 1}{2\sqrt{x}} = 0$

$\ln(\sqrt{x}) + 1 = 0$

$\ln(\sqrt{x}) = -1$

$\sqrt{x} = e^{-1}$

$x = e^{-2} = \dfrac{1}{e^2}$

$f\left(\dfrac{1}{e^2}\right) = e^{-1} \cdot \ln(e^{-1}) = \dfrac{1}{e} \cdot -1 = -\dfrac{1}{e}$, dus $A\left(\dfrac{1}{e^2}, -\dfrac{1}{e}\right)$.

b $f'(x) = \dfrac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$ geeft

$$f''(x) = \frac{2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - (\ln(\sqrt{x}) + 1) \cdot \frac{2}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{\frac{1}{\sqrt{x}} - \frac{(\ln(\sqrt{x}) + 1)}{\sqrt{x}}}{4x} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{1 - (\ln(\sqrt{x}) + 1)}{4x\sqrt{x}} = \frac{-\ln(\sqrt{x})}{4x\sqrt{x}}$$

$f''(x) = 0$ geeft $-\ln(\sqrt{x}) = 0$

$\ln(\sqrt{x}) = 0$

$\sqrt{x} = 1$

$x = 1$

Stel $k: y = ax + b$ met $a = f'(1) = \dfrac{\ln(\sqrt{1}) + 1}{2\sqrt{1}} = \dfrac{0 + 1}{2} = \frac{1}{2}$.

$$\left.\begin{array}{l} y = \frac{1}{2}x + b \\ f(1) = \sqrt{1}\ln(\sqrt{1}) = 0 \end{array}\right\} \begin{array}{l} \frac{1}{2} \cdot 1 + b = 0 \\ b = -\frac{1}{2} \end{array}$$

Dus $k: y = \frac{1}{2}x - \frac{1}{2}$.

83 **a** Op $t = 0$ is $T = 80$, dus $20 + p\,e^0 = 80$

$20 + p \cdot 1 = 80$

$p = 60$

b $T = 45$ geeft $20 + 60\,e^{-\frac{1}{6}t} = 45$

$60\,e^{-\frac{1}{6}t} = 25$

$e^{-\frac{1}{6}t} = \frac{25}{60}$

$-\frac{1}{6}t = \ln(\frac{25}{60})$

$t = -6\ln(\frac{25}{60}) = 5{,}252...$

Dus na 5 minuten en $0{,}252... \cdot 60 \approx 15$ seconden is de temperatuur van de koffie 45 °C.

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c $T = 20 + 60\,e^{-\frac{1}{6}t}$ geeft $\frac{dT}{dt} = 60\,e^{-\frac{1}{6}t} \cdot -\frac{1}{6} = -10\,e^{-\frac{1}{6}t}$

$t = 2$ geeft $\frac{dT}{dt} = -10e^{-\frac{1}{3}} \approx -7{,}2$

Twee minuten nadat de koffie is ingeschonken neemt de temperatuur van de koffie af met een snelheid van 7,2 °C per minuut.

d $T = 20 + 60\,e^{-\frac{1}{6}t}$

$60\,e^{-\frac{1}{6}t} = T - 20$

$e^{-\frac{1}{6}t} = \frac{1}{60}T - \frac{1}{3}$

$-\frac{1}{6}t = \ln(\frac{1}{60}T - \frac{1}{3})$

$t = -6\ln(\frac{1}{60}T - \frac{1}{3})$

Dus $a = -6$, $b = \frac{1}{60}$ en $c = -\frac{1}{3}$.

84 **a** $f_2(x) = \ln\left(\frac{2x}{2x+1}\right)$ geeft

$$f_2'(x) = \frac{1}{\left(\frac{2x}{2x+1}\right)} \cdot \frac{(2x+1)\cdot 2 - 2x \cdot 2}{(2x+1)^2} = \frac{2x+1}{2x} \cdot \frac{4x+2-4x}{(2x+1)^2} = \frac{2(2x+1)}{2x(2x+1)^2} = \frac{1}{x(2x+1)}$$

$rc_k = \frac{1}{3}$, dus $f_2'(x) = \frac{1}{3}$

$\frac{1}{x(2x+1)} = \frac{1}{3}$

$x(2x+1) = 3$

$2x^2 + x - 3 = 0$

$D = 1^2 - 4 \cdot 2 \cdot -3 = 25$

$x = \frac{-1+5}{4} = 1 \vee x = \frac{-1-5}{4} = -1\frac{1}{2}$

vold. niet

$f_2(1) = \ln\left(\frac{2}{2+1}\right) = \ln(\frac{2}{3})$, dus $A(1, \ln(\frac{2}{3}))$.

b Voor f_a geldt $y = \ln\left(\frac{ax}{2x+1}\right)$, dus voor f_a^{inv} geldt $x = \ln\left(\frac{ay}{2y+1}\right)$.

$x = \ln\left(\frac{ay}{2y+1}\right)$ geeft $\ln\left(\frac{ay}{2y+1}\right) = x$

$\frac{ay}{2y+1} = e^x$

$ay = (2y+1)\,e^x$

$ay = 2y\,e^x + e^x$

$ay - 2y\,e^x = e^x$

$y(a - 2\,e^x) = e^x$

$y = \frac{e^x}{a - 2\,e^x}$

Dus $f_a^{\text{inv}}(x) = \frac{e^x}{a - 2\,e^x}$.

Dus voor $a = 3$ is $g(x) = \frac{e^x}{3 - 2\,e^x}$ de inverse van f_a.

Bladzijde 43

85 **a** $f(x) = 1$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} = 1$

$\ln^2(x) + 2\ln(x) - 3 = 0$

Stel $\ln(x) = u$.

$u^2 + 2u - 3 = 0$

$(u-1)(u+3) = 0$

$u = 1 \vee u = -3$

$\ln(x) = 1 \vee \ln(x) = -3$

$x = e \vee x = e^{-3} = \frac{1}{e^3}$

$f(x) \leq 1$ geeft $\frac{1}{e^3} \leq x \leq e$

b $f(x) = e^{\ln^2(x) + 2\ln(x) - 3}$ geeft

$$f'(x) = e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(2\ln(x) \cdot \frac{1}{x} + \frac{2}{x}\right) = e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(\frac{2\ln(x) + 2}{x}\right)$$

$f'(x) = 0$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(\frac{2\ln(x) + 2}{x}\right) = 0$

$$\frac{2\ln(x) + 2}{x} = 0$$

$$2\ln(x) + 2 = 0$$

$$\ln(x) = -1$$

$$x = e^{-1}$$

min. is $f(e^{-1}) = e^{(-1)^2 + 2 \cdot -1 - 3} = e^{-4} = \frac{1}{e^4}$, dus $B_f = \left[\frac{1}{e^4}, \rightarrow\right\rangle$.

86 **a** $f(x) = x^x = (e^{\ln(x)})^x = e^{x\ln(x)}$ geeft $f'(x) = e^{x\ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x}\right) = (\ln(x) + 1) \cdot x^x$

b $g_a(x) = x^{\frac{a}{x}} = (e^{\ln(x)})^{\frac{a}{x}} = e^{\frac{a}{x}\ln(x)} = e^{ax^{-1} \cdot \ln(x)}$ geeft

$$g_a'(x) = e^{ax^{-1} \cdot \ln(x)} \cdot \left(-ax^{-2} \cdot \ln(x) + ax^{-1} \cdot \frac{1}{x}\right) = x^{\frac{a}{x}} \cdot \left(\frac{-a\ln(x)}{x^2} + \frac{a}{x^2}\right) = \left(\frac{a - a\ln(x)}{x^2}\right) \cdot x^{\frac{a}{x}}$$

Voor raken geldt $f(x) = g_a(x) \wedge f'(x) = g_a'(x)$.

$f(x) = g_a(x)$ geeft $x^x = x^{\frac{a}{x}}$

$$x = \frac{a}{x}$$

$$a = x^2$$

$f'(x) = g_a'(x)$ met $a = x^2$ geeft $(\ln(x) + 1) \cdot x^x = \left(\frac{x^2 - x^2\ln(x)}{x^2}\right) \cdot x^{\frac{x^2}{x}}$

$$(\ln(x) + 1) \cdot x^x = \left(\frac{x^2(1 - \ln(x))}{x^2}\right) \cdot x^x$$

$$\ln(x) + 1 = 1 - \ln(x)$$

$$2\ln(x) = 0$$

$$x = 1$$

Dus $a = 1^2 = 1$.

Stel $k: y = mx + n$ met $m = f'(1) = (\ln(1) + 1) \cdot 1^1 = 1$.

$$\left.\begin{array}{l} y = x + n \\ f(1) = 1, \text{dus } A(1, 1) \end{array}\right\} \begin{array}{l} 1 + n = 1 \\ n = 0 \end{array}$$

Dus $k: y = x$.

Diagnostische toets

Bladzijde 46

1 **a** ${}^2\log(18) - {}^2\log(36) + 3 = {}^2\log(\frac{18}{36}) + 3 = {}^2\log(\frac{1}{2}) + 3 = -1 + 3 = 2$

b $2 \cdot {}^3\log(5) - {}^3\log(75) + {}^2\log(8\sqrt{2}) = {}^3\log(5^2) - {}^3\log(75) + {}^2\log(2^3 \cdot 2^{\frac{1}{2}}) = {}^3\log(25) - {}^3\log(75) + {}^2\log(2^{3\frac{1}{2}}) =$
${}^3\log(\frac{25}{75}) + 3\frac{1}{2} = {}^3\log(\frac{1}{3}) + 3\frac{1}{2} = -1 + 3\frac{1}{2} = 2\frac{1}{2}$

2 **a** $4 + {}^2\log(a) + \log(100) = 4 + {}^2\log(a) + 2 = 6 + {}^2\log(a) = {}^2\log(2^6) + {}^2\log(a) = {}^2\log(64) + {}^2\log(a) = {}^2\log(64a)$

b $2 - {}^3\log(6a) = {}^3\log(3^2) - {}^3\log(6a) = {}^3\log(9) - {}^3\log(6a) = {}^3\log\left(\frac{9}{6a}\right) = {}^3\log\left(\frac{3}{2a}\right)$

3 **a** $3 \cdot {}^3\log(36) = 3 \cdot {}^3\log(9 \cdot 4) = 3 \cdot ({}^3\log(9) + {}^3\log(4)) = 3 \cdot (2 + {}^3\log(4)) = 6 + 3 \cdot {}^3\log(4) = 6 + {}^3\log(4^3) =$
$6 + {}^3\log(64)$

b $3 + \log(800) = 3 + \log(100 \cdot 8) = 3 + \log(100) + \log(8) = 3 + 2 + \log(8) = 5 + \log(8)$

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4 **a** $y = {}^5\log(x) \xrightarrow{\text{translatie }(0,\ 2)} y = {}^5\log(x) + 2$
Er geldt ${}^5\log(x) + 2 = {}^5\log(x) + {}^5\log(5^2) = {}^5\log(x) + {}^5\log(25) = {}^5\log(25x)$.
Dus de vermenigvuldiging ten opzichte van de y-as met $\frac{1}{25}$ levert dezelfde beeldgrafiek op.

b $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = {}^3\log(3x)$
Er geldt ${}^3\log(3x) = {}^3\log(3) + {}^3\log(x) = 1 + {}^3\log(x)$.
Dus de translatie (0, 1) levert dezelfde beeldgrafiek op.

5 **a** $2 \cdot {}^2\log(x-1) = 1 + {}^2\log(18)$
${}^2\log((x-1)^2) = {}^2\log(2) + {}^2\log(18)$
${}^2\log((x-1)^2) = {}^2\log(2 \cdot 18)$
${}^2\log((x-1)^2) = {}^2\log(36)$
$(x-1)^2 = 36$
$x - 1 = 6 \vee x - 1 = -6$
$x = 7 \vee x = -5$
vold. niet

b ${}^3\log(2x-1) + {}^{\frac{1}{3}}\log(x+2) = 0$
${}^3\log(2x-1) - {}^3\log(x+2) = 0$
${}^3\log(2x-1) = {}^3\log(x+2)$
$2x - 1 = x + 2$
$x = 3$

c ${}^2\log(2x-1) = {}^4\log(x)$
$${}^2\log(2x-1) = \frac{{}^2\log(x)}{{}^2\log(4)}$$
$${}^2\log(2x-1) = \frac{{}^2\log(x)}{2}$$
$2 \cdot {}^2\log(2x-1) = {}^2\log(x)$
${}^2\log((2x-1)^2) = {}^2\log(x)$
$(2x-1)^2 = x$
$4x^2 - 4x + 1 = x$
$4x^2 - 5x + 1 = 0$
$D = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$
$$x = \frac{5+3}{8} = 1 \vee x = \frac{5-3}{8} = \frac{1}{4}$$
vold. niet

d $\log^2(x) = \log(x) + 2$
Stel $\log(x) = u$.
$u^2 = u + 2$
$u^2 - u - 2 = 0$
$(u+1)(u-2) = 0$
$u = -1 \vee u = 2$
$\log(x) = -1 \vee \log(x) = 2$
$x = \frac{1}{10} \vee x = 100$

e ${}^2\log(x) = 3 - {}^2\log(x+2)$
${}^2\log(x) = {}^2\log(8) - {}^2\log(x+2)$
${}^2\log(x) + {}^2\log(x+2) = {}^2\log(8)$
${}^2\log(x(x+2)) = {}^2\log(8)$
$x(x+2) = 8$
$x^2 + 2x - 8 = 0$
$(x-2)(x+4) = 0$
$x = 2 \vee x = -4$
vold. niet

f ${}^2\log^2(x) + 12 = 7 \cdot {}^2\log(x)$
Stel ${}^2\log(x) = u$.
$u^2 + 12 = 7u$
$u^2 - 7u + 12 = 0$
$(u-3)(u-4) = 0$
$u = 3 \vee u = 4$
${}^2\log(x) = 3 \vee {}^2\log(x) = 4$
$x = 8 \vee x = 16$

6 **a** $g_{dag} = 1{,}1$
$g_{week} = 1{,}1^7 \approx 1{,}949$
Het groeipercentage per week is 94,9%.

b $g_{dag} = 1{,}1$
$g_{8\text{ uur}} = 1{,}1^{\frac{1}{3}} \approx 1{,}032$
Het groeipercentage per 8 uur is 3,2%.

7 **a** $g_{jaar} = 0{,}64$
$g_{maand} = 0{,}64^{\frac{1}{12}} \approx 0{,}963$
De afname per maand is 3,7%.

b $g_{jaar} = 0{,}64$
$g_{5\text{ jaar}} = 0{,}64^5 \approx 0{,}107$
De afname per 5 jaar is 89,3%.

8 **a** $g_{maand} = 1{,}002$
$1{,}002^T = 2$
$T = {}^{1{,}002}\log(2) = 346{,}920...$
De verdubbelingstijd is 346,920... : 12 ≈ 29 jaar.

b $g_{week} = 0{,}8$
$0{,}8^T = \frac{1}{2}$
$T = {}^{0{,}8}\log(\frac{1}{2}) = 3{,}106...$
De halveringstijd is 3,106... · 7 ≈ 22 dagen.

9 **a** $y = 4 \cdot \log(3x + 12) - 1$
$4 \cdot \log(3x + 12) = y + 1$
$\log(3x + 12) = 0{,}25y + 0{,}25$
$3x + 12 = 10^{0{,}25y + 0{,}25}$
$3x + 12 = 10^{0{,}25y} \cdot 10^{0{,}25}$
$3x = -12 + (10^{0{,}25})^y \cdot 10^{0{,}25}$
$x \approx -4 + 0{,}59 \cdot 1{,}78^y$
Dus $x = -4 + 0{,}59 \cdot 1{,}78^y$.

b $p = 34 \cdot 2{,}7^{5q+2}$
$34 \cdot 2{,}7^{5q+2} = p$
$2{,}7^{5q+2} = \frac{1}{34}p$
$5q + 2 = {}^{2{,}7}\log(\frac{1}{34}p)$
$5q + 2 = {}^{2{,}7}\log(\frac{1}{34}) + {}^{2{,}7}\log(p)$
$$5q = -2 + \frac{\log(\frac{1}{34})}{\log(2{,}7)} + \frac{\log(p)}{\log(2{,}7)}$$
$$q = -\frac{2}{5} + \frac{\log(\frac{1}{34})}{5\log(2{,}7)} + \frac{\log(p)}{5\log(2{,}7)}$$
$q \approx -1{,}11 + 0{,}46 \cdot \log(p)$
Dus $q = -1{,}11 + 0{,}46 \cdot \log(p)$.

Bladzijde 47

10 **a** $\frac{3e^3 - e^3}{e^2} = \frac{2e^3}{e^2} = 2e$

b $\frac{e^{3x} - e^x}{e^x} = e^{2x} - 1$

c $(e^{3x} - 5)^2 = (e^{3x})^2 - 2 \cdot e^{3x} \cdot 5 + 25 = e^{6x} - 10\,e^{3x} + 25$

11 **a** $3x\,e^x - e^x = 0$
$e^x(3x - 1) = 0$
$3x - 1 = 0$
$3x = 1$
$x = \frac{1}{3}$

b $e^{2x-1} - \sqrt[3]{e^2} = 0$
$e^{2x-1} = \sqrt[3]{e^2}$
$e^{2x-1} = e^{\frac{2}{3}}$
$2x - 1 = \frac{2}{3}$
$2x = 1\frac{2}{3}$
$x = \frac{5}{6}$

c $e^{2x} + 2\,e^x = 3$
$(e^x)^2 + 2\,e^x - 3 = 0$
Stel $e^x = u$.
$u^2 + 2u - 3 = 0$
$(u - 1)(u + 3) = 0$
$u = 1 \vee u = -3$
$e^x = 1 \vee e^x = -3$
$x = 0$

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12 **a** $f(x) = 2\,e^x - 3x^2$ geeft $f'(x) = 2\,e^x - 6x$

b $f(x) = \dfrac{x^2+1}{e^x}$ geeft $f'(x) = \dfrac{e^x \cdot 2x - (x^2+1)\cdot e^x}{(e^x)^2} = \dfrac{-x^2+2x-1}{e^x}$

c $f(x) = (x^2+1)\,e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2+1)\cdot e^x = (x^2+2x+1)\,e^x$

d $f(x) = \dfrac{e^x}{x^2+1}$ geeft $f'(x) = \dfrac{(x^2+1)\cdot e^x - e^x \cdot 2x}{(x^2+1)^2} = \dfrac{(x^2-2x+1)\,e^x}{(x^2+1)^2}$

e $f(x) = x^2 \cdot e^{2x-1}$ geeft $f'(x) = 2x \cdot e^{2x-1} + x^2 \cdot e^{2x-1} \cdot 2 = (2x^2+2x)\,e^{2x-1}$

f $f(x) = e^{x^2+9}$ geeft $f'(x) = e^{x^2+9} \cdot 2x = 2x\,e^{x^2+9}$

13 **a** $f(x) = (x^2-x+1)\,e^x$ geeft $f'(x) = (2x-1)\cdot e^x + (x^2-x+1)\cdot e^x = (x^2+x)\,e^x$

$f'(x) = 0$ geeft $(x^2+x)\,e^x = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0 \vee x = -1$

max. is $f(-1) = \dfrac{3}{e}$ en min. is $f(0) = 1$.

b $f'(x) = (x^2+x)\,e^x$ geeft $f''(x) = (2x+1)\cdot e^x + (x^2+x)\cdot e^x = (x^2+3x+1)\,e^x$

$f''(x) = 0$ geeft $(x^2+3x+1)\,e^x = 0$

$x^2+3x+1 = 0$

$D = 3^2 - 4\cdot 1 \cdot 1 = 5$

$x = \dfrac{-3+\sqrt{5}}{2} \vee x = \dfrac{-3-\sqrt{5}}{2}$

$x = -1\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee x = -1\frac{1}{2} - \frac{1}{2}\sqrt{5}$

c Stel k: $y = ax + b$ met $a = f'(1) = (1+1)e = 2e$.

k: $y = 2ex + b$
$f(1) = e$, dus $A(1, e)$ $\Big\}$ $2e + b = e$

$b = -e$

Dus k: $y = 2ex - e$.

14 **a** $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$

b $\ln(10) - 4\ln(2) = \ln(10) - \ln(2^4) = \ln(10) - \ln(16) = \ln(\frac{10}{16}) = \ln(\frac{5}{8})$

15 **a** $2\,e^{5x} = 16$

$e^{5x} = 8$

$5x = \ln(8)$

$x = \frac{1}{5}\ln(8)$

b $\ln^2(5x) = 16$

$\ln(5x) = 4 \vee \ln(5x) = -4$

$5x = e^4 \vee 5x = e^{-4}$

$x = \frac{1}{5}e^4 \vee x = \dfrac{1}{5e^4}$

c $2\ln^2(x) - \ln(x) = 0$

$\ln(x)(2\ln(x) - 1) = 0$

$\ln(x) = 0 \vee 2\ln(x) - 1 = 0$

$x = 1 \vee 2\ln(x) = 1$

$x = 1 \vee \ln(x) = \frac{1}{2}$

$x = 1 \vee x = e^{\frac{1}{2}} = \sqrt{e}$

d $\ln(9x+1) - \ln(x+2) = \ln(4)$

$\ln(9x+1) = \ln(x+2) + \ln(4)$

$\ln(9x+1) = \ln(4(x+2))$

$9x + 1 = 4(x+2)$

$9x + 1 = 4x + 8$

$5x = 7$

$x = 1\frac{2}{5}$

16 **a** $f(x) = 2^{3x-4}$ geeft $f'(x) = 2^{3x-4} \cdot \ln(2) \cdot 3 = 3 \cdot 2^{3x-4} \cdot \ln(2)$

b $f(x) = x \cdot 3^x$ geeft $f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln(3) = (1 + x\ln(3)) \cdot 3^x$

c $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln(x^{1\frac{1}{3}}) = 1\frac{1}{3}\ln(x)$ geeft $f'(x) = \dfrac{1\frac{1}{3}}{x} = \dfrac{4}{3x}$

d $f(x) = {}^2\log(4x) = {}^2\log(4) + {}^2\log(x)$ geeft $f'(x) = \dfrac{1}{x\ln(2)}$

e $f(x) = {}^3\log(5x-6)$ geeft $f'(x) = \dfrac{1}{(5x-6)\ln(3)} \cdot 5 = \dfrac{5}{(5x-6)\ln(3)}$

f $f(x) = \ln(3x^2+3)$ geeft $f'(x) = \dfrac{1}{3x^2+3} \cdot 6x = \dfrac{2x}{x^2+1}$

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17 $f(x) = 3^{x-1} + 3^{-x+1}$ geeft $f'(x) = 3^{x-1} \cdot \ln(3) + 3^{-x+1} \cdot \ln(3) \cdot -1 = (3^{x-1} - 3^{-x+1})\ln(3)$

$f'(x) = 0$ geeft $(3^{x-1} - 3^{-x+1})\ln(3) = 0$

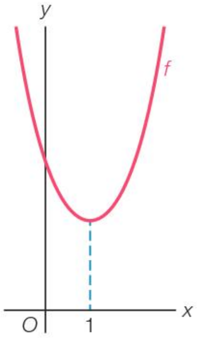
$3^{x-1} - 3^{-x+1} = 0$

$3^{x-1} = 3^{-x+1}$

$x - 1 = -x + 1$

$2x = 2$

$x = 1$



min. is $f(1) = 3^0 + 3^0 = 1 + 1 = 2$, dus $B_f = [2, \rightarrow\rangle$.

18 **a** $f(x) = x^2 \ln(x) - \frac{1}{2}x^2$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} - x = 2x\ln(x)$

$f'(x) = 0$ geeft $2x\ln(x) = 0$

$x = 0 \vee \ln(x) = 0$

$x = 0 \quad \vee \quad x = 1$

vold. niet

$f(1) = 1 \cdot 0 - \frac{1}{2} \cdot 1 = -\frac{1}{2}$, dus $A(1, -\frac{1}{2})$.

b $f'(x) = 2x\ln(x)$ geeft $f''(x) = 2 \cdot \ln(x) + 2x \cdot \frac{1}{x} = 2\ln(x) + 2$

$f''(x) = 0$ geeft $2\ln(x) + 2 = 0$

$2\ln(x) = -2$

$\ln(x) = -1$

$x = e^{-1} = \frac{1}{e}$

Stel k: $y = ax + b$ met $a = f'\left(\frac{1}{e}\right) = 2 \cdot \frac{1}{e} \cdot -1 = -\frac{2}{e}$.

$$\left.\begin{array}{l} k: y = -\frac{2}{e}x + b \\ f\left(\frac{1}{e}\right) = -\frac{3}{2e^2} \end{array}\right\} -\frac{2}{e} \cdot \frac{1}{e} + b = -\frac{3}{2e^2}$$

$$-\frac{4}{2e^2} + b = -\frac{3}{2e^2}$$

$$b = \frac{1}{2e^2}$$

Dus $k: y = -\frac{2}{e}x + \frac{1}{2e^2}$.

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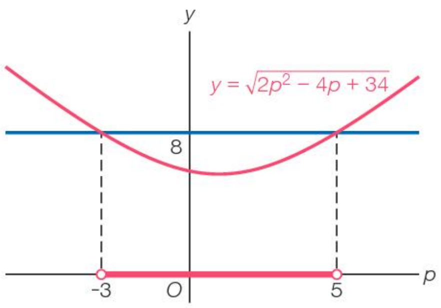
10 Meetkunde met vectoren

Voorkennis Afstanden en middens

Bladzijde 52

1 $P(\frac{1}{2}(5 + 11), \frac{1}{2}(-1 + 5)) = P(8, 2)$
$Q(\frac{1}{2}(11 + -3), \frac{1}{2}(5 + 7)) = Q(4, 6)$
$d(A, P) = \sqrt{(8-5)^2 + (2 - -1)^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$
$d(A, Q) = \sqrt{(4-5)^2 + (6 - -1)^2} = \sqrt{(-1)^2 + 7^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$
$d(P, Q) = \sqrt{(4-8)^2 + (6-2)^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$
De omtrek van driehoek APQ is $3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$.

2 **a** $p = 3$ geeft $A(-2, 3)$, $B(4, 5)$ en
$d(A, B) = \sqrt{(4 - -2)^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$.
b $\frac{1}{2}(-2 + p + 1) = -6$
$-2 + p + 1 = -12$
$p = -11$
$p = -11$ geeft $A(-2, -11)$, $B(-10, 5)$ en
$d(A, B) = \sqrt{(-10 - -2)^2 + (5 - -11)^2} = \sqrt{(-8)^2 + 16^2} = \sqrt{64 + 256} = \sqrt{320} = 8\sqrt{5}$.
c $M(\frac{1}{2}(-2 + p + 1), \frac{1}{2}(p + 5)) = M(\frac{1}{2}p - \frac{1}{2}, \frac{1}{2}p + 2\frac{1}{2})$
M op $y = -x$ geeft $\frac{1}{2}p + 2\frac{1}{2} = -(\frac{1}{2}p - \frac{1}{2})$
$\frac{1}{2}p + 2\frac{1}{2} = -\frac{1}{2}p + \frac{1}{2}$
$p = -2$
$p = -2$ geeft $A(-2, -2)$, $M(-1\frac{1}{2}, 1\frac{1}{2})$ en
$d(A, M) = \sqrt{(-1\frac{1}{2} - -2)^2 + (1\frac{1}{2} - -2)^2} = \sqrt{(\frac{1}{2})^2 + (3\frac{1}{2})^2} = \sqrt{\frac{1}{4} + 12\frac{1}{4}} = \sqrt{12\frac{1}{2}} = 2\frac{1}{2}\sqrt{2}$.
d $d(A, B) = \sqrt{(p + 1 - -2)^2 + (5-p)^2} = \sqrt{(p+3)^2 + (5-p)^2} = \sqrt{p^2 + 6p + 9 + 25 - 10p + p^2}$
$= \sqrt{2p^2 - 4p + 34}$
e $d(A, B) = 8$ geeft $\sqrt{2p^2 - 4p + 34} = 8$
$2p^2 - 4p + 34 = 64$
$2p^2 - 4p - 30 = 0$
$p^2 - 2p - 15 = 0$
$(p+3)(p-5) = 0$
$p = -3 \vee p = 5$



Dus $d(A, B) < 8$ voor $-3 < p < 5$.

10.1 Vectoren

Bladzijde 53

1 $d(O, A) = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$
$d(A, B) = \sqrt{(2-5)^2 + (6-2)^2} = \sqrt{(-3)^2 + 4^2} = 5$
Dus de lengte van de wandeling is $5 + \sqrt{29}$.

Bladzijde 56

2 **a** $|\vec{a}| = 8\sin(17°) \approx 2{,}34$ en $|\vec{b}| = 8\cos(17°) \approx 7{,}65$

b $1{,}5 = |\vec{v}| \cdot \sin(26°)$ geeft $|\vec{v}| = \dfrac{1{,}5}{\sin(26°)} \approx 3{,}42$ en $|\vec{b}| = 3{,}42...\cos(26°) \approx 3{,}08.$

3 **a** $|\vec{a}| = \left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$

b $|\vec{b}| = \left|\begin{pmatrix} \sqrt{3} \\ \sqrt{6} \end{pmatrix}\right| = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = \sqrt{3+6} = \sqrt{9} = 3$

c $|\vec{c}| = \left|\begin{pmatrix} 0 \\ 5 \end{pmatrix}\right| = \sqrt{0^2 + 5^2} = \sqrt{0+25} = \sqrt{25} = 5$

d $|\vec{d}| = \left|\begin{pmatrix} -0{,}3 \\ 0{,}4 \end{pmatrix}\right| = \sqrt{(-0{,}3)^2 + 0{,}4^2} = \sqrt{0{,}09 + 0{,}16} = \sqrt{0{,}25} = 0{,}5$

e $|\vec{e}| = 2 \cdot \left|\begin{pmatrix} 1 \\ 3 \end{pmatrix}\right| = 2 \cdot \sqrt{1^2 + 3^2} = 2 \cdot \sqrt{1+9} = 2\sqrt{10}$

f $|\vec{f}| = 0{,}6 \cdot \left|\begin{pmatrix} 6 \\ 8 \end{pmatrix}\right| = 0{,}6 \cdot \sqrt{6^2 + 8^2} = 0{,}6 \cdot \sqrt{36+64} = 0{,}6 \cdot \sqrt{100} = 0{,}6 \cdot 10 = 6$

4 **a** $\vec{c} = 2\vec{a} + 3\vec{b} = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$

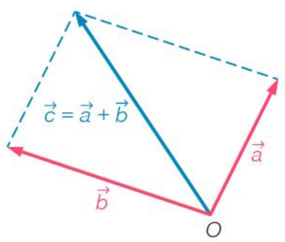
b $\vec{d} = \vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

c $\vec{e} = 4\vec{a} - 3\vec{b} = 4 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$

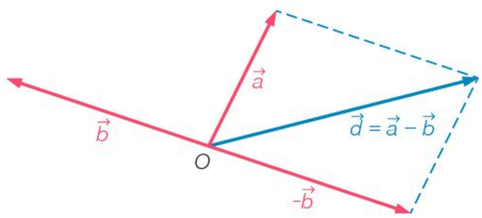
d $\vec{f} = 2 \cdot (\vec{a} + \vec{b}) = 2 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}\right) = 2 \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}$

Bladzijde 57

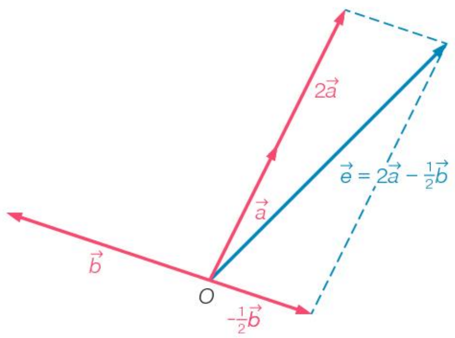
5 **a**



b



c



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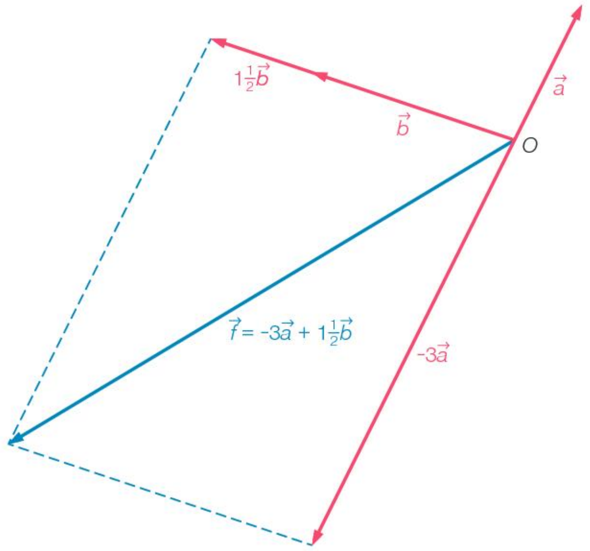
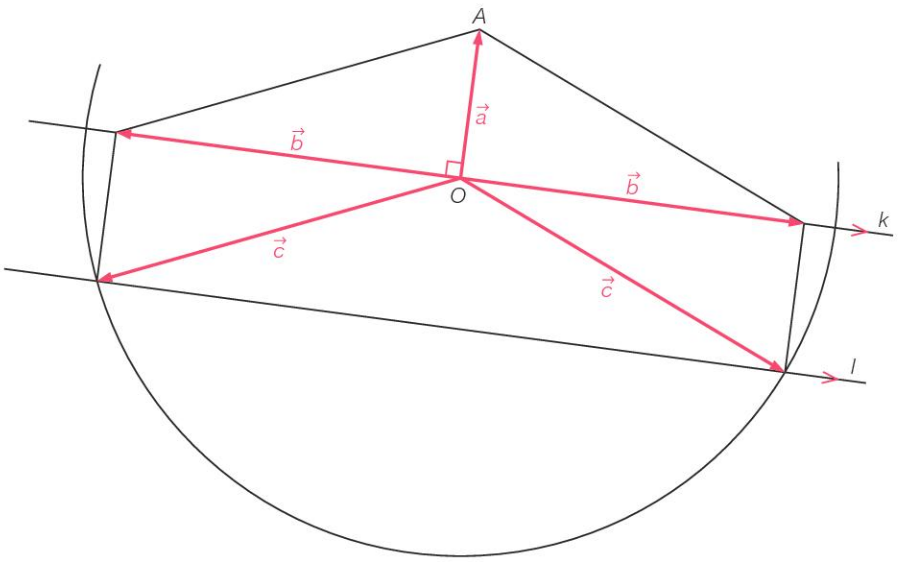
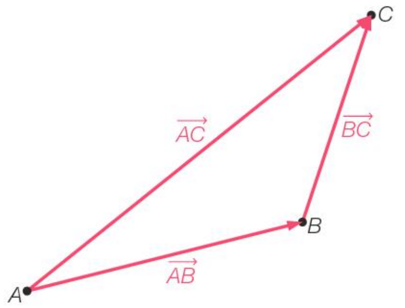
d



6 Teken de cirkel met middelpunt O en straal 5.
Teken de lijn l evenwijdig met k, op afstand 2 van k.
De snijpunten van l met de cirkel zijn de eindpunten van de vectoren c.
Met de parallellogrammen krijg je de vectoren b.



7 **a**



b $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$
c $\overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{DF}$
d $\overrightarrow{AK} + \overrightarrow{KL} = \overrightarrow{AL}$
e $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$

8 **a** $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$, dus $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$.

b $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \vec{a} + \frac{1}{2}(\vec{b} - \vec{a}) = \vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} + \vec{b})$

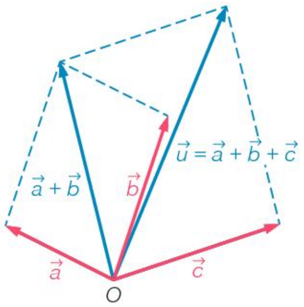
c $\overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}7\\2\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix}$

$\overrightarrow{QP} = \vec{p} - \vec{q} = \begin{pmatrix}7\\2\end{pmatrix} - \begin{pmatrix}3\\5\end{pmatrix} = \begin{pmatrix}4\\-3\end{pmatrix}$

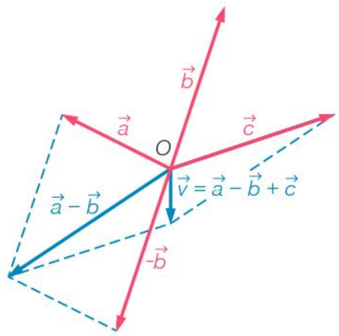
$\overrightarrow{OR} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2} \cdot \left(\begin{pmatrix}7\\2\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}\right) = \frac{1}{2} \cdot \begin{pmatrix}10\\7\end{pmatrix} = \begin{pmatrix}5\\3\frac{1}{2}\end{pmatrix}$

$\overrightarrow{QR} = \vec{r} - \vec{q} = \begin{pmatrix}5\\3\frac{1}{2}\end{pmatrix} - \begin{pmatrix}3\\5\end{pmatrix} = \begin{pmatrix}2\\-1\frac{1}{2}\end{pmatrix}$

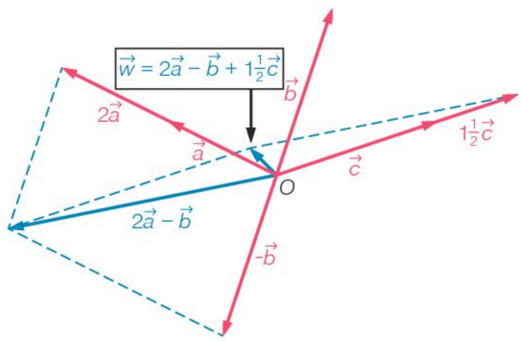
9 **a**



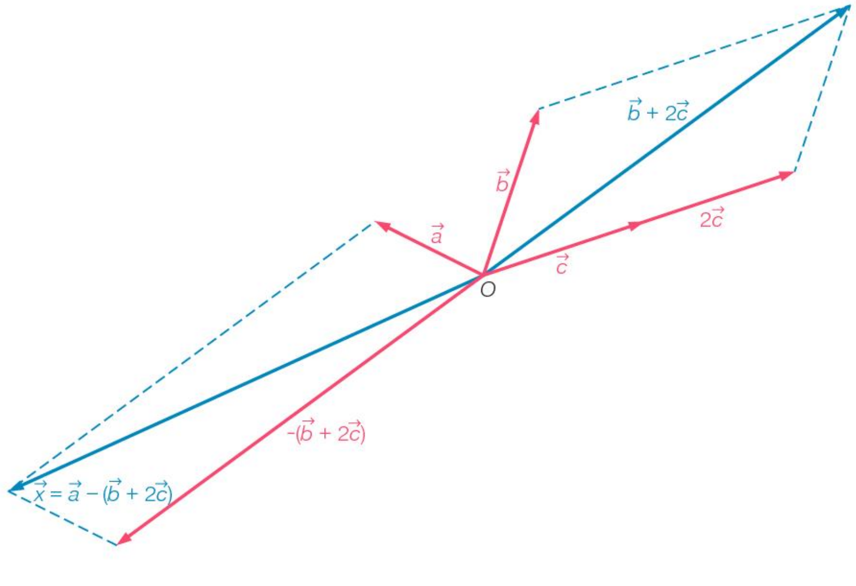
b



c



d



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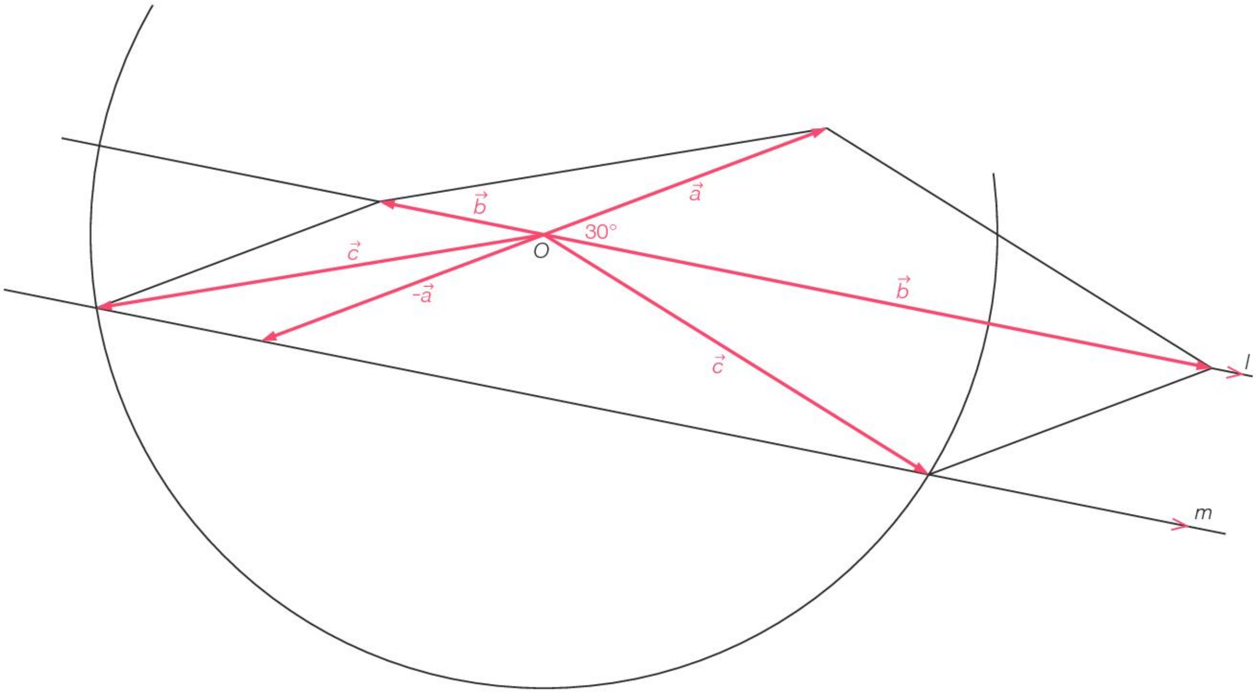
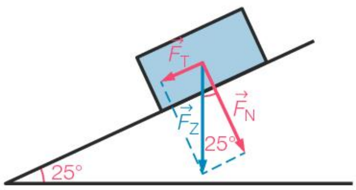
10



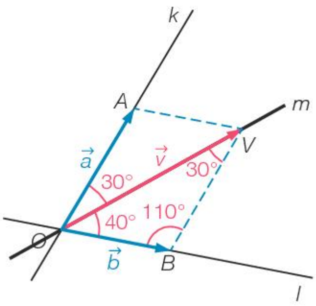
Bladzijde 58

11



De component langs de helling is $F_T = 750\sin(25°) \approx 317$ N.
De component loodrecht op de helling is $F_N = 750\cos(25°) \approx 680$ N.

12



De sinusregel in $\triangle OBV$ geeft $\dfrac{12}{\sin(110°)} = \dfrac{|\vec{b}|}{\sin(30°)} = \dfrac{|\vec{a}|}{\sin(40°)}$.

Dus $|\vec{a}| = \dfrac{12\sin(40°)}{\sin(110°)} \approx 8{,}2$ en $|\vec{b}| = \dfrac{12\sin(30°)}{\sin(110°)} \approx 6{,}4$.

10.2 Vectoren en rotaties

Bladzijde 60

13 **a** $\vec{a}_R = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ en $\vec{a}_L = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

b $\vec{b}_R = \begin{pmatrix} q \\ -p \end{pmatrix}$

Bladzijde 61

14 $\vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM}_L = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Dus $D(-1, 5)$.

Bladzijde 62

15 **a** $\vec{b} = \vec{a} + \overrightarrow{AB} = \vec{a} + \overrightarrow{AD}_R$

$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, dus $\overrightarrow{AD}_R = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

Dit geeft $\vec{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$.

Dus $B(9, 5)$.

b $\vec{n} = \vec{b} + \overrightarrow{BN} = \vec{b} + \overrightarrow{BM}_R$

$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}(\vec{d} - \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$, dus $\overrightarrow{BM}_R = \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix}$.

Dit geeft $\vec{n} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 9\frac{1}{2} \\ 8\frac{1}{2} \end{pmatrix}$.

Dus $N(9\frac{1}{2}, 8\frac{1}{2})$.

16 $\vec{n} = \frac{1}{2}(\vec{b} + \vec{d})$ met $\vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM}_L$

$\vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\overrightarrow{AM}_L = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Dit geeft $\vec{d} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ en $\vec{n} = \frac{1}{2}\left(\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} 5\frac{1}{2} \\ 5\frac{1}{2} \end{pmatrix}$.

Dus $N(5\frac{1}{2}, 5\frac{1}{2})$.

17 $\vec{q} = \vec{a} + \overrightarrow{AB} + \overrightarrow{BP} + \overrightarrow{PQ} = \vec{a} + \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{AB} = \vec{a} + 1\frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC}$

$\overrightarrow{BC} = \overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ en $\overrightarrow{AB} = \overrightarrow{AD}_R = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\vec{q} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 1\frac{1}{3}\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 + 5\frac{1}{3} - 1 \\ 0 + 4 + 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix}$

$\vec{r} = \vec{q} + \frac{1}{3}\overrightarrow{BC} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \begin{pmatrix} -1 \\ 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 6\frac{1}{3} \\ 6\frac{2}{3} \end{pmatrix}$

Dus $Q(7\frac{1}{3}, 5\frac{1}{3})$ en $R(6\frac{1}{3}, 6\frac{2}{3})$.

18 **a** $\vec{p} = \vec{a} + \overrightarrow{AP} = \vec{a} + \overrightarrow{AC}_R$

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{AD}_R$

$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ d \end{pmatrix}$ en $\overrightarrow{AD}_R = \begin{pmatrix} d \\ a \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -a \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} -a + d \\ a + d \end{pmatrix}$ en $\overrightarrow{AC}_R = \begin{pmatrix} a + d \\ a - d \end{pmatrix}$

$\vec{p} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} a + d \\ a - d \end{pmatrix} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix}$

Dus $P(2a + d, a - d)$.

b $\vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{AC} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix} + \begin{pmatrix} -a + d \\ a + d \end{pmatrix} = \begin{pmatrix} a + 2d \\ 2a \end{pmatrix}$

Dus $Q(a + 2d, 2a)$.

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Bladzijde 63

19 **a** $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c})$ met $\vec{c} = \vec{d} + \overrightarrow{DC} = \vec{d} + \overrightarrow{AD}_R$

$\overrightarrow{AD} = \begin{pmatrix} -a \\ d \end{pmatrix}$ en $\overrightarrow{DC} = \overrightarrow{AD}_R = \begin{pmatrix} d \\ a \end{pmatrix}$

$\vec{c} = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a + d \end{pmatrix}$

$\vec{m} = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ a + d \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a + d \\ a + d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}d \\ \frac{1}{2}a + \frac{1}{2}d \end{pmatrix}$

Dus $M(\frac{1}{2}a + \frac{1}{2}d, \frac{1}{2}a + \frac{1}{2}d)$.

Voor alle waarden van a en d is $x_M = y_M$, dus M ligt op de lijn $y = x$.

b $OM = \sqrt{(\frac{1}{2}a + \frac{1}{2}d)^2 + (\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{4}a^2 + \frac{1}{2}ad + \frac{1}{4}d^2)}$

$= \sqrt{\frac{1}{2}(a^2 + 2ad + d^2)} = \sqrt{\frac{1}{2}(a + d)^2} = \sqrt{\frac{1}{2}} \cdot (a + d) = \frac{1}{2}\sqrt{2} \cdot (a + d)$

20 $\vec{t} = \vec{b} + \overrightarrow{BS} + \overrightarrow{ST} = \vec{b} + \overrightarrow{BA}_L + \overrightarrow{SP}_R$ en $\vec{u} = \vec{t} + \overrightarrow{TU} = \vec{t} + \overrightarrow{SP}$

$\overrightarrow{BA} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a - b \\ -c \end{pmatrix}$, dus $\overrightarrow{BA}_L = \begin{pmatrix} c \\ a - b \end{pmatrix}$.

$\overrightarrow{SP} = \overrightarrow{BP} - \overrightarrow{BS} = \vec{b}_L - \overrightarrow{BA}_L = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} c \\ a - b \end{pmatrix} = \begin{pmatrix} -2c \\ -a + 2b \end{pmatrix}$, dus $\overrightarrow{SP}_R = \begin{pmatrix} -a + 2b \\ 2c \end{pmatrix}$.

$\vec{t} = \begin{pmatrix} b \\ c \end{pmatrix} + \begin{pmatrix} c \\ a - b \end{pmatrix} + \begin{pmatrix} -a + 2b \\ 2c \end{pmatrix} = \begin{pmatrix} -a + 3b + c \\ a - b + 3c \end{pmatrix}$

$\vec{u} = \begin{pmatrix} -a + 3b + c \\ a - b + 3c \end{pmatrix} + \begin{pmatrix} -2c \\ -a + 2b \end{pmatrix} = \begin{pmatrix} -a + 3b - c \\ b + 3c \end{pmatrix}$

Dus $T(-a + 3b + c, a - b + 3c)$ en $U(-a + 3b - c, b + 3c)$.

21 $\vec{n} = \vec{m} + \overrightarrow{MN} = \vec{m} + \overrightarrow{AB}$

$\overrightarrow{AD} = \begin{pmatrix} 0 \\ 2a \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ 2a \end{pmatrix}$ en $\overrightarrow{AB} = \overrightarrow{AD}_R = \begin{pmatrix} 2a \\ a \end{pmatrix}$

$\vec{m} = \frac{1}{2}(\vec{d} + \vec{b}) = \frac{1}{2}(\vec{d} + \vec{a} + \overrightarrow{AB}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 2a \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 3a \\ 3a \end{pmatrix} = \begin{pmatrix} 1\frac{1}{2}a \\ 1\frac{1}{2}a \end{pmatrix}$

$\vec{n} = \begin{pmatrix} 1\frac{1}{2}a \\ 1\frac{1}{2}a \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} = \begin{pmatrix} 3\frac{1}{2}a \\ 2\frac{1}{2}a \end{pmatrix}$

$\left.\begin{array}{l} y = mx \\ \text{door } N(3\frac{1}{2}a, 2\frac{1}{2}a) \end{array}\right\} 3\frac{1}{2}am = 2\frac{1}{2}a$

$m = \frac{5}{7}$

Bladzijde 64

22 **a** $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} b \\ c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} b - a \\ c \end{pmatrix}$

$\vec{d} = \vec{c}_L = \begin{pmatrix} -c \\ b \end{pmatrix}$ en $\vec{b} = \vec{a}_L = \begin{pmatrix} 0 \\ a \end{pmatrix}$ geeft $\overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} -c \\ b - a \end{pmatrix}$.

b Uit vraag a volgt $\overrightarrow{BD} = \overrightarrow{AC}_L$, dus $AC = BD$ en $AC \perp BD$.

23 $M(\frac{1}{2}a, 0)$

$\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b - a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a - b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a + b + c \\ a - b + c \end{pmatrix}$

$\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_L = \frac{1}{2}\begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b - c \\ b + c \end{pmatrix}$

$\overrightarrow{MP} = \vec{p} - \vec{m} = \frac{1}{2}\begin{pmatrix} a + b + c \\ a - b + c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b + c \\ a - b + c \end{pmatrix}$

$\overrightarrow{MQ} = \vec{q} - \vec{m} = \frac{1}{2}\begin{pmatrix} b - c \\ b + c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a + b - c \\ b + c \end{pmatrix}$

$\overrightarrow{MQ} = \overrightarrow{MP}_L$, dus $MP = MQ$ en $MP \perp MQ$.

Dus $\triangle MPQ$ is een gelijkbenige rechthoekige driehoek.

24 $\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AB}_{\text{R}} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$

$\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_{\text{L}} = \frac{1}{2}\begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix}$

$\vec{r} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_{\text{R}} = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}$

$\overrightarrow{QR} = \vec{r} - \vec{q} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a-b+c \\ -a-b-c \end{pmatrix}$

$\overrightarrow{QR} = \overrightarrow{OP}_{\text{L}}$, dus $OP \perp QR$.

$\overrightarrow{AQ} = \vec{q} - \vec{a} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a+\frac{1}{2}b-\frac{1}{2}c \\ \frac{1}{2}b+\frac{1}{2}c \end{pmatrix}$

$\overrightarrow{PR} = \vec{r} - \vec{p} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}b-\frac{1}{2}c \\ -a+\frac{1}{2}b-\frac{1}{2}c \end{pmatrix}$

$\overrightarrow{PR} = \overrightarrow{AQ}_{\text{L}}$, dus $AQ \perp PR$.

$\overrightarrow{BR} = \vec{r} - \vec{b} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a-b \\ -\frac{1}{2}a-c \end{pmatrix}$

$\overrightarrow{PQ} = \vec{q} - \vec{p} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}a-c \\ -\frac{1}{2}a+b \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{BR}_{\text{R}}$, dus $BR \perp PQ$.

Bladzijde 65

25 **a** $\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AB}_{\text{R}} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$

$\vec{q} = \vec{c} + \frac{1}{2}\overrightarrow{CB} + \frac{1}{2}\overrightarrow{CB}_{\text{L}} = \begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-d \\ c-e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c+e \\ b-d \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix}$

$\vec{r} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{c}_{\text{L}} = \frac{1}{2}\begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -e \\ d \end{pmatrix} = \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix}$

$\vec{s} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_{\text{R}} = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}$

b $\overrightarrow{PR} = \vec{r} - \vec{p} = \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a-b-c+d-e \\ -a+b-c+d+e \end{pmatrix}$

$\overrightarrow{QS} = \vec{s} - \vec{q} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a-b+c-d-e \\ -a-b-c+d-e \end{pmatrix}$

$\overrightarrow{QS} = \overrightarrow{PR}_{\text{L}}$, dus $PR = QS$ en $PR \perp QS$.

26 $\vec{k} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix}$

$\vec{l} = \frac{1}{2}(\vec{q} + \vec{r}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix}$

$\vec{m} = \frac{1}{2}(\vec{r} + \vec{s}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix}$

$\vec{n} = \frac{1}{2}(\vec{p} + \vec{s}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix}$

$\overrightarrow{KM} = \vec{m} - \vec{k} = \frac{1}{4}\begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix} - \frac{1}{4}\begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -2b-2e \\ -2a-2c+2d \end{pmatrix}$

$\overrightarrow{LN} = \vec{n} - \vec{l} = \frac{1}{4}\begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix} - \frac{1}{4}\begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2a+2c-2d \\ -2b-2e \end{pmatrix}$

$\overrightarrow{LN} = \overrightarrow{KM}_{\text{L}}$, dus $KM = LN$ en $KM \perp LN$.

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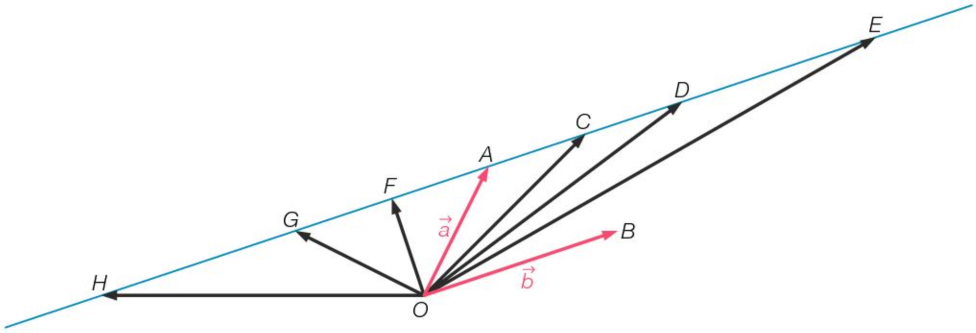
10.3 Vectoren en lijnen

Bladzijde 67

27 **a**



b De eindpunten van de in a getekende vectoren liggen op de lijn door het eindpunt van de vector $\vec{a}$ en die evenwijdig is met de vector $\vec{b}$.

Bladzijde 69

28 **a** Je kunt ook $\vec{q}$ als steunvector nemen.

$$\left.\begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{q} + t \cdot (\vec{q} - \vec{p}) \\ \vec{q} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \text{ en } \vec{q} - \vec{p} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{array}\right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

b $\begin{pmatrix} 2 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ is ook een richtingsvector van l.

$t = -\frac{1}{2}$ geeft $\begin{pmatrix} 5 \\ 6 \end{pmatrix} + -\frac{1}{2} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, dus (4, 5) is een punt van l en dus is ook $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ een steunvector van l.

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

c (3, 4) op l.
$x = 3$ geeft $6 + t = 3$, dus $t = -3$.
$t = -3$ en $y = 4$ geeft $p - 3 \cdot 1 = 4$, dus $p = 7$.

29 **a**

$$\left.\begin{array}{l} k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{a}) \\ \vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 6 \\ 1 \end{pmatrix} \end{array}\right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

b

$$\left.\begin{array}{l} l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t \cdot (\vec{d} - \vec{c}) \\ \vec{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{array}\right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

c

$$\left.\begin{array}{l} m: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{e} + t \cdot (\vec{f} - \vec{e}) \\ \vec{e} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ en } \vec{f} - \vec{e} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{array}\right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

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30 $x = 7$ geeft $-2 + 3t = 7$

$3t = 9$

$t = 3$

$t = 3$ geeft $y = 5 - 4 \cdot 3 = 5 - 12 = -7$

Dus A ligt op k.

$x = -13$ geeft $-2 + 3t = -13$

$3t = -11$

$t = -3\frac{2}{3}$

$t = -3\frac{2}{3}$ geeft $y = 5 - 4 \cdot -3\frac{2}{3} = 5 + 14\frac{2}{3} = 19\frac{2}{3}$

Dus B ligt niet op k.

$x = -3\frac{1}{2}$ geeft $-2 + 3t = -3\frac{1}{2}$

$3t = -1\frac{1}{2}$

$t = -\frac{1}{2}$

$t = -\frac{1}{2}$ geeft $y = 5 - 4 \cdot -\frac{1}{2} = 5 + 2 = 7$

Dus C ligt op k.

31 lijn k

t	0	1
punt	(1, 2)	(4, 3)

lijn l

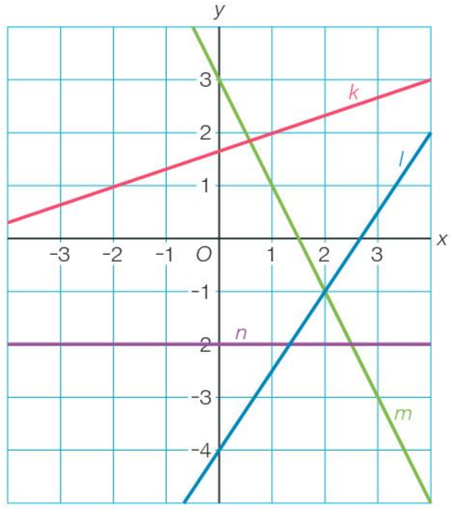
u	0	1
punt	(2, −1)	(4, 2)

lijn m

v	0	1
punt	(0, 3)	(1, 1)

lijn n

w	0	1
punt	(1, −2)	(2, −2)

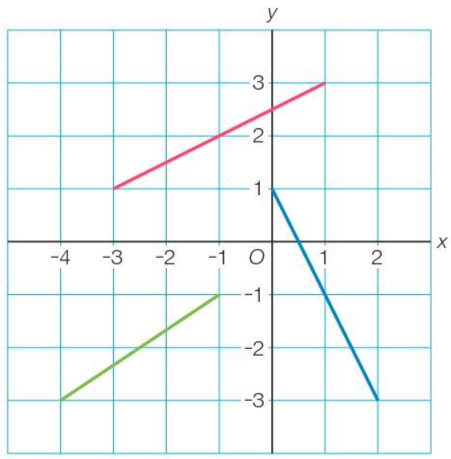


32

t	−1	1
punt	(−3, 1)	(1, 3)

u	1	3
punt	(0, 1)	(2, −3)

v	−1	0
punt	(−4, −3)	(−1, −1)



33 **a** $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{c})$

$\vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ en $\vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

$\Bigg\} \; k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

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b $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}-5\\-2\end{pmatrix}\right) = \frac{1}{2} \cdot \begin{pmatrix}-6\\0\end{pmatrix} = \begin{pmatrix}-3\\0\end{pmatrix}$

$$\left.\begin{array}{l} l\colon \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + t \cdot (\vec{b} - \vec{m}) \\ \vec{b} = \begin{pmatrix}3\\4\end{pmatrix} \text{ en } \vec{b} - \vec{m} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}-3\\0\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix} \end{array}\right\} l\colon \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} + t \cdot \begin{pmatrix}6\\4\end{pmatrix}$$

c $\vec{n} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}\right) = \frac{1}{2} \cdot \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$

$$\left.\begin{array}{l} m\colon \begin{pmatrix}x\\y\end{pmatrix} = \vec{n} + t \cdot (\vec{a} - \vec{c}) \\ \vec{n} = \begin{pmatrix}1\\3\end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}-5\\-2\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix} \end{array}\right\} m\colon \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} + t \cdot \begin{pmatrix}4\\4\end{pmatrix}$$

d Stel eerst een vectorvoorstelling op van de lijn door BC.

$$\left.\begin{array}{l} \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + t \cdot (\vec{b} - \vec{c}) \\ \vec{b} = \begin{pmatrix}3\\4\end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}-5\\-2\end{pmatrix} = \begin{pmatrix}8\\6\end{pmatrix} \end{array}\right\} \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} + t \cdot \begin{pmatrix}8\\6\end{pmatrix}$$

Dus een vectorvoorstelling van het lijnstuk BC is $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} + t \cdot \begin{pmatrix}8\\6\end{pmatrix} \wedge -1 \le t \le 0$.

Bladzijde 70

34 **a** De lijn door de middens van AD en BC is evenwijdig met AB en CD en heeft dus dezelfde richting als AB en CD. Dus een richtingsvector van deze lijn is $\overrightarrow{DC} = \vec{c} - \vec{d}$.
De vector vanuit O naar het midden van BC is $\frac{1}{2}(\vec{b} + \vec{c})$. Dus $\frac{1}{2}(\vec{b} + \vec{c})$ is een steunvector van de lijn door de middens van AD en BC. Dus $\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}(\vec{b} + \vec{c}) + t \cdot (\vec{c} - \vec{d})$ is een vectorvoorstelling van de lijn door de middens van AD en BC.

b
- $k\colon \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{d})$
- $l\colon \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + u \cdot (\frac{1}{2}(\vec{c} + \vec{d}) - \vec{b})$
- $m\colon \begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}(\vec{a} + \vec{c}) + v \cdot (\vec{c} - \vec{b})$

c I gaat door B en is evenwijdig met AC.
II gaat door het midden van AB en is evenwijdig met BD.
III gaat door A en het midden van BC.

35 **a** Substitutie van $x = 1 - 3t$ en $y = 2 + t$ in $y = 4x - 28$ geeft $2 + t = 4(1 - 3t) - 28$
$2 + t = 4 - 12t - 28$
$13t = -26$
$t = -2$
$t = -2$ geeft $x = 1 - 2 \cdot -3 = 7$ en $y = 2 - 2 \cdot 1 = 0$
Dus $S(7, 0)$.

b Substitutie van $x = 7 + 3u$ en $y = -3 + u$ in $2x - 5y = 11$ geeft $2(7 + 3u) - 5(-3 + u) = 11$
$14 + 6u + 15 - 5u = 11$
$u = -18$
$u = -18$ geeft $x = 7 - 18 \cdot 3 = -47$ en $y = -3 - 18 \cdot 1 = -21$
Dus $T(-47, -21)$.

36 **a** Substitutie van $x = 3t$ en $y = 2 + 2t$ in $2x - 5y = 6$ geeft $2 \cdot 3t - 5(2 + 2t) = 6$
$6t - 10 - 10t = 6$
$-4t = 16$
$t = -4$
$t = -4$ geeft $x = -4 \cdot 3 = -12$ en $y = 2 - 4 \cdot 2 = -6$
Dus het snijpunt is $(-12, -6)$.

b Substitutie van $x = 2 - u$ en $y = -5 + 3u$ in $3x + 4y = 10$ geeft $3(2 - u) + 4(-5 + 3u) = 10$

$$6 - 3u - 20 + 12u = 10$$
$$9u = 24$$
$$u = 2\tfrac{2}{3}$$

$u = 2\frac{2}{3}$ geeft $x = 2 + 2\frac{2}{3} \cdot -1 = -\frac{2}{3}$ en $y = -5 + 2\frac{2}{3} \cdot 3 = 3$

Dus het snijpunt is $(-\frac{2}{3}, 3)$.

37 **a** $\begin{cases} x = 4 + 2t \\ y = 1 - 5t \end{cases} \begin{vmatrix} 5 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 5x = 20 + 10t \\ 2y = 2 - 10t \end{cases}$ +

$5x + 2y = 22$

Dus k: $5x + 2y = 22$.

b $\begin{cases} x = 3 - t \\ y = 2 + 3t \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 3x = 9 - 3t \\ y = 2 + 3t \end{cases}$ +

$3x + y = 11$

Dus l: $3x + y = 11$.

c $\begin{cases} x = 3t - 1 \\ y = 5t + 1 \end{cases} \begin{vmatrix} 5 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 5x = 15t - 5 \\ 3y = 15t + 3 \end{cases}$ −

$5x - 3y = -8$

Dus m: $5x - 3y = -8$.

Bladzijde 71

38 **a** Neem je $x = 4t$ in $y = 2x + 3$, dan krijg je $y = 2 \cdot 4t + 3$ oftewel $y = 8t + 3$.

Dus k: $x = 4t \wedge y = 8t + 3$ is een parametervoorstelling van k.

b $x = t + 5$ geeft $y = 2 \cdot (t + 5) + 3$

$$y = 2t + 10 + 3$$
$$y = 2t + 13$$

Dus k: $x = t + 5 \wedge y = 2t + 13$.

c $x = -3t + 2$ geeft $y = 2 \cdot (-3t + 2) + 3$

$$y = -6t + 4 + 3$$
$$y = -6t + 7$$

Dus k: $x = -3t + 2 \wedge y = -6t + 7$.

d $y = 4t$ geeft $4t = 2x + 3$

$$-2x = -4t + 3$$
$$x = 2t - 1\tfrac{1}{2}$$

Dus k: $x = 2t - 1\frac{1}{2} \wedge y = 4t$.

39 **a** $\begin{cases} x = 5 + 4t \\ y = 3 - 2t \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} x = 5 + 4t \\ 2y = 6 - 4t \end{cases}$ +

$x + 2y = 11$

Dus k: $x + 2y = 11$.

b Neem $x = 2t$ in $3x - 2y = 6$.

Dit geeft $3 \cdot 2t - 2y = 6$

$$6t - 2y = 6$$
$$-2y = -6t + 6$$
$$y = 3t - 3$$

Dus l: $x = 2t \wedge y = 3t - 3$.

c m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -9 \\ 2 \end{pmatrix}$

d n: $x = 4 + 3t \wedge y = -1 + 2t$

40 **a** $\begin{cases} x = 5t + p \\ y = 4t + 3 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 4x = 20t + 4p \\ 5y = 20t + 15 \end{cases}$ −

$4x - 5y = 4p - 15$

Er moet gelden $4p - 15 = -9$ oftewel $4p = 6$, dus $p = 1\frac{1}{2}$.

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b $\begin{cases} x = 2t - 1 \\ y = -3t + 5 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 3x = 6t - 3 \\ 2y = -6t + 10 \end{cases}$ +

$3x + 2y = 7$

$3x + 2y = 7$
$2y = -3x + 7$
$y = -1\frac{1}{2}x + 3\frac{1}{2}$

41 **a** Substitutie van $x = 4 + 2t$ en $y = 5 + 3t$ in $x - 2y = 10$ geeft $4 + 2t - 2(5 + 3t) = 10$
$4 + 2t - 10 - 6t = 10$
$-4t = 16$
$t = -4$

$t = -4$ geeft $x = 4 - 4 \cdot 2 = -4$ en $y = 5 - 4 \cdot 3 = -7$
Dus $S(-4, -7)$.

b $\begin{cases} -2 + 3u = -3a + 1 \\ 5 - 4u = 2a \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix}$ geeft $\begin{cases} -8 + 12u = -12a + 4 \\ 15 - 12u = 6a \end{cases}$ +

$7 = -6a + 4$
$6a = -3$
$a = -\frac{1}{2}$

Dus $A(-3 \cdot -\frac{1}{2} + 1, 2 \cdot -\frac{1}{2}) = A(2\frac{1}{2}, -1)$.

c $\begin{cases} 2v + q = 5 \\ -v - 2q = 2 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 2v + q = 5 \\ -2v - 4q = 4 \end{cases}$ +

$-3q = 9$
$q = -3$

42 **a** (3, 4) op l_c: $2x + 5y = c$ geeft $c = 2 \cdot 3 + 5 \cdot 4 = 26$, dus l: $2x + 5y = 26$.
(3, 4) op $k_{a,b}$: $x = at - 3 \wedge y = bt + 1$ geeft $at - 3 = 3$ oftewel $at = 6$ en $bt + 1 = 4$ oftewel $bt = 3$.
Dus $at = 2bt$ oftewel $a = 2b$.
Kies bijvoorbeeld $b = 1$, dan is $a = 2$.
Dit geeft k: $x = 2t - 3 \wedge y = t + 1$.

$\begin{cases} x = 2t - 3 \\ y = t + 1 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} x = 2t - 3 \\ 2y = 2t + 2 \end{cases}$ −

$x - 2y = -5$

Omdat $\frac{2}{1} \neq \frac{5}{-2}$ snijden k: $x - 2y = -5$ en l: $2x + 5y = 26$ elkaar en vallen ze niet samen.
Dus mogelijke waarden zijn $a = 2$, $b = 1$ en $c = 26$.

b $\begin{cases} x = at - 3 \\ y = bt + 1 \end{cases} \begin{vmatrix} b \\ a \end{vmatrix}$ geeft $\begin{cases} bx = abt - 3b \\ ay = abt + a \end{cases}$ −

$bx - ay = -a - 3b$

$k_{a,b}$: $bx - ay = -a - 3b$ en l_c: $2x + 5y = c$ vallen samen als geldt $\frac{b}{2} = \frac{-a}{5} = \frac{-a - 3b}{c}$.

Uit $\frac{b}{2} = \frac{-a}{5}$ volgt $5b = -2a$.
Kies bijvoorbeeld $a = 5$, dan is $b = -2$.

$\left.\begin{array}{l} \frac{-a}{5} = \frac{-a - 3b}{c} \\ a = 5 \text{ en } b = -2 \end{array}\right\}$ $\frac{-5}{5} = \frac{-5 - 3 \cdot -2}{c}$

$\frac{-5}{5} = \frac{1}{c}$
$c = -1$

Dus mogelijke waarden zijn $a = 5$, $b = -2$ en $c = -1$.

43 $x = -t - 3 \wedge y = pt - 8$ geeft $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ p \end{pmatrix}$

$\begin{pmatrix} -1 \\ p \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ geeft $p = -2\frac{1}{2}$

$(1, q)$ op $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -2\frac{1}{2} \end{pmatrix}$ geeft $-3 - t = 1$

$-t = 4$

$t = -4$

$t = -4$ geeft $q = -8 - 4 \cdot -2\frac{1}{2} = -8 + 10 = 2$

Dus $p = -2\frac{1}{2}$ en $q = 2$.

10.4 Vectoren en hoeken

Bladzijde 73

44 **a** $A(a_x, a_y)$, dus $\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ en $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$.

Kwadrateren geeft $|\vec{a}|^2 = a_x^2 + a_y^2$.

$B(b_x, b_y)$, dus $\vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$ en $|\vec{b}| = \sqrt{b_x^2 + b_y^2}$.

Kwadrateren geeft $|\vec{b}|^2 = b_x^2 + b_y^2$.

b Voor de punten $A(x_A, y_A)$ en $B(x_B, y_B)$ geldt $d(A, B) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$.

Dus voor de punten $A(a_x, a_y)$ en $B(b_x, b_y)$ geldt $d(A, B) = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.

De lengte van het lijnstuk AB wordt behalve als $d(A, B)$ ook genoteerd als AB, dus $AB = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.

Kwadrateren geeft $AB^2 = (b_x - a_x)^2 + (b_y - a_y)^2$

$= b_x^2 - 2a_xb_x + a_x^2 + b_y^2 - 2a_yb_y + a_y^2$

$= a_x^2 + a_y^2 + b_x^2 + b_y^2 - 2a_xb_x - 2a_yb_y$

Bladzijde 75

45 **a** $\angle(k, l) \approx 180° - 108{,}4° = 71{,}6°$

b $\cos(\angle(\vec{r}_m, \vec{r}_n)) = \dfrac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -3 \\ 1 \end{pmatrix}\right|} = \dfrac{2 \cdot -3 + 3 \cdot 1}{\sqrt{13} \cdot \sqrt{10}} = \dfrac{-3}{\sqrt{130}}$

Dus $\angle(\vec{r}_m, \vec{r}_n) \approx 105{,}3°$.

46 **a** $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -6 \end{pmatrix} = 2 \cdot -1 + 3 \cdot -6 = -2 - 18 = -20$

b $\vec{c} \cdot \vec{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = -1 \cdot 2 + 0 \cdot 4 = -2 + 0 = -2$

c $\vec{e} \cdot \vec{f} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \cdot 3 + -3 \cdot 2 = 6 - 6 = 0$

d $\vec{g} \cdot \vec{h} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2\sqrt{2} \\ -\sqrt{3} \end{pmatrix} = \sqrt{2} \cdot 2\sqrt{2} + \sqrt{3} \cdot -\sqrt{3} = 4 - 3 = 1$

47 **a** $\cos(\angle(\vec{a}, \vec{b})) = \dfrac{\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right|} = \dfrac{3 \cdot 2 + 4 \cdot 1}{\sqrt{25} \cdot \sqrt{5}} = \dfrac{10}{5\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

Dus $\angle(\vec{a}, \vec{b}) \approx 26{,}6°$.

b $\cos(\angle(\vec{c}, \vec{d})) = \dfrac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 3 \end{pmatrix}\right|} = \dfrac{4 \cdot -1 + -1 \cdot 3}{\sqrt{17} \cdot \sqrt{10}} = \dfrac{-7}{\sqrt{170}}$

Dus $\angle(\vec{c}, \vec{d}) \approx 125{,}5°$.

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c $\cos(\angle(\vec{e}, \vec{f})) = \dfrac{\begin{pmatrix}0\\3\end{pmatrix} \cdot \begin{pmatrix}2\\5\end{pmatrix}}{\left|\begin{pmatrix}0\\3\end{pmatrix}\right| \cdot \left|\begin{pmatrix}2\\5\end{pmatrix}\right|} = \dfrac{0 \cdot 2 + 3 \cdot 5}{3 \cdot \sqrt{29}} = \dfrac{15}{3\sqrt{29}} = \dfrac{5}{\sqrt{29}}$

Dus $\angle(\vec{e}, \vec{f}) \approx 21{,}8°$.

d $\cos(\angle(\vec{g}, \vec{h})) = \dfrac{\begin{pmatrix}2\\-5\end{pmatrix} \cdot \begin{pmatrix}5\\2\end{pmatrix}}{\left|\begin{pmatrix}2\\-5\end{pmatrix}\right| \cdot \left|\begin{pmatrix}5\\2\end{pmatrix}\right|} = \dfrac{2 \cdot 5 + -5 \cdot 2}{\sqrt{29} \cdot \sqrt{29}} = \dfrac{0}{29} = 0$

Dus $\angle(\vec{g}, \vec{h}) = 90°$.

48 **a** $\cos(\angle(k, l)) = \dfrac{\left|\begin{pmatrix}1\\2\end{pmatrix} \cdot \begin{pmatrix}2\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}1\\2\end{pmatrix}\right| \cdot \left|\begin{pmatrix}2\\1\end{pmatrix}\right|} = \dfrac{|1 \cdot 2 + 2 \cdot 1|}{\sqrt{5} \cdot \sqrt{5}} = \dfrac{4}{5}$

Dus $\angle(k, l) \approx 36{,}9°$.

b $\cos(\angle(m, n)) = \dfrac{\left|\begin{pmatrix}3\\1\end{pmatrix} \cdot \begin{pmatrix}-1\\2\end{pmatrix}\right|}{\left|\begin{pmatrix}3\\1\end{pmatrix}\right| \cdot \left|\begin{pmatrix}-1\\2\end{pmatrix}\right|} = \dfrac{|3 \cdot -1 + 1 \cdot 2|}{\sqrt{10} \cdot \sqrt{5}} = \dfrac{1}{\sqrt{50}}$

Dus $\angle(m, n) \approx 81{,}9°$.

c $\cos(\angle(p, q)) = \dfrac{\left|\begin{pmatrix}-1\\1\end{pmatrix} \cdot \begin{pmatrix}2\\1\end{pmatrix}\right|}{\left|\begin{pmatrix}-1\\1\end{pmatrix}\right| \cdot \left|\begin{pmatrix}2\\1\end{pmatrix}\right|} = \dfrac{|-1 \cdot 2 + 1 \cdot 1|}{\sqrt{2} \cdot \sqrt{5}} = \dfrac{1}{\sqrt{10}}$

Dus $\angle(p, q) \approx 71{,}6°$.

Bladzijde 76

49 **a** $\cos(\angle(k, l)) = \dfrac{\left|\begin{pmatrix}3\\-1\end{pmatrix} \cdot \begin{pmatrix}3\\5\end{pmatrix}\right|}{\left|\begin{pmatrix}3\\-1\end{pmatrix}\right| \cdot \left|\begin{pmatrix}3\\5\end{pmatrix}\right|} = \dfrac{|3 \cdot 3 + -1 \cdot 5|}{\sqrt{10} \cdot \sqrt{34}} = \dfrac{4}{\sqrt{340}}$

Dus $\angle(k, l) \approx 77{,}5°$.

b $n\colon x = 3u - 2 \wedge y = 10u + 3$ oftewel $n\colon \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2\\3\end{pmatrix} + u \cdot \begin{pmatrix}3\\10\end{pmatrix}$

$\cos(\angle(m, n)) = \dfrac{\left|\begin{pmatrix}2\\-1\end{pmatrix} \cdot \begin{pmatrix}3\\10\end{pmatrix}\right|}{\left|\begin{pmatrix}2\\-1\end{pmatrix}\right| \cdot \left|\begin{pmatrix}3\\10\end{pmatrix}\right|} = \dfrac{|2 \cdot 3 + -1 \cdot 10|}{\sqrt{5} \cdot \sqrt{109}} = \dfrac{4}{\sqrt{545}}$

Dus $\angle(m, n) \approx 80{,}1°$.

50 **a** $\overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix}4\\5\end{pmatrix} - \begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$

$\cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) = \dfrac{\begin{pmatrix}-2\\-2\end{pmatrix} \cdot \begin{pmatrix}-1\\3\end{pmatrix}}{\left|\begin{pmatrix}-2\\-2\end{pmatrix}\right| \cdot \left|\begin{pmatrix}-1\\3\end{pmatrix}\right|} = \dfrac{-2 \cdot -1 + -2 \cdot 3}{\sqrt{8} \cdot \sqrt{10}} = \dfrac{-4}{\sqrt{80}}$

$\angle(\overrightarrow{BA}, \overrightarrow{BC}) \approx 116{,}6°$

Dus $\angle B \approx 116{,}6°$.

Alternatieve uitwerking

$\tan(\alpha) = \text{rc}_{AB} = 1$ geeft $\alpha = 45°$

$\tan(\beta) = \text{rc}_{BC} = -3$ geeft $\beta = -71{,}56...°$

$\alpha - \beta = 45° - -71{,}56...° = 116{,}56...°$

Dus $\angle B \approx 116{,}6°$.

b $\vec{r}_{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

$\vec{r}_{BE} = \vec{e} - \vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$$\cos(\angle(AD, BE)) = \frac{\left|\begin{pmatrix} -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} -1 \\ 6 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 5 \end{pmatrix}\right|} = \frac{|-1 \cdot -5 + 6 \cdot 1|}{\sqrt{37} \cdot \sqrt{26}} = \frac{11}{\sqrt{962}}$$

$\angle(AD, BE) \approx 69{,}2°$

Dus de hoek tussen de diagonalen AD en BE is $69{,}2°$.

51 **a** $\vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$\cos(\angle(AC, BC)) = \frac{\left|\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ -5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 4 \end{pmatrix}\right|} = \frac{|2 \cdot 1 + -5 \cdot 4|}{\sqrt{29} \cdot \sqrt{17}} = \frac{18}{\sqrt{493}}$$

Dus $\angle(AC, BC) \approx 35{,}8°$.

b $\overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) = \frac{\begin{pmatrix} -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix}}{\left|\begin{pmatrix} -3 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ 4 \end{pmatrix}\right|} = \frac{-3 \cdot -1 + 1 \cdot -4}{\sqrt{10} \cdot \sqrt{17}} = \frac{-1}{\sqrt{170}}$$

$\angle(\overrightarrow{BA}, \overrightarrow{BC}) \approx 94{,}4°$

Dus $\angle ABC \approx 94{,}4°$.

52 **a** $S(3, -1)$ en $(2, 1)$ op $k_{a,b}$ geeft $\vec{r}_{k_{a,b}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left|\begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -4 \end{pmatrix}\right|} = \frac{|1 \cdot 3 + -2 \cdot -4|}{\sqrt{5} \cdot \sqrt{25}} = \frac{11}{5\sqrt{5}}$$

Dus $\angle(k, l) \approx 10{,}3°$.

b $$\cos(\angle(k_{a,1}, l)) = \frac{\left|\begin{pmatrix} a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix}\right|}{\left|\begin{pmatrix} a \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 3 \\ -4 \end{pmatrix}\right|} = \frac{|a \cdot 3 + 1 \cdot -4|}{\sqrt{a^2 + 1} \cdot \sqrt{25}} = \frac{|3a - 4|}{5\sqrt{a^2 + 1}}$$

Voer in $y_1 = \frac{|3x - 4|}{5\sqrt{x^2 + 1}}$ en $y_2 = \cos(80)$.

De optie snijpunt geeft $x = 0{,}936...$ en $x = 1{,}973...$

Dus $a \approx 0{,}94 \vee a \approx 1{,}97$.

53 $\vec{a} \cdot \vec{b} = \begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} q \\ -p \end{pmatrix} = pq - pq = 0$

$\vec{a} \cdot \vec{b} = 0$, dus $\cos(\angle(\vec{a}, \vec{b})) = 0$ en hieruit volgt $\angle(\vec{a}, \vec{b}) = 90°$.

54 **a** Een richtingsvector gaat van het punt $(0, 2)$ naar het punt $(3, 0)$.

Dit is de vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Dus $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

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b De vector $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Dus $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op de lijn k.

c De vector $\vec{n}_l = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ staat loodrecht op de lijn l.

Bladzijde 78

55 **a** $\vec{r}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

$\left.\begin{array}{l} k\text{: } 4x - 3y = c \\ (2, -1) \text{ op } k \end{array}\right\} c = 4 \cdot 2 - 3 \cdot -1 = 11$

Dus k: $4x - 3y = 11$.

$\vec{r}_l = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.

$\left.\begin{array}{l} l\text{: } 7x + 4y = c \\ (0, 0) \text{ op } l \end{array}\right\} c = 0$

Dus l: $7x + 4y = 0$.

b $\vec{n}_m = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$(-3, 0)$ op m, dus $\vec{s}_m = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

Dus m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$\vec{n}_n = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$(0, 0)$ op n, dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Dus n: $\begin{pmatrix} x \\ y \end{pmatrix} = u \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

Bladzijde 79

56 **a** $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$\left.\begin{array}{l} l\text{: } 5x + 2y = c \\ \text{door } A(4, 1) \end{array}\right\} c = 5 \cdot 4 + 2 \cdot 1 = 22$

Dus l: $5x + 2y = 22$.

b $n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$\left.\begin{array}{l} n\text{: } 3x + 2y = c \\ \text{door } B(5, -1) \end{array}\right\} c = 3 \cdot 5 + 2 \cdot -1 = 13$

Dus n: $3x + 2y = 13$.

57 **a** $\vec{n}_k = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$A(5, 2)$ op l, dus $\vec{s}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

Dus l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

b $n \perp m$, dus $\vec{r}_n = \vec{n}_m = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

$B(1, -3)$ op n, dus $\vec{s}_n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

Dus n: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + u \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

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58 **a** $\begin{cases}3+t=-1+3u\\3+2t=5-4u\end{cases}$ oftewel $\begin{cases}t-3u=-4\\2t+4u=2\end{cases}\begin{vmatrix}2\\1\end{vmatrix}$ geeft $\begin{cases}2t-6u=-8\\2t+4u=2\end{cases}$ −

$-10u=-10$

$u=1$

$u=1$ geeft $x=-1+3=2 \wedge y=5-4=1$

Dus $S(2, 1)$.

b m door $A(-3, 5)$ en $B(3, -1)$, dus $\vec{r}_m=\begin{pmatrix}3--3\\-1-5\end{pmatrix}=\begin{pmatrix}6\\-6\end{pmatrix}\triangleq\begin{pmatrix}1\\-1\end{pmatrix}$.

m door $A(-3, 5)$, dus $\vec{s}_m=\begin{pmatrix}-3\\5\end{pmatrix}$.

Dus m: $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-3\\5\end{pmatrix}+t\cdot\begin{pmatrix}1\\-1\end{pmatrix}$.

n staat loodrecht op p: $-x+5y=4$, dus n: $5x+y=c$.

$\left.\begin{matrix}n\text{: } 5x+y=c\\ \text{door } C(-4, -2)\end{matrix}\right\} c=5\cdot -4+-2=-22$

Dus n: $5x+y=-22$.

Substitutie van $x=-3+t$ en $y=5-t$ in $5x+y=-22$ geeft $5(-3+t)+5-t=-22$

$-15+5t+5-t=-22$

$4t=-12$

$t=-3$

$t=-3$ geeft $x=-3+-3=-6$ en $y=5--3=8$

Dus $T(-6, 8)$.

c $\vec{r}_q=\begin{pmatrix}4\\-1\end{pmatrix}$, dus $\vec{n}_q=\begin{pmatrix}1\\4\end{pmatrix}$.

$\left.\begin{matrix}q\text{: } x+4y=c\\ \text{door } (5, 3)\end{matrix}\right\} c=5+4\cdot 3=17$

Dus q: $x+4y=17$.

Substitutie van $x=2w-4$ en $y=w-6$ in $x+4y=17$ geeft $2w-4+4(w-6)=17$

$2w-4+4w-24=17$

$6w=45$

$w=7\frac{1}{2}$

$w=7\frac{1}{2}$ geeft $x=2\cdot 7\frac{1}{2}-4=11$ en $y=7\frac{1}{2}-6=1\frac{1}{2}$

Dus $U(11, 1\frac{1}{2})$.

59 **a** Snijden van k met de x-as geeft $y=0$

$3+t=0$

$t=-3$

$t=-3$ geeft $x=-2+-3\cdot 5=-17$, dus $A(-17, 0)$.

$l\perp k$, dus $\vec{n}_l=\vec{r}_k=\begin{pmatrix}5\\1\end{pmatrix}$.

$\left.\begin{matrix}l\text{: } 5x+y=c\\ \text{door } A(-17, 0)\end{matrix}\right\} c=5\cdot -17+0=-85$

Dus l: $5x+y=-85$.

b $\vec{r}_m=\vec{b}-\vec{c}=\begin{pmatrix}2\\5\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$

$n /\!/ m$, dus $\vec{r}_n=\vec{r}_m=\begin{pmatrix}3\\1\end{pmatrix}$ en $(0, 0)$ op n, dus $\vec{s}_n=\begin{pmatrix}0\\0\end{pmatrix}$.

Dus n: $\begin{pmatrix}x\\y\end{pmatrix}=t\cdot\begin{pmatrix}3\\1\end{pmatrix}$.

c $\vec{r}_q=\vec{e}-\vec{o}=\begin{pmatrix}1\\-4\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix}$

$p\perp q$, dus $\vec{n}_q=\vec{r}_p=\begin{pmatrix}1\\-4\end{pmatrix}$.

$\left.\begin{matrix}q\text{: } x-4y=c\\ \text{door } D(-4, 7)\end{matrix}\right\} c=-4-4\cdot 7=-32$

Dus q: $x-4y=-32$.

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60 **a** $\vec{n}_l = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left|\begin{pmatrix} -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} -4 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 5 \\ 2 \end{pmatrix}\right|} = \frac{|-4 \cdot 5 + 1 \cdot 2|}{\sqrt{17} \cdot \sqrt{29}} = \frac{18}{\sqrt{493}}$$

Dus $\angle(k, l) \approx 35{,}8°$.

b $\vec{n}_m = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$ en $\vec{n}_n = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$$\cos(\angle(m, n)) = \frac{\left|\begin{pmatrix} 3 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 3 \\ -7 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right|} = \frac{|3 \cdot 1 + -7 \cdot 2|}{\sqrt{58} \cdot \sqrt{5}} = \frac{11}{\sqrt{290}}$$

Dus $\angle(m, n) = \angle(\vec{n}_m, \vec{n}_n) \approx 49{,}8°$.

c $\tan(\alpha) = \text{rc}_p = 2$ geeft $\alpha = 63{,}43...°$
$\tan(\beta) = \text{rc}_q = -1$ geeft $\beta = -45°$
$\alpha - \beta = 63{,}43...° - -45° = 108{,}43...°$
Dus $\angle(p, q) = 180 - 108{,}43...° \approx 71{,}6°$.

61 **a** $k_p \,//\, l_p$, dus $\vec{r}_{k_p} = \vec{r}_{l_p} = \begin{pmatrix} 2p \\ p+5 \end{pmatrix}$ en $\vec{n}_{l_p} = \begin{pmatrix} p+5 \\ -2p \end{pmatrix}$.

$\left.\begin{array}{l} l_p\colon (p+5)x - 2py = c \\ \text{door } B(-1, -2) \end{array}\right\} c = -(p+5) + 4p$

$\left.\begin{array}{l} l_p\colon (p+5)x - 2py = c \\ \text{door } C(p, 4) \end{array}\right\} c = (p+5)p - 8p$

$-(p+5) + 4p = (p+5)p - 8p$
$-p - 5 + 4p = p^2 + 5p - 8p$
$p^2 - 6p + 5 = 0$
$(p-1)(p-5) = 0$
$p = 1 \vee p = 5$
$p = 1$ geeft l_1: $(1+5)x - 2y = -(1+5) + 4$ oftewel l_1: $6x - 2y = -2$, dus l_1: $3x - y = -1$.
$p = 5$ geeft l_5: $(5+5)x - 2 \cdot 5y = -(5+5) + 4 \cdot 5$ oftewel l_5: $10x - 10y = 10$, dus l_5: $x - y = 1$.

b $D(-2, p-4)$ op m_p: $(p+3)x + (p-1)y = -12$ geeft $(p+3) \cdot -2 + (p-1)(p-4) = -12$
$-2p - 6 + p^2 - 4p - p + 4 = -12$
$p^2 - 7p + 10 = 0$
$(p-2)(p-5) = 0$
$p = 2 \vee p = 5$

$p = 2$ geeft $D(-2, -2)$, m_2: $5x + y = -12$ en $n_{2,q}$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_2 \perp n_{2,q}$, dus $\vec{n}_{m_2} = \vec{r}_{n_{2,q}}$, dus $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $5 = qu$ en $1 = u(3-q)$
$1 = 3u - qu$
$1 = 3u - 5$
$3u = 6$
$u = 2$

$5 = qu$ en $u = 2$ geeft $q = 2\frac{1}{2}$.

$p = 5$ geeft $D(-2, 1)$, m_5: $8x + 4y = -12$ oftewel m_5: $2x + y = -3$ en $n_{5,q}$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_5 \perp n_{5,q}$, dus $\vec{n}_{m_5} = \vec{r}_{n_{5,q}}$, dus $\begin{pmatrix} 2 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $2 = qu$ en $1 = u(3-q)$
$1 = 3u - qu$
$1 = 3u - 2$
$3u = 3$
$u = 1$

$2 = qu$ en $u = 1$ geeft $q = 2$.
Dus $(p = 2 \wedge q = 2\frac{1}{2}) \vee (p = 5 \wedge q = 2)$.

Bladzijde 80

62 **a** P op BD, dus $\vec{p} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix}$.

$\overrightarrow{CP} = \vec{p} - \vec{c} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix}$

b $CP \perp BD$, dus $\overrightarrow{CP} \cdot \vec{r}_{BD} = 0$

$\begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$

$t - 1 + 9t + 12 = 0$

$10t = -11$

$t = -1\frac{1}{10}$

$t = -1\frac{1}{10}$ geeft $P(2 - 1\frac{1}{10}, 5 + 3 \cdot -1\frac{1}{10}) = P(\frac{9}{10}, 1\frac{7}{10})$

c $CP = OP$, dus $|\overrightarrow{CP}| = |\overrightarrow{OP}|$

$\left|\begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix}\right| = \left|\begin{pmatrix} 2+t \\ 5+3t \end{pmatrix}\right|$

$\sqrt{(t-1)^2 + (3t+4)^2} = \sqrt{(2+t)^2 + (5+3t)^2}$

kwadrateren geeft

$(t-1)^2 + (3t+4)^2 = (2+t)^2 + (5+3t)^2$

$t^2 - 2t + 1 + 9t^2 + 24t + 16 = 4 + 4t + t^2 + 25 + 30t + 9t^2$

$-12t = 12$

$t = -1$

$t = -1$ geeft $P(2 + -1, 5 + 3 \cdot -1) = P(1, 2)$

63 **a** $y = 0$ geeft $4x = 12$, dus $x = 3$ en $A(3, 0)$.

$x = 0$ geeft $3y = 12$, dus $y = 4$ en $B(0, 4)$.

P op l, dus $\vec{p} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix}$.

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix}$

$\overrightarrow{BP} = \vec{p} - \vec{b} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix}$

$AP \perp BP$, dus $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$

$\begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \cdot \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} = 0$

$4t^2 + 12t + 6t + 18 + 9t^2 + 15t + 3t + 5 = 0$

$13t^2 + 36t + 23 = 0$

$D = 36^2 - 4 \cdot 13 \cdot 23 = 100$

$t = \dfrac{-36 + 10}{26} = -1 \vee t = \dfrac{-36 - 10}{26} = -1\frac{10}{13}$

$t = -1$ geeft $P(6 + 2 \cdot -1, -1 - 3 \cdot -1) = P(4, 2)$

$t = -1\frac{10}{13}$ geeft $P(6 + 2 \cdot -1\frac{10}{13}, -1 - 3 \cdot -1\frac{10}{13}) = P(2\frac{6}{13}, 4\frac{4}{13})$

b $AP = BP$, dus $|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$\left|\begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix}\right| = \left|\begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix}\right|$

$\sqrt{(2t+3)^2 + (-3t-1)^2} = \sqrt{(2t+6)^2 + (-3t-5)^2}$

kwadrateren geeft

$(2t+3)^2 + (-3t-1)^2 = (2t+6)^2 + (-3t-5)^2$

$4t^2 + 12t + 9 + 9t^2 + 6t + 1 = 4t^2 + 24t + 36 + 9t^2 + 30t + 25$

$-36t = 51$

$t = -1\frac{5}{12}$

$t = -1\frac{5}{12}$ geeft $P(6 + 2 \cdot -1\frac{5}{12}, -1 - 3 \cdot -1\frac{5}{12}) = P(3\frac{1}{6}, 3\frac{1}{4})$

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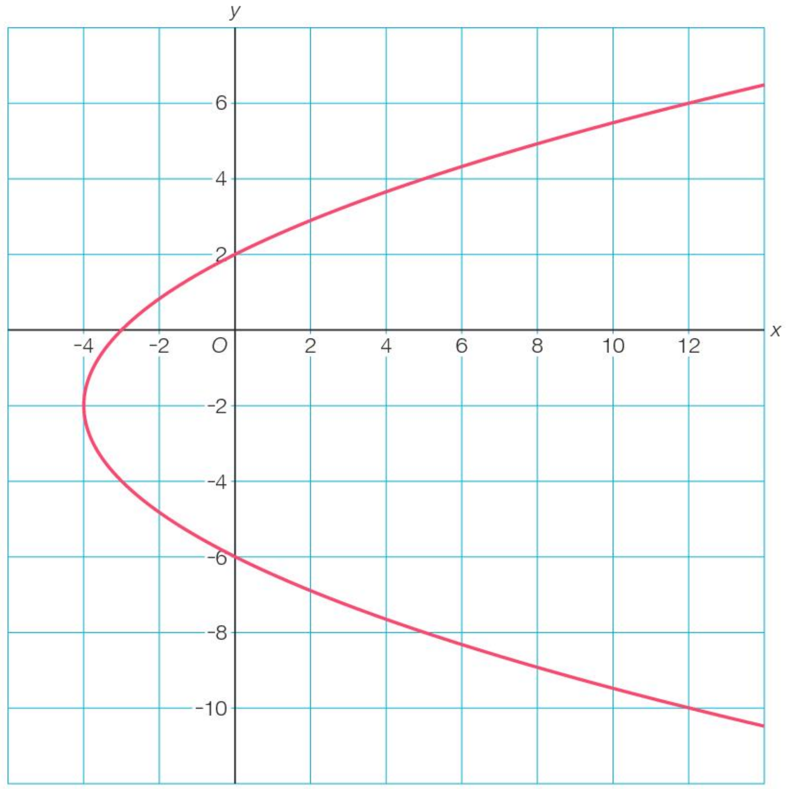
10.5 Vectoren bij snelheid en versnelling

Bladzijde 82

64 **a** $x(0) = 0 - 0 = 0$ en $y(0) = 0 - 6 = -6$, dus op $t = 0$ is P in het punt $(0, -6)$.

b

t	−2	−1	0	1	2	3	4	5	6
x	12	5	0	−3	−4	−3	0	5	12
y	−10	−8	−6	−4	−2	0	2	4	6



c De verplaatsing van P op $[2, 3]$ is $\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Bladzijde 85

65 **a** Evenwijdig met de y-as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$t^2 - 4 = 0 \wedge 2t - 2 \neq 0$$
$$t^2 = 4 \wedge 2t \neq 2$$
$$(t = 2 \vee t = -2) \wedge t \neq 1$$
$$t = 2 \vee t = -2$$

$x(2) = \frac{1}{3} \cdot 2^3 - 4 \cdot 2 = -5\frac{1}{3}$ en $y(2) = 2^2 - 2 \cdot 2 = 0$

$x(-2) = \frac{1}{3} \cdot (-2)^3 - 4 \cdot -2 = 5\frac{1}{3}$ en $y(-2) = (-2)^2 - 2 \cdot -2 = 8$

Dus $(-5\frac{1}{3}, 0)$ en $(5\frac{1}{3}, 8)$.

b Door de oorsprong, dus $x(t) = 0 \wedge y(t) = 0$

$$\tfrac{1}{3}t^3 - 4t = 0 \wedge t^2 - 2t = 0$$
$$\tfrac{1}{3}t(t^2 - 12) = 0 \wedge t(t - 2) = 0$$
$$(t = 0 \vee t^2 = 12) \wedge (t = 0 \vee t = 2)$$
$$t = 0$$

Stel $l: ax + by = 0$.

$\vec{r}_l = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 0 - 4 \\ 0 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Dus $l: x - 2y = 0$.

c Naar links bewegen betekent $x'(t) < 0$ en omhoog bewegen betekent $y'(t) > 0$,
dus $x'(t) < 0 \wedge y'(t) > 0$ en dit geeft $t^2 - 4 < 0 \wedge 2t - 2 > 0$.

d $t^2 - 4 < 0 \wedge 2t - 2 > 0$
$t^2 < 4 \wedge 2t > 2$
$-2 < t < 2 \wedge t > 1$
$1 < t < 2$

66 **a** $x(t) = t^2 - 2$ geeft $x'(t) = 2t$
$y(t) = t^3 - 4t$ geeft $y'(t) = 3t^2 - 4$
Evenwijdig met de x-as, dus $y'(t) = 0 \wedge x'(t) \neq 0$
$3t^2 - 4 = 0 \wedge 2t \neq 0$
$3t^2 = 4 \wedge t \neq 0$
$t^2 = \frac{4}{3} \wedge t \neq 0$
$t = \sqrt{\frac{4}{3}} = \frac{2}{3}\sqrt{3} \vee t = -\frac{2}{3}\sqrt{3}$
$x(\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3}$ en $y(\frac{2}{3}\sqrt{3}) = (\frac{2}{3}\sqrt{3})^3 - 4 \cdot \frac{2}{3}\sqrt{3} = \frac{8}{9}\sqrt{3} - \frac{8}{3}\sqrt{3} = -1\frac{7}{9}\sqrt{3}$
$x(-\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3}$ en $y(-\frac{2}{3}\sqrt{3}) = (-\frac{2}{3}\sqrt{3})^3 - 4 \cdot -\frac{2}{3}\sqrt{3} = -\frac{8}{9}\sqrt{3} + \frac{8}{3}\sqrt{3} = 1\frac{7}{9}\sqrt{3}$
Dus de punten zijn $(-\frac{2}{3}, -1\frac{7}{9}\sqrt{3})$ en $(-\frac{2}{3}, 1\frac{7}{9}\sqrt{3})$.

b $v(-1) = \sqrt{(x'(-1))^2 + (y'(-1))^2} = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$

c $x'(t) > 0 \wedge y'(t) < 0$ geeft $2t > 0 \wedge 3t^2 - 4 < 0$
$t > 0 \wedge 3t^2 < 4$
$t > 0 \wedge t^2 < \frac{4}{3}$
$t > 0 \wedge -\sqrt{\frac{4}{3}} < t < \sqrt{\frac{4}{3}}$
$t > 0 \wedge -\frac{2}{3}\sqrt{3} < t < \frac{2}{3}\sqrt{3}$
$0 < t < \frac{2}{3}\sqrt{3}$

d $x(t) = 2$ geeft $t^2 - 2 = 2$
$t^2 = 4$
$t = 2 \vee t = -2$
$y(2) = 2^3 - 4 \cdot 2 = 0$ en $y(-2) = (-2)^3 - 4 \cdot -2 = 0$.
$t = 2$ geeft het punt $(2, 0)$ en $t = -2$ geeft het punt $(2, 0)$, dus de baan snijdt zichzelf in het punt $(2, 0)$.

$$\vec{v}(2) = \begin{pmatrix} x'(2) \\ y'(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\vec{v}(-2) = \begin{pmatrix} x'(-2) \\ y'(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right|} = \frac{|1 \cdot 1 + 2 \cdot -2|}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5}$$

Dus $\varphi \approx 53{,}1°$.

67 **a** $x(t) = 0$ geeft $2t - \frac{1}{6}t^3 = 0$
$\frac{1}{6}t(12 - t^2) = 0$
$t = 0 \vee t^2 = 12$
$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$
$t = 0$ geeft $(0, 0)$
$y(2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot 2\sqrt{3} = 3 - 4\sqrt{3}$, dus $B(0, 3 - 4\sqrt{3})$ en
$y(-2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot -2\sqrt{3} = 3 + 4\sqrt{3}$, dus $A(0, 3 + 4\sqrt{3})$.
$d(A, B) = 3 + 4\sqrt{3} - (3 - 4\sqrt{3}) = 8\sqrt{3}$

b $x'(t) = 2 - \frac{1}{2}t^2$ en $y'(t) = \frac{1}{2}t - 2$
Evenwijdig met de x-as, dus $y'(t) = 0 \wedge x'(t) \neq 0$
$\frac{1}{2}t - 2 = 0 \wedge 2 - \frac{1}{2}t^2 \neq 0$
$\frac{1}{2}t = 2 \wedge \frac{1}{2}t^2 \neq 2$
$t = 4 \wedge t^2 \neq 4$
$t = 4 \wedge t \neq 2 \wedge t \neq -2$
$t = 4$
$x(4) = 2 \cdot 4 - \frac{1}{6} \cdot 4^3 = -2\frac{2}{3}$ en $y(4) = \frac{1}{4} \cdot 4^2 - 2 \cdot 4 = -4$, dus $(-2\frac{2}{3}, -4)$.

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Evenwijdig met de y-as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$2 - \tfrac{1}{2}t^2 = 0 \wedge \tfrac{1}{2}t - 2 \neq 0$$
$$(t = 2 \vee t = -2) \wedge t \neq 4$$
$$t = 2 \vee t = -2$$

$x(2) = 2 \cdot 2 - \frac{1}{6} \cdot 2^3 = 2\frac{2}{3}$ en $y(2) = \frac{1}{4} \cdot 2^2 - 2 \cdot 2 = -3$

$x(-2) = 2 \cdot -2 - \frac{1}{6} \cdot (-2)^3 = -2\frac{2}{3}$ en $y(-2) = \frac{1}{4} \cdot (-2)^2 - 2 \cdot -2 = 5$

Dus $(2\frac{2}{3}, -3)$ en $(-2\frac{2}{3}, 5)$.

c Bij de oorsprong hoort $t = 0$.

$v(0) = \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

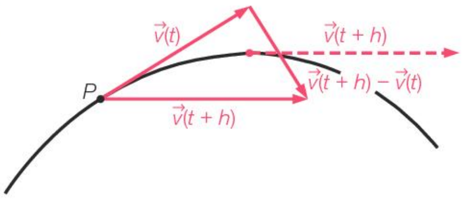
d $v(t) = \sqrt{(x'(t))^2 + (y'(t))} = \sqrt{(2 - \frac{1}{2}t^2)^2 + (\frac{1}{2}t - 2)^2}$

Voer in $y_1 = \sqrt{(2 - \frac{1}{2}x^2)^2 + (\frac{1}{2}x - 2)^2}$.

De optie minimum geeft $x = 2{,}109...$ en $y = 0{,}971....$

Dus de minimale baansnelheid is ongeveer 0,97 voor $t \approx 2{,}11$.

68



Bladzijde 86

69 $\vec{v}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix}$

$\vec{v}(t) \cdot \vec{a}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix} = 4t + 3t^5 - 6t^2 = 3t^5 - 6t^2 + 4t$

$|\vec{v}(t)| = \sqrt{t^6 - 4t^3 + 4t^2 + 4}$

Dit geeft $a(t) = \dfrac{3t^5 - 6t^2 + 4t}{\sqrt{t^6 - 4t^3 + 4t^2 + 4}}$.

Bladzijde 87

70 **a** Snijden met de x-as, dus $y(t) = 0$ geeft $t^3 - 3t = 0$

$$t(t^2 - 3) = 0$$
$$t = 0 \vee t^2 = 3$$
$$t = 0 \vee t = \sqrt{3} \vee t = -\sqrt{3}$$

De baan passeert D twee keer, dat is dus op $t = -\sqrt{3}$ en $t = \sqrt{3}$. Dus op $t = 0$ is P in C.

Snijden met de y-as, dus $x(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$
$$t = 2 \vee t = -2$$

$y(2) = 2^3 - 3 \cdot 2 = 2$, dus de baan passeert A op $t = 2$ en B op $t = -2$.

Dus de punten worden in de volgorde B, D, C, D, A doorlopen.

b $\vec{r}(t) = \begin{pmatrix} t^2 - 4 \\ t^3 - 3t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 2t \\ 3t^2 - 3 \end{pmatrix}$ en

$v(t) = \sqrt{(2t)^2 + (3t^2 - 3)^2} = \sqrt{4t^2 + 9t^4 - 18t^2 + 9} = \sqrt{9t^4 - 14t^2 + 9}$.

$a(t) = v'(t) = \dfrac{1}{2\sqrt{9t^4 - 14t^2 + 9}} \cdot (36t^3 - 28t) = \dfrac{18t^3 - 14t}{\sqrt{9t^4 - 14t^2 + 9}}$

c $v(2) = \sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9} = \sqrt{97}$

$a(2) = \dfrac{18 \cdot 2^3 - 14 \cdot 2}{\sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9}} = \dfrac{116}{\sqrt{97}} = 1\frac{19}{97}\sqrt{97}$

d $a(t)=0$ geeft $18t^3-14t=0$

$2t(9t^2-7)=0$

$t=0 \vee t^2=\frac{7}{9}$

$t=0 \vee t=\sqrt{\frac{7}{9}} \vee t=-\sqrt{\frac{7}{9}}$

$t=0 \vee t=\frac{1}{3}\sqrt{7} \vee t=-\frac{1}{3}\sqrt{7}$

$v(0)=\sqrt{0+9}=3$

$v(\frac{1}{3}\sqrt{7})=\sqrt{9\cdot(\frac{7}{9})^2-14\cdot\frac{7}{9}+9}=\sqrt{\frac{32}{9}}=1\frac{1}{3}\sqrt{2}$

$v(-\frac{1}{3}\sqrt{7})=\sqrt{9\cdot(\frac{7}{9})^2-14\cdot\frac{7}{9}+9}=1\frac{1}{3}\sqrt{2}$

e Voer in $y_1=\sqrt{9x^4-14x^2+9}$.
De optie minimum geeft $y=1{,}8856...$ voor $x=-0{,}8819...$ en $x\approx 0{,}8819...$
Dus de minimale baansnelheid is ongeveer 1,886 voor $t\approx -0{,}882$ en voor $t\approx 0{,}882$.

f $v(t)=2$ geeft $\sqrt{9t^4-14t^2+9}=2$

kwadrateren geeft

$9t^4-14t^2+9=4$

$9t^4-14t^2+5=0$

Stel $t^2=u$.

$9u^2-14u+5=0$

$D=(-14)^2-4\cdot 9\cdot 5=16$

$u=\frac{14+4}{18}=1 \vee \frac{14-4}{18}=\frac{5}{9}$

$t^2=1 \vee t^2=\frac{5}{9}$

$t=1 \vee t=-1 \vee t=\sqrt{\frac{5}{9}} \vee t=-\sqrt{\frac{5}{9}}$

$t=1 \vee t=-1 \vee t=\frac{1}{3}\sqrt{5} \vee t=-\frac{1}{3}\sqrt{5}$

g $t=\sqrt{3}$ geeft $x=(\sqrt{3})^2-4=-1$ en $y=0$
$t=-\sqrt{3}$ geeft $x=(-\sqrt{3})^2-4=-1$ en $y=0$
Dus de baan snijdt zichzelf in $D(-1, 0)$.

$\vec{v}(\sqrt{3})=\begin{pmatrix}2\sqrt{3}\\6\end{pmatrix}$ en $\vec{v}(-\sqrt{3})=\begin{pmatrix}-2\sqrt{3}\\6\end{pmatrix}$.

$$\cos(\varphi)=\frac{\left|\begin{pmatrix}2\sqrt{3}\\6\end{pmatrix}\cdot\begin{pmatrix}-2\sqrt{3}\\6\end{pmatrix}\right|}{\left|\begin{pmatrix}2\sqrt{3}\\6\end{pmatrix}\right|\cdot\left|\begin{pmatrix}-2\sqrt{3}\\6\end{pmatrix}\right|}=\frac{|2\sqrt{3}\cdot -2\sqrt{3}+6\cdot 6|}{(\sqrt{12+36})^2}=\frac{24}{48}=\frac{1}{2}$$

Dus $\varphi=60°$.

Bladzijde 88

71 **a** Snijden met de x-as, dus $y(t)=0$ geeft $\frac{1}{2}t^2-2t=0$

$\frac{1}{2}t(t-4)=0$

$t=0 \vee t=4$

$t=0$ geeft $(-2, 0)$, dus $t=0$ hoort bij A en $t=4$ hoort bij B.
Dus de punten worden in de volgorde C, A, D, B doorlopen.

b $\vec{r}(t)=\begin{pmatrix}\frac{1}{2}t^2-2\\\frac{1}{2}t^2-2t\end{pmatrix}$ geeft $\vec{v}(t)=\begin{pmatrix}t\\t-2\end{pmatrix}$ en $v(t)=\sqrt{t^2+(t-2)^2}=\sqrt{t^2+t^2-4t+4}=\sqrt{2t^2-4t+4}$.

$a(t)=v'(t)=\frac{1}{2\sqrt{2t^2-4t+4}}\cdot(4t-4)=\frac{2t-2}{\sqrt{2t^2-4t+4}}$

c $\vec{v}(t)=\begin{pmatrix}t\\t-2\end{pmatrix}$ geeft $\vec{a}(t)=\begin{pmatrix}1\\1\end{pmatrix}$

$\vec{v}(t)\perp\vec{a}(t)$ geeft $\vec{v}(t)\cdot\vec{a}(t)=0$

$\begin{pmatrix}t\\t-2\end{pmatrix}\cdot\begin{pmatrix}1\\1\end{pmatrix}=0$

$t+t-2=0$

$2t=2$

$t=1$

$x(1)=\frac{1}{2}-2=-1\frac{1}{2}$ en $y(1)=\frac{1}{2}-2=-1\frac{1}{2}$, dus in $(-1\frac{1}{2}, -1\frac{1}{2})$.

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d $a(t) = 0$ geeft $2t - 2 = 0$

$$2t = 2$$
$$t = 1$$

$v(1) = \sqrt{2 - 4 + 4} = \sqrt{2}$

Dus de minimale baansnelheid is $\sqrt{2}$ voor $t = 1$.

e Bij $t = 4$ hoort B, dus de baansnelheid in B is $v(4) = \sqrt{4^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$.

Snijden met de y-as, dus $x(t) = 0$ geeft $\frac{1}{2}t^2 - 2 = 0$

$$\tfrac{1}{2}t^2 = 2$$
$$t^2 = 4$$
$$t = 2 \vee t = -2$$

Bij $t = -2$ hoort dus C en dit geeft $v(-2) = \sqrt{(-2)^2 + (-2-2)^2} = \sqrt{20} = 2\sqrt{5}$.

Dus de baansnelheden in B en C zijn gelijk.

f $E(-1\frac{1}{2}, 2\frac{1}{2})$ invullen in $\begin{cases} x(t) = \frac{1}{2}t^2 - 2 \\ y(t) = \frac{1}{2}t^2 - 2t \end{cases}$ geeft $\begin{cases} \frac{1}{2}t^2 - 2 = -1\frac{1}{2} \\ \frac{1}{2}t^2 - 2t = 2\frac{1}{2} \end{cases}$ –

$$2t - 2 = -4$$
$$2t = -2$$
$$t = -1$$

$\vec{r}_k = \vec{v}(-1) = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ geeft $\vec{n}_k = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\left.\begin{array}{l} k\colon 3x - y = c \\ \text{door } (-1\frac{1}{2}, 2\frac{1}{2}) \end{array}\right\}$ $c = 3 \cdot -1\frac{1}{2} - 2\frac{1}{2} = -7$

Dus $k\colon 3x - y = -7$.

72 Q en R op een verticale lijn, dus $x_Q = x_R$.

Dit geeft $\frac{1}{2}t_Q^{\,2} - 2 = \frac{1}{2}t_R^{\,2} - 2$, dus $t_Q = -t_R$.

$\left.\begin{array}{l} y_R - y_Q = 4 \text{ geeft } \frac{1}{2}t_R^{\,2} - 2t_R - (\frac{1}{2}t_Q^{\,2} - 2t_Q) = 4 \\ t_Q = -t_R \end{array}\right\}$ $\frac{1}{2}t_R^{\,2} - 2t_R - \frac{1}{2}t_R^{\,2} - 2t_R = 4$

$$-4t_R = 4$$
$$t_R = -1$$

Dus $t_Q = 1$.

$y(1) = \frac{1}{2} - 2 = -1\frac{1}{2}$ en $y(-1) = \frac{1}{2} - 2 = 2\frac{1}{2}$.

Dus $Q(-1\frac{1}{2}, -1\frac{1}{2})$ en $R(-1\frac{1}{2}, 2\frac{1}{2})$.

73 **a** $\vec{r}(t) = \begin{pmatrix} 25t \\ -5t^2 + 15t + 3 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$

$v(0) = \sqrt{25^2 + 15^2} \approx 29{,}2$ m/s

b Een richtingsvector van een horizontale lijn is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\vec{v}(0) = \begin{pmatrix} 25 \\ 15 \end{pmatrix}$

$$\cos(\varphi) = \frac{\left|\begin{pmatrix} 25 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|}{\left|\begin{pmatrix} 25 \\ 15 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|} = \frac{|25 \cdot 1 + 15 \cdot 0|}{\sqrt{850}} = \frac{25}{\sqrt{850}}$$

Dus $\varphi \approx 31°$.

c $y(t) = -5t^2 + 15t + 3$ geeft $y'(t) = -10t + 15$

$y'(t) = 0$ geeft $-10t + 15 = 0$

$$-10t = -15$$
$$t = 1{,}5$$

$y(1{,}5) = -5 \cdot 1{,}5^2 + 15 \cdot 1{,}5 + 3 = 14{,}25$

Dus de maximale hoogte is 14,25 meter.

d $y(t) = 0$ geeft $-5t^2 + 15t + 3 = 0$

$$D = 15^2 - 4 \cdot -5 \cdot 3 = 285$$

$$t = \frac{-15 + \sqrt{285}}{-10} \vee t = \frac{-15 - \sqrt{285}}{-10}$$

$$t = 1{,}5 - 0{,}1\sqrt{285} \vee t = 1{,}5 + 0{,}1\sqrt{285}$$

vold. niet

$x(1{,}5 + 0{,}1\sqrt{285}) = 25(1{,}5 + 0{,}1\sqrt{285}) = 79{,}70...$

Dus de speer komt 77,7 meter achter de afworplijn neer.

e $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$

De versnellingsvector is de valversnelling van -10 m/s^2.

Bladzijde 89

74 **a** Snijden met de x-as, dus $y(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$\vec{r}(t) = \begin{pmatrix} \frac{1}{3}t^3 - 4t \\ t^2 - 4 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$

$\vec{v}(2) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ en $\vec{v}(-2) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

De snelheidsvectoren in A en B zijn verticaal, dus de raaklijnen aan de baan in A en B zijn verticaal.

b $y(t) = 5$ geeft $t^2 - 4 = 5$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$x(3) = \frac{1}{3} \cdot 3^3 - 4 \cdot 3 = -3$, dus $t = 3$ hoort bij C.

$\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$ geeft $v(t) = \sqrt{(t^2 - 4)^2 + (2t)^2} = \sqrt{t^4 - 8t^2 + 16 + 4t^2} = \sqrt{t^4 - 4t^2 + 16}$

$$a(t) = v'(t) = \frac{1}{2\sqrt{t^4 - 4t^2 + 16}} \cdot (4t^3 - 8t) = \frac{2t^3 - 4t}{\sqrt{t^4 - 4t^2 + 16}}$$

De baansnelheid in C is $v(3) = \sqrt{3^4 - 4 \cdot 3^2 + 16} = \sqrt{61}$.

De baanversnelling in C is $a(3) = \frac{2 \cdot 3^3 - 4 \cdot 3}{\sqrt{3^4 - 4 \cdot 3^2 + 16}} = \frac{42}{\sqrt{61}} = \frac{42}{61}\sqrt{61}$.

c $v(t) = 6\frac{1}{2}$ geeft $\sqrt{t^4 - 4t^2 + 16} = 6\frac{1}{2}$

kwadrateren geeft

$$t^4 - 4t^2 + 16 = 42\tfrac{1}{4}$$

$$t^4 - 4t^2 - 26\tfrac{1}{4} = 0$$

Stel $t^2 = u$.

$$u^2 - 4u - 26\tfrac{1}{4} = 0$$

$$D = (-4^2) - 4 \cdot 1 \cdot -26\tfrac{1}{4} = 121$$

$$u = \frac{4 + 11}{2} = 7\tfrac{1}{2} \vee u = \frac{4 - 11}{2} = -3\tfrac{1}{2}$$

$$t^2 = 7\tfrac{1}{2} \vee t^2 = -3\tfrac{1}{2}$$

$$t = \sqrt{7\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{30} \vee t = -\sqrt{7\tfrac{1}{2}} = -\tfrac{1}{2}\sqrt{30}$$

$x(\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot \frac{1}{2}\sqrt{30} - 4 \cdot \frac{1}{2}\sqrt{30} = 1\frac{1}{4}\sqrt{30} - 2\sqrt{30} = -\frac{3}{4}\sqrt{30}$ en $y(\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$.

$x(-\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot -\frac{1}{2}\sqrt{30} - 4 \cdot -\frac{1}{2}\sqrt{30} = -1\frac{1}{4}\sqrt{30} + 2\sqrt{30} = \frac{3}{4}\sqrt{30}$ en $y(-\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$.

Dus de punten zijn $(-\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$ en $(\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$.

d Voer in $y_1 = \frac{2x^3 - 4x}{\sqrt{x^4 - 4x^2 + 16}}$.

De optie minimum geeft $x = 0{,}8510...$ en $y = -0{,}5882...$.

Dus de baanversnelling heeft voor $t \approx 0{,}851$ een minimum van ongeveer $-0{,}588$.

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e Snijden met de y-as, dus $x(t) = 0$ geeft $\frac{1}{3}t^3 - 4t = 0$

$$\frac{1}{3}t(t^2 - 12) = 0$$
$$t = 0 \vee t^2 = 12$$
$$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$$

$y(2\sqrt{3}) = 12 - 4 = 8$ en $y(-2\sqrt{3}) = 12 - 4 = 8$.

Dus de baan snijdt zichzelf in $S(0, 8)$.

$\vec{v}(2\sqrt{3}) = \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix}$ en $\vec{v}(-2\sqrt{3}) = \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}$.

$$\cos(\varphi) = \frac{\left|\begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}\right|}{\left|\begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}\right|} = \frac{|8 \cdot 8 + 4\sqrt{3} \cdot -4\sqrt{3}|}{(\sqrt{64 + 48})^2} = \frac{16}{112} = \frac{1}{7}$$

Dus $\varphi \approx 82°$.

Diagnostische toets

Bladzijde 92

1 **a** $\vec{c} = \vec{a} + 2\vec{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

$|\vec{c}| = 10$

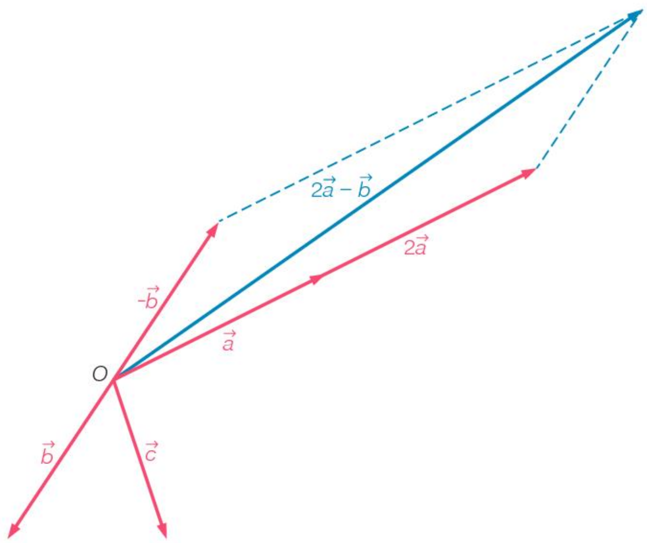
b $\vec{d} = 2\vec{a} - 3\vec{b} = 2 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 3 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$

$|\vec{d}| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$

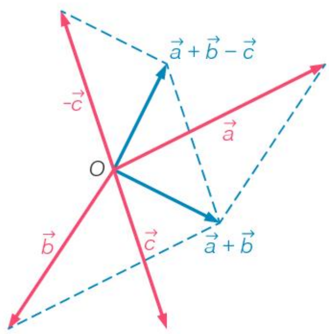
c $\vec{e} = \vec{b} - \frac{1}{2}\vec{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$|\vec{e}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

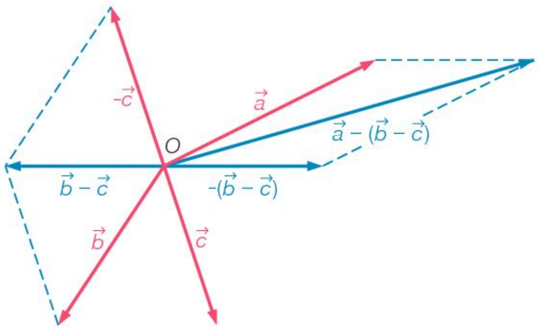
2 **a**



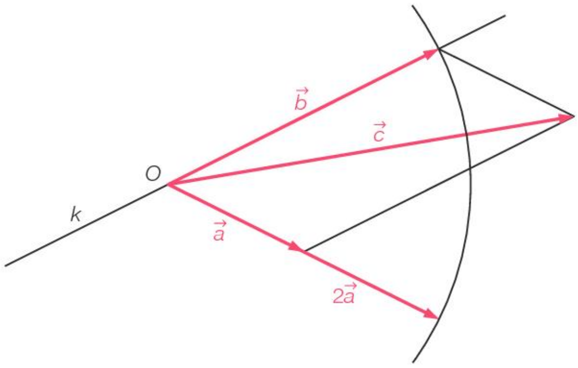
b

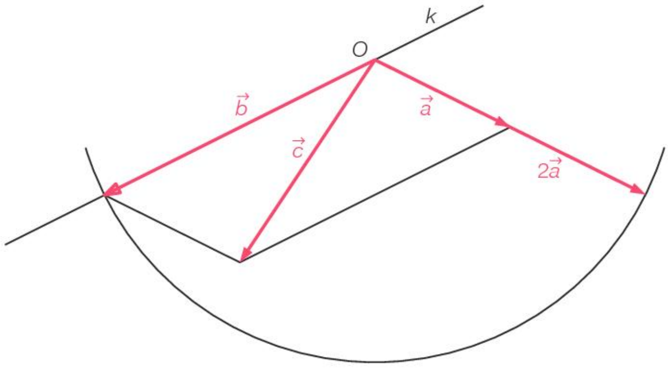


c

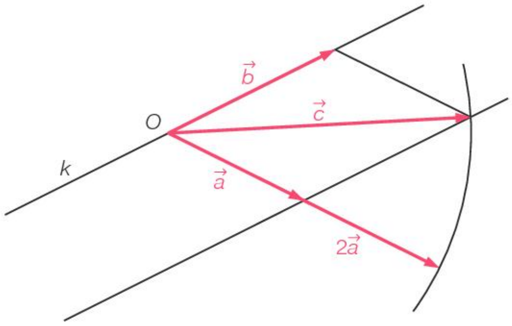


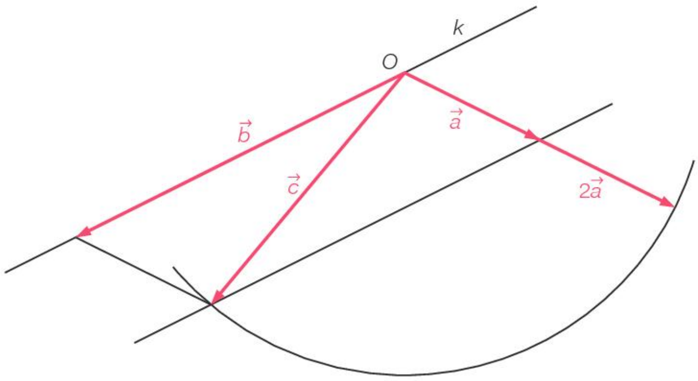
3 **a**





b





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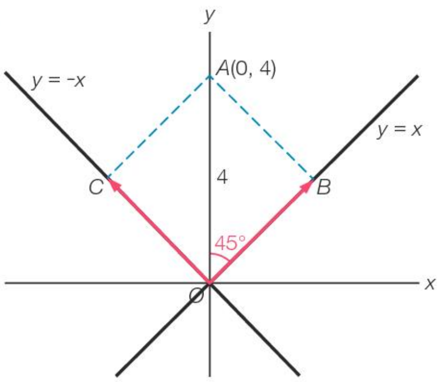
4



In $\triangle OAB$ is $OA = 4$ en $\angle O = 45°$.

Dus $OB = \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$.

Ook $OC = 2\sqrt{2}$, dus de lengte van beide componenten is $2\sqrt{2}$.

5 M is het midden van zijde AB.

$\vec{c} = \vec{b} + \overrightarrow{BC} = \vec{b} + \overrightarrow{MA}_R$ en $\vec{d} = \vec{b} + \overrightarrow{AD} = \vec{b} + \overrightarrow{BC}$

$M(\tfrac{1}{2}(4+8), \tfrac{1}{2}(0+6)) = M(6, 3)$

$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}-2\\-3\end{pmatrix}$, dus $\overrightarrow{MA}_R = \begin{pmatrix}-3\\2\end{pmatrix}$.

$\vec{c} = \begin{pmatrix}8\\6\end{pmatrix} + \begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$

$\vec{d} = \begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$

Dus $C(5, 8)$ en $D(1, 2)$.

6 $\overrightarrow{CM} = \vec{m} - \vec{c} = \tfrac{1}{2}(\vec{a} + \vec{b}) - \vec{c} = \tfrac{1}{2}\left(\begin{pmatrix}a\\0\end{pmatrix} + \begin{pmatrix}b\\0\end{pmatrix}\right) - \begin{pmatrix}c\\d\end{pmatrix} = \begin{pmatrix}\tfrac{1}{2}a + \tfrac{1}{2}b\\0\end{pmatrix} - \begin{pmatrix}c\\d\end{pmatrix} = \begin{pmatrix}\tfrac{1}{2}a + \tfrac{1}{2}b - c\\-d\end{pmatrix}$

$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{CA}_R$ en $\vec{h} = \vec{c} + \overrightarrow{CH} = \vec{c} + \overrightarrow{CB}_L$

$\overrightarrow{CA} = \vec{a} - \vec{c} = \begin{pmatrix}a\\0\end{pmatrix} - \begin{pmatrix}c\\d\end{pmatrix} = \begin{pmatrix}a-c\\-d\end{pmatrix}$ en $\overrightarrow{CB} = \vec{b} - \vec{c} = \begin{pmatrix}b\\0\end{pmatrix} - \begin{pmatrix}c\\d\end{pmatrix} = \begin{pmatrix}b-c\\-d\end{pmatrix}$

$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{CA}_R = \begin{pmatrix}c\\d\end{pmatrix} + \begin{pmatrix}-d\\c-a\end{pmatrix} = \begin{pmatrix}c-d\\-a+c+d\end{pmatrix}$

$\vec{h} = \vec{c} + \overrightarrow{CH} = \vec{c} + \overrightarrow{CB}_L = \begin{pmatrix}c\\d\end{pmatrix} + \begin{pmatrix}d\\b-c\end{pmatrix} = \begin{pmatrix}c+d\\b-c+d\end{pmatrix}$

$\overrightarrow{DH} = \vec{h} - \vec{d} = \begin{pmatrix}c+d\\b-c+d\end{pmatrix} - \begin{pmatrix}c-d\\-a+c+d\end{pmatrix} = \begin{pmatrix}2d\\a+b-2c\end{pmatrix} = 2 \cdot \begin{pmatrix}d\\\tfrac{1}{2}a + \tfrac{1}{2}b - c\end{pmatrix}$

$\begin{pmatrix}\tfrac{1}{2}a + \tfrac{1}{2}b - c\\-d\end{pmatrix}_L = \begin{pmatrix}d\\\tfrac{1}{2}a + \tfrac{1}{2}b - c\end{pmatrix}$, dus $CM \perp DH$ en $DH = 2 \cdot CM$.

7 **a** $k: \begin{pmatrix}x\\y\end{pmatrix} = \vec{b} + t \cdot (\vec{a} - \vec{c})$

$\vec{b} = \begin{pmatrix}-4\\5\end{pmatrix}$ en $\vec{a} - \vec{c} = \begin{pmatrix}6\\8\end{pmatrix} - \begin{pmatrix}1\\-4\end{pmatrix} = \begin{pmatrix}5\\12\end{pmatrix}$

$\Rightarrow k: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-4\\5\end{pmatrix} + t \cdot \begin{pmatrix}5\\12\end{pmatrix}$

b $\vec{m} = \tfrac{1}{2}(\vec{b} + \vec{c}) = \tfrac{1}{2}\left(\begin{pmatrix}-4\\5\end{pmatrix} + \begin{pmatrix}1\\-4\end{pmatrix}\right) = \tfrac{1}{2}\begin{pmatrix}-3\\1\end{pmatrix} = \begin{pmatrix}-1\tfrac{1}{2}\\\tfrac{1}{2}\end{pmatrix}$

$l: \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + t \cdot (\vec{m} - \vec{a})$

$\vec{a} = \begin{pmatrix}6\\8\end{pmatrix}$ en $\vec{m} - \vec{a} = \begin{pmatrix}-1\tfrac{1}{2}\\\tfrac{1}{2}\end{pmatrix} - \begin{pmatrix}6\\8\end{pmatrix} = \begin{pmatrix}-7\tfrac{1}{2}\\-7\tfrac{1}{2}\end{pmatrix} \triangleq \begin{pmatrix}1\\1\end{pmatrix}$

$\Rightarrow l: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix} + t \cdot \begin{pmatrix}1\\1\end{pmatrix}$

c Stel eerst een vectorvoorstelling op van de lijn n door A en C.

$n: \begin{pmatrix}x\\y\end{pmatrix} = \vec{a} + t \cdot (\vec{a} - \vec{c})$

$\vec{a} = \begin{pmatrix}6\\8\end{pmatrix}$ en $\vec{a} - \vec{c} = \begin{pmatrix}5\\12\end{pmatrix}$

$\Rightarrow n: \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix} + t \cdot \begin{pmatrix}5\\12\end{pmatrix}$

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix} + t \cdot \begin{pmatrix}5\\12\end{pmatrix} \wedge -1 \le t \le 0$.

Bladzijde 93

8 Substitutie van $x = -2 + 2t$ en $y = 3 - t$ in $2x - 3y = 22$ geeft $2(-2 + 2t) - 3(3 - t) = 22$

$$-4 + 4t - 9 + 3t = 22$$
$$7t = 35$$
$$t = 5$$

$t = 5$ geeft $x = -2 + 5 \cdot 2 = 8$ en $y = 3 - 5 = -2$, dus $S(8, -2)$.

9 **a** k: $x = 1 + 3t \wedge y = 2 + 4t$

b $x = t$ geeft $3t - y = 6$ oftewel $y = 3t - 6$.

Dus l: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

c $\begin{cases} x = 4 - 2t & |3| \\ y = 6 + 3t & |2| \end{cases}$ geeft $\begin{cases} 3x = 12 - 6t \\ 2y = 12 + 6t \end{cases}$ +
$3x + 2y = 24$

Dus m: $3x + 2y = 24$.

10 **a** $2 + t = 2p$ geeft $t = 2p - 2$

$\left.\begin{array}{l} 3 - 2t = 3 - p \\ t = 2p - 2 \end{array}\right\} 3 - 2(2p - 2) = 3 - p$

$3 - 4p + 4 = 3 - p$

$-3p = -4$

$p = 1\frac{1}{3}$

Dus $A(2 \cdot 1\frac{1}{3}, 3 - 1\frac{1}{3}) = A(2\frac{2}{3}, 1\frac{2}{3})$.

b $3 - 2t = 6$

$-2t = 3$

$t = -1\frac{1}{2}$

$\left.\begin{array}{l} q + t = 1 \\ t = -1\frac{1}{2} \end{array}\right\} q = 2\frac{1}{2}$

11 **a** $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

$\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$

$$\cos(\angle(AB, BC)) = \frac{\left|\begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -5 \end{pmatrix}\right|}{\left|\begin{pmatrix} 6 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -7 \\ -5 \end{pmatrix}\right|} = \frac{|6 \cdot -7 + 1 \cdot -5|}{\sqrt{37} \cdot \sqrt{74}} = \frac{47}{\sqrt{2738}}$$

Dus $\angle(AB, BC) \approx 26{,}1°$.

b $\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ en $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{AB}, \overrightarrow{AC})) = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} -1 \\ -4 \end{pmatrix}\right|} = \frac{-6 - 4}{\sqrt{37} \cdot \sqrt{17}} = \frac{-10}{\sqrt{629}}$$

Dus $\angle BAC = \angle(\overrightarrow{AB}, \overrightarrow{AC}) \approx 113{,}5°$.

12 **a** $\vec{r}_k = \vec{r}_l = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$\left.\begin{array}{l} k\text{: } 3x + 5y = c \\ \text{door } (3, 4) \end{array}\right\} c = 3 \cdot 3 + 5 \cdot 4 = 29$

Dus k: $3x + 5y = 29$.

b $(1, 1)$ op m, dus $\vec{s}_m = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$m \perp n$, dus $\vec{r}_m = \vec{n}_n = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Dus m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

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c $p \perp r$, dus $\vec{n}_p = \vec{r}_r = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

$\left.\begin{array}{l} p\text{: } 4x - y = c \\ \text{door } (-2, 3) \end{array}\right\} c = 4 \cdot -2 - 3 = -11$

Dus p: $4x - y = -11$.

13 Neem $x = 3t$ bij k, dan is $y = \frac{1}{3} \cdot 3t + 3 = t + 3$.
Dus een parametervoorstelling van k is k: $x = 3t \wedge y = t + 3$.

$\vec{r}_k = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ en $\vec{r}_l = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ geeft $\vec{r}_k \cdot \vec{r}_l = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 3 - 3 = 0$

Dus $\angle(k, l) = 90°$.

14 **a** $x(t) = 4t - t^2$ geeft $x'(t) = 4 - 2t$

$y(t) = \frac{2}{3}t^3 - t^2$ geeft $y'(t) = 2t^2 - 2t$

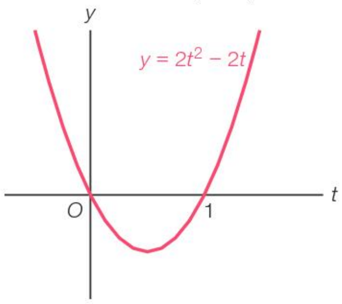
$x'(t) > 0$ geeft $4 - 2t > 0$

$-2t > -4$

$t < 2$

$y'(t) > 0$ geeft $2t^2 - 2t > 0$

$2t(t - 1) > 0$



$2t^2 - 2t > 0$ geeft $t < 0 \vee t > 1$

$t < 2 \wedge (t < 0 \vee t > 1)$ geeft $t < 0 \vee 1 < t < 2$

Dus voor $t < 0$ en voor $1 < t < 2$ beweegt P zowel naar rechts als omhoog.

b $y'(t) = 0$ geeft $t = 0 \vee t = 1$ (zie a) met de bijbehorende punten $(0, 0)$ en $(3, -\frac{1}{3})$.

$x'(t) = 0$ geeft $t = 2$ (zie a) met het bijbehorende punt $(4, 1\frac{1}{3})$.

Dus evenwijdig met de x-as in $(0, 0)$ en $(3, -\frac{1}{3})$ en evenwijdig met de y-as in $(4, 1\frac{1}{3})$.

c $y(t) = 0$ geeft $\frac{2}{3}t^3 - t^2 = 0$

$t^2(\frac{2}{3}t - 1) = 0$

$t = 0 \vee \frac{2}{3}t = 1$

$t = 0 \quad \vee \quad t = 1\frac{1}{2}$

vold. niet

$\vec{v}(1\frac{1}{2}) = \begin{pmatrix} x'(1\frac{1}{2}) \\ y'(1\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 4 - 2 \cdot 1\frac{1}{2} \\ 2 \cdot (1\frac{1}{2})^2 - 2 \cdot 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}$

Bij de x-as hoort richtingsvector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\cos(\varphi) = \frac{\left|\begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|} = \frac{1}{\sqrt{3\frac{1}{4}}}$$

Dus $\varphi \approx 56{,}3°$.

d $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(4 - 2t)^2 + (2t^2 - 2t)^2}$

$v(1\frac{1}{2}) = \sqrt{(4 - 3)^2 + (4\frac{1}{2} - 3)^2} = \sqrt{1 + 2\frac{1}{4}} = \sqrt{3\frac{1}{4}} = \frac{1}{2}\sqrt{13}$

e Voer in $y_1 = \sqrt{(4 - 2x)^2 + (2x^2 - 2x)^2}$.

De optie minimum geeft $x = 1{,}317...$ en $y = 1{,}600...$

Dus de minimale baansnelheid van P is ongeveer 1,60 voor $t \approx 1{,}32$.

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f $\vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} -2 \\ 4t - 2 \end{pmatrix}$

$\vec{a}(t)$ is evenwijdig met de x-as geeft $y''(t) = 0$

$$4t - 2 = 0$$
$$4t = 2$$
$$t = \tfrac{1}{2}$$

$x(\tfrac{1}{2}) = 4 \cdot \tfrac{1}{2} - (\tfrac{1}{2})^2 = 1\tfrac{3}{4}$ en $y(\tfrac{1}{2}) = \tfrac{2}{3} \cdot (\tfrac{1}{2})^3 - (\tfrac{1}{2})^2 = -\tfrac{1}{6}$

Dus in het punt $(1\tfrac{3}{4}, -\tfrac{1}{6})$.

g $v(t) = \sqrt{(4 - 2t)^2 + (2t^2 - 2t)^2} = \sqrt{16 - 16t + 4t^2 + 4t^4 - 8t^3 + 4t^2} = \sqrt{4t^4 - 8t^3 + 8t^2 - 16t + 16}$

$$a(t) = v'(t) = \frac{1}{2\sqrt{4t^4 - 8t^3 + 8t^2 - 16t + 16}} \cdot (16t^3 - 24t^2 + 16t - 16) = \frac{8t^3 - 12t^2 + 8t - 8}{\sqrt{4t^4 - 8t^3 + 8t^2 - 16t + 16}}$$

$$a(3) = \frac{216 - 108 + 24 - 8}{\sqrt{324 - 216 + 72 - 48 + 16}} = \frac{124}{\sqrt{148}} = \frac{124}{2\sqrt{37}} = 1\tfrac{25}{37}\sqrt{37}$$

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11 Integraalrekening

Voorkennis Differentiaalrekening

Bladzijde 98

1 **a** $f(x) = 4\ln^2(x) - 3\ln(x)$ geeft $f'(x) = 8\ln(x) \cdot \frac{1}{x} - 3 \cdot \frac{1}{x} = \frac{8\ln(x) - 3}{x}$

b $f(x) = \frac{10 \cdot 3^x}{\ln(3)} = \frac{10}{\ln(3)} \cdot 3^x$ geeft $f'(x) = \frac{10}{\ln(3)} \cdot 3^x \cdot \ln(3) = 10 \cdot 3^x$

c $f(x) = x^2 \cdot \sqrt{2x - 1}$ geeft

$f'(x) = 2x \cdot \sqrt{2x-1} + x^2 \cdot \frac{1}{2\sqrt{2x-1}} \cdot 2 = 2x\sqrt{2x-1} + \frac{x^2}{\sqrt{2x-1}} = \frac{2x(2x-1) + x^2}{\sqrt{2x-1}} = \frac{5x^2 - 2x}{\sqrt{2x-1}}$

d $f(x) = \frac{e^{2x} + 3}{4\,e^x} = \frac{1}{4}e^x + \frac{3}{4}e^{-x}$ geeft $f'(x) = \frac{1}{4}e^x + \frac{3}{4}e^{-x} \cdot -1 = \frac{e^x}{4} - \frac{3}{4\,e^x} = \frac{e^{2x} - 3}{4\,e^x}$

2 **a** $f(x) = x\ln(x) - x$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$

b $f(x) = x\sqrt{4x - 3}$ geeft

$f'(x) = 1 \cdot \sqrt{4x-3} + x \cdot \frac{1}{2\sqrt{4x-3}} \cdot 4 = \sqrt{4x-3} + \frac{2x}{\sqrt{4x-3}} = \frac{4x - 3 + 2x}{\sqrt{4x-3}} = \frac{6x - 3}{\sqrt{4x-3}}$

c $f(x) = \frac{e^{4x} - 5}{2\,e^{3x}} = \frac{1}{2}e^x - \frac{5}{2}e^{-3x}$ geeft $f'(x) = \frac{1}{2}e^x - \frac{5}{2}e^{-3x} \cdot -3 = \frac{1}{2}e^x + \frac{15}{2\,e^{3x}} = \frac{e^{4x} + 15}{2\,e^{3x}}$

d $f(x) = x^2\ln^2(x) - x^2\ln(x)$ geeft

$f'(x) = 2x \cdot \ln^2(x) + x^2 \cdot 2\ln(x) \cdot \frac{1}{x} - 2x \cdot \ln(x) - x^2 \cdot \frac{1}{x} = 2x\ln^2(x) + 2x\ln(x) - 2x\ln(x) - x$

$= 2x\ln^2(x) - x$

e $f(x) = 6x^2\sqrt{x^2 + 1}$ geeft

$f'(x) = 12x \cdot \sqrt{x^2+1} + 6x^2 \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x = 12x\sqrt{x^2+1} + \frac{6x^3}{\sqrt{x^2+1}} = \frac{12x(x^2+1) + 6x^3}{\sqrt{x^2+1}} = \frac{18x^3 + 12x}{\sqrt{x^2+1}}$

f $f(x) = \frac{5 \cdot 2^x}{\ln(2)} - \frac{e^{2x} - 1}{e^x} = \frac{5}{\ln(2)} \cdot 2^x - (e^x - e^{-x}) = \frac{5}{\ln(2)} \cdot 2^x - e^x + e^{-x}$ geeft

$f'(x) = \frac{5}{\ln(2)} \cdot 2^x \cdot \ln(2) - e^x + e^{-x} \cdot -1 = 5 \cdot 2^x - \left(e^x + \frac{1}{e^x}\right) = 5 \cdot 2^x - \frac{e^{2x} + 1}{e^x}$

3 **a** $f(x) = x + \frac{1}{1 + e^x} - \ln(1 + e^x) = x + (1 + e^x)^{-1} - \ln(1 + e^x)$ geeft

$f'(x) = 1 - (1 + e^x)^{-2} \cdot e^x - \frac{1}{1 + e^x} \cdot e^x = 1 - \frac{e^x}{(1 + e^x)^2} - \frac{e^x}{1 + e^x}$

$= \frac{(1 + e^x)^2 - e^x - e^x(1 + e^x)}{(1 + e^x)^2} = \frac{1 + 2\,e^x + e^{2x} - e^x - e^x - e^{2x}}{(1 + e^x)^2} = \frac{1}{(1 + e^x)^2}$

b $f(x) = \sin(x) - \frac{1}{3}\sin^3(x)$ geeft

$f'(x) = \cos(x) - \sin^2(x) \cdot \cos(x) = \cos(x) - (1 - \cos^2(x)) \cdot \cos(x) = \cos(x) - \cos(x) + \cos^3(x)$

$= \cos^3(x)$

c $f(x) = x + 2 - \frac{4}{x + 2} - 4\ln(x + 2) = x + 2 - 4(x + 2)^{-1} - 4\ln(x + 2)$ geeft

$f'(x) = 1 + 4(x + 2)^{-2} - 4 \cdot \frac{1}{x + 2} = 1 + \frac{4}{(x + 2)^2} - \frac{4}{x + 2}$

$= \frac{(x + 2)^2 + 4 - 4(x + 2)}{(x + 2)^2} = \frac{x^2 + 4x + 4 + 4 - 4x - 8}{(x + 2)^2} = \frac{x^2}{(x + 2)^2}$

d $f(x) = (0{,}2x^2 - 0{,}4x + 1{,}2)\sqrt{2x+3}$ geeft

$$f'(x) = (0{,}4x - 0{,}4) \cdot \sqrt{2x+3} + (0{,}2x^2 - 0{,}4x + 1{,}2) \cdot \frac{1}{2\sqrt{2x+3}} \cdot 2$$

$$= (0{,}4x - 0{,}4)\sqrt{2x+3} + \frac{0{,}2x^2 - 0{,}4x + 1{,}2}{\sqrt{2x+3}} = \frac{(0{,}4x - 0{,}4)(2x+3)}{\sqrt{2x+3}} + \frac{0{,}2x^2 - 0{,}4x + 1{,}2}{\sqrt{2x+3}}$$

$$= \frac{0{,}8x^2 + 1{,}2x - 0{,}8x - 1{,}2 + 0{,}2x^2 - 0{,}4x + 1{,}2}{\sqrt{2x+3}} = \frac{x^2}{\sqrt{2x+3}}$$

11.1 Primitieven en integralen

Bladzijde 99

1 $g(x) = x^3 + c$ geeft $g'(x) = 3x^2$ en dit is $f(x)$.
$g(2) = 15$ geeft $2^3 + c = 15$ oftewel $8 + c = 15$, dus $c = 7$.

Bladzijde 100

2 **a** $F(x) = \frac{1}{9}(3x+4)^3$ geeft $F'(x) = \frac{1}{3}(3x+4)^2 \cdot 3 = (3x+4)^2$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

b $F(x) = x - \ln(x+2)$ geeft $F'(x) = 1 - \frac{1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+1}{x+2}$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

c $F(x) = \frac{-1}{2(x^2+1)} = -\frac{1}{2}(x^2+1)^{-1}$ geeft $F'(x) = \frac{1}{2}(x^2+1)^{-2} \cdot 2x = \frac{x}{(x^2+1)^2}$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

3 **a** $F(x) = (x^2+1)^6 + 1$ geeft $F'(x) = 6(x^2+1)^5 \cdot 2x = 12x(x^2+1)^5$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

b $G(x) = (\frac{1}{2}x - \frac{1}{4})\mathrm{e}^{2x} - 2$ geeft $G'(x) = \frac{1}{2} \cdot \mathrm{e}^{2x} + (\frac{1}{2}x - \frac{1}{4}) \cdot \mathrm{e}^{2x} \cdot 2 = (\frac{1}{2} + x - \frac{1}{2}) \cdot \mathrm{e}^{2x} = x\,\mathrm{e}^{2x}$
Dus $G'(x) = g(x)$ oftewel G is een primitieve van g.

c $H(x) = \ln^2(x) + 2\ln(x) + 3$ geeft $H'(x) = 2\ln(x) \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2\ln(x) + 2}{x}$

Dus $H'(x) = h(x)$ oftewel H is een primitieve van h.

d $J(x) = \frac{\mathrm{e}^{3x} - 10}{2\,\mathrm{e}^x} - 4 = \frac{1}{2}\mathrm{e}^{2x} - 5\,\mathrm{e}^{-x} - 4$ geeft $J'(x) = \frac{1}{2}\mathrm{e}^{2x} \cdot 2 - 5\,\mathrm{e}^{-x} \cdot -1 = \mathrm{e}^{2x} + \frac{5}{\mathrm{e}^x} = \frac{\mathrm{e}^{3x} + 5}{\mathrm{e}^x}$

Dus $J'(x) = j(x)$ oftewel J is een primitieve van j.

e $K(x) = \sin^3(x)$ geeft $K'(x) = 3\sin^2(x) \cdot \cos(x) = 3(1 - \cos^2(x)) \cdot \cos(x) = 3\cos(x) - 3\cos^3(x)$
Dus $K'(x) = k(x)$ oftewel K is een primitieve van k.

4 $F(x) = \ln(x + \sqrt{x^2-1})$ geeft

$$F'(x) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}$$

$$= \frac{\sqrt{x^2-1} + x}{x + \sqrt{x^2-1}} \cdot \frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

5 **a** $F(x) = \frac{1}{5}x^5$ geeft $F'(x) = x^4$

Dus $F(x) = \frac{1}{5}x^5$ is een primitieve van $f(x) = x^4$.

b $G(x) = \frac{1}{4}\mathrm{e}^{4x+1}$ geeft $G'(x) = \frac{1}{4}\mathrm{e}^{4x+1} \cdot 4 = \mathrm{e}^{4x+1}$

Dus $G(x) = \frac{1}{4}\mathrm{e}^{4x+1}$ is een primitieve van $g(x) = \mathrm{e}^{4x+1}$.

c $H(x) = \frac{3^x}{\ln(3)} = \frac{1}{\ln(3)} \cdot 3^x$ geeft $H'(x) = \frac{1}{\ln(3)} \cdot 3^x \cdot \ln(3) = 3^x$

Dus $H(x) = \frac{3^x}{\ln(3)}$ is een primitieve van $h(x) = 3^x$.

d $K(x) = \frac{1}{2}\sin(2x)$ geeft $K'(x) = \frac{1}{2} \cdot \cos(2x) \cdot 2 = \cos(2x)$

Dus $K(x) = \frac{1}{2}\sin(2x)$ is een primitieve van $k(x) = \cos(2x)$.

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Bladzijde 101

6 $F(x) = a \cdot \frac{1}{n+1}x^{n+1} + c$ geeft $F'(x) = a \cdot \frac{1}{n+1} \cdot (n+1)x^n = ax^n$
Dus $F(x) = a \cdot \frac{1}{n+1}x^{n+1} + c$ zijn de primitieven van $f(x) = ax^n$.

$F(x) = \frac{g^x}{\ln(g)} + c = \frac{1}{\ln(g)} \cdot g^x + c$ geeft $F'(x) = \frac{1}{\ln(g)} \cdot g^x \cdot \ln(g) = g^x$
Dus $F(x) = \frac{g^x}{\ln(g)} + c$ zijn de primitieven van $f(x) = g^x$.
$F(x) = \mathrm{e}^x + c$ geeft $F'(x) = \mathrm{e}^x$
Dus $F(x) = \mathrm{e}^x + c$ zijn de primitieven van $f(x) = \mathrm{e}^x$.

$F(x) = \ln|x| + c = \begin{cases} \ln(x) + c \text{ voor } x > 0 \\ \ln(-x) + c \text{ voor } x < 0 \end{cases}$ geeft $F'(x) = \begin{cases} \frac{1}{x} \text{ voor } x > 0 \\ -\frac{1}{x} \cdot -1 = \frac{1}{x} \text{ voor } x < 0 \end{cases}$

Dus $F(x) = \ln|x| + c$ zijn de primitieven van $f(x) = \frac{1}{x}$.

$F(x) = x\ln(x) - x + c$ geeft $F'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x)$
Dus $F(x) = x\ln(x) - x + c$ zijn de primitieven van $f(x) = \ln(x)$.
$F(x) = \frac{1}{\ln(g)}(x\ln(x) - x) + c$ geeft

$F'(x) = \frac{1}{\ln(g)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1\right) = \frac{1}{\ln(g)} \cdot \ln(x) = \frac{\ln(x)}{\ln(g)} = {}^{g}\log(x)$
Dus $F(x) = \frac{1}{\ln(g)}(x\ln(x) - x) + c$ zijn de primitieven van $f(x) = {}^{g}\log(x)$.
$F(x) = -\cos(x) + c$ geeft $F'(x) = \sin(x)$
Dus $F(x) = -\cos(x) + c$ zijn de primitieven van $f(x) = \sin(x)$.
$F(x) = \sin(x) + c$ geeft $F'(x) = \cos(x)$
Dus $F(x) = \sin(x) + c$ zijn de primitieven van $f(x) = \cos(x)$.

Bladzijde 102

7 **a** $f(x) = ax^{-1}$ geeft $F(x) = a \cdot \frac{1}{-1+1}x^{-1+1} + c$ kan niet kloppen, want $\frac{1}{-1+1} = \frac{1}{0}$ bestaat niet en $x^0 = 1$ voor $x \neq 0$.

b $f(x) = x^{-1} = \frac{1}{x}$ geeft $F(x) = \ln|x| + c$

c $g(x) = a \cdot F(x)$ geeft $g'(x) = a \cdot F'(x) = a \cdot f(x)$
Dus als F een primitieve is van f, dan is $a \cdot F$ een primitieve van $a \cdot f$.

8 **a** $f(x) = 6x^2$ geeft $F(x) = 6 \cdot \frac{1}{3}x^3 + c = 2x^3 + c$

b $f(x) = 2x^3 + 5x^4$ geeft $F(x) = 2 \cdot \frac{1}{4}x^4 + 5 \cdot \frac{1}{5}x^5 + c = \frac{1}{2}x^4 + x^5 + c$

c $f(x) = \frac{x^4 - 2x}{2x^3} = \frac{1}{2}x - x^{-2}$ geeft $F(x) = \frac{1}{4}x^2 + x^{-1} + c = \frac{1}{4}x^2 + \frac{1}{x} + c$

d $f(x) = 10^x$ geeft $F(x) = \frac{10^x}{\ln(10)} + c$

e $f(x) = 5 \cdot 2^x$ geeft $F(x) = 5 \cdot \frac{2^x}{\ln(2)} + c = \frac{5 \cdot 2^x}{\ln(2)} + c$

f $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3}x^3 - \cos(x) + c$

g $f(x) = \frac{x^3 + 2}{x^4} = \frac{1}{x} + 2x^{-4}$ geeft $F(x) = \ln|x| + 2 \cdot -\frac{1}{3} \cdot x^{-3} + c = \ln|x| - \frac{2}{3x^3} + c$

h $f(x) = x\sqrt{x} - 2\cos(x) = x^{1\frac{1}{2}} - 2\cos(x)$ geeft
$F(x) = \frac{1}{2\frac{1}{2}}x^{2\frac{1}{2}} - 2\sin(x) + c = \frac{2}{5}x^2 \cdot \sqrt{x} - 2\sin(x) + c$

9 **a** $f(x) = x^3 - 3x$ geeft $F(x) = \frac{1}{4}x^4 - 1\frac{1}{2}x^2 + c$

b $f(x) = 5\,e^x$ geeft $F(x) = 5\,e^x + c$

c $f(x) = \dfrac{x^4 - 6}{2x^3} = \frac{1}{2}x - 3x^{-3}$ geeft $F(x) = \frac{1}{4}x^2 + \frac{3}{2}x^{-2} + c = \frac{1}{4}x^2 + \dfrac{3}{2x^2} + c$

d $f(x) = 3^x + x^3$ geeft $F(x) = \dfrac{3^x}{\ln(3)} + \frac{1}{4}x^4 + c$

e $f(x) = 2\ln(x)$ geeft $F(x) = 2(x\ln(x) - x) + c = 2x\ln(x) - 2x + c$

f $f(x) = \ln(2x) = \ln(2) + \ln(x)$ geeft $F(x) = \ln(2) \cdot x + x\ln(x) - x + c = x\ln(2) + x\ln(x) - x + c$

10 **a** $f(x) = e^{x+1} = e^x \cdot e = e \cdot e^x$ geeft $F(x) = e \cdot e^x + c = e^{x+1} + c$

b $f(x) = \dfrac{8}{x^3} = 8x^{-3}$ geeft $F(x) = 8 \cdot -\frac{1}{2}x^{-2} + c = -\dfrac{4}{x^2} + c$

c $f(x) = \dfrac{-x^2 + 2x + 3}{x^4} = -x^{-2} + 2x^{-3} + 3x^{-4}$ geeft

$F(x) = x^{-1} - x^{-2} - x^{-3} + c = \dfrac{1}{x} - \dfrac{1}{x^2} - \dfrac{1}{x^3} + c = \dfrac{x^2 - x - 1}{x^3} + c$

d $f(x) = \ln(x\sqrt{x}) = \ln(x^{1\frac{1}{2}}) = 1\frac{1}{2}\ln(x)$ geeft $F(x) = 1\frac{1}{2}(x\ln(x) - x) + c = 1\frac{1}{2}x\ln(x) - 1\frac{1}{2}x + c$

e $f(x) = {}^2\log\left(\dfrac{1}{x}\right) = {}^2\log(x^{-1}) = -{}^2\log(x)$ geeft

$F(x) = -\dfrac{1}{\ln(2)}(x\ln(x) - x) + c = -\dfrac{x\ln(x) - x}{\ln(2)} + c$

f $f(x) = 5 \cdot \log(2x) = 5 \cdot \log(2) + 5 \cdot \log(x)$ geeft

$F(x) = 5 \cdot \log(2) \cdot x + 5 \cdot \dfrac{1}{\ln(10)} \cdot (x\ln(x) - x) + c = 5x\log(2) + \dfrac{5x\ln(x) - 5x}{\ln(10)} + c$

11 $F_q(x) = (3x + q)x\sqrt{x} = 3x^{2\frac{1}{2}} + qx^{1\frac{1}{2}}$ geeft $F_q'(x) = 7\frac{1}{2}x^{1\frac{1}{2}} + 1\frac{1}{2}qx^{\frac{1}{2}} = 7\frac{1}{2}x\sqrt{x} + 1\frac{1}{2}q\sqrt{x}$

Dit moet gelijk zijn aan $f_p(x) = p\sqrt{x} + 2px\sqrt{x}$, dus $2p = 7\frac{1}{2}$ en $p = 1\frac{1}{2}q$

$p = 3\frac{3}{4}$ en $q = \frac{2}{3}p = \frac{2}{3} \cdot \frac{15}{4} = 2\frac{1}{2}$

Dus voor $p = 3\frac{3}{4}$ en $q = 2\frac{1}{2}$.

12 **a** $f(x) = 2x - 3$ geeft $F(x) = x^2 - 3x + c$

b $F(1) = 2$ geeft $1 - 3 + c = 2$, dus $c = 4$.

Dus $F(x) = x^2 - 3x + 4$.

c De grafiek van $F(x) = x^2 - 3x + c$ raakt de x-as, dus

$$\left.\begin{array}{l} D = 0 \\ D = (-3)^2 - 4 \cdot 1 \cdot c \end{array}\right\} \begin{array}{l} 9 - 4c = 0 \\ -4c = -9 \\ c = 2\frac{1}{4} \end{array}$$

Dus $F(x) = x^2 - 3x + 2\frac{1}{4}$.

13 $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$ geeft $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$

$F(1) = 7$ geeft $\frac{1}{5} - \frac{2}{3} + 1 + c = 7$, dus $c = 6\frac{7}{15}$.

Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

14 $F(x) = (ax^2 + bx + c)\,e^{2x}$ geeft

$F'(x) = (2ax + b) \cdot e^{2x} + (ax^2 + bx + c) \cdot e^{2x} \cdot 2 = (2ax^2 + (2a + 2b)x + b + 2c)\,e^{2x}$

Dit moet gelijk zijn aan $f(x) = x^2 e^{2x}$, dus $2a = 1 \wedge 2a + 2b = 0 \wedge b + 2c = 0$.

$2a = 1$ geeft $a = \frac{1}{2}$

$2a + 2b = 0$ en $a = \frac{1}{2}$ geeft $1 + 2b = 0$, dus $b = -\frac{1}{2}$.

$b + 2c = 0$ en $b = -\frac{1}{2}$ geeft $-\frac{1}{2} + 2c = 0$, dus $c = \frac{1}{4}$.

Dus $a = \frac{1}{2}$, $b = -\frac{1}{2}$ en $c = \frac{1}{4}$.

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Bladzijde 103

15 **a** $A(p) = O(\text{rechthoek}) + O(\text{driehoek}) = p \cdot b + \frac{1}{2}p \cdot (ap + b - b) = bp + \frac{1}{2}p \cdot ap = \frac{1}{2}ap^2 + bp$

b $A'(p) = ap + b$

Dus $A'(p) = f(p)$.

Bladzijde 105

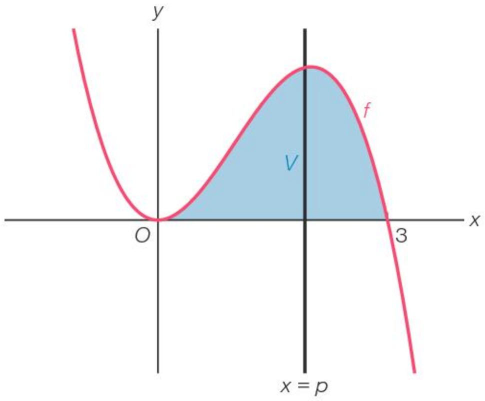
16 **a** $f(x) = 0$ geeft $3x^2 - x^3 = 0$

$x^2(3 - x) = 0$

$x = 0 \vee x = 3$

$O(V) = \int_0^3 (3x^2 - x^3)\,dx = \left[x^3 - \frac{1}{4}x^4\right]_0^3 = 3^3 - \frac{1}{4} \cdot 3^4 - 0 = 27 - 20\frac{1}{4} = 6\frac{3}{4}$

b



$\int_0^p (3x^2 - x^3)\,dx = \frac{1}{2} \cdot 6\frac{3}{4}$

$\left[x^3 - \frac{1}{4}x^4\right]_0^p = 3\frac{3}{8}$

$p^3 - \frac{1}{4}p^4 - 0 = 3\frac{3}{8}$

$p^3 - \frac{1}{4}p^4 = 3\frac{3}{8}$

Voer in $y_1 = x^3 - \frac{1}{4}x^4$ en $y_2 = 3\frac{3}{8}$.

De optie snijpunt geeft $x \approx 1{,}84$.

Dus $p \approx 1{,}84$.

17 $O(W) = \frac{1}{2}O(V)$

$\int_{\frac{1}{3}\pi}^{p} \sin(x)\,dx = \frac{1}{2} \cdot \int_0^{\pi} \sin(x)\,dx$

$\left[-\cos(x)\right]_{\frac{1}{3}\pi}^{p} = \frac{1}{2}\left[-\cos(x)\right]_0^{\pi}$

$-\cos(p) + \cos(\frac{1}{3}\pi) = \frac{1}{2}(-\cos(\pi) + \cos(0))$

$-\cos(p) + \frac{1}{2} = \frac{1}{2}(1 + 1)$

$-\cos(p) + \frac{1}{2} = 1$

$-\cos(p) = \frac{1}{2}$

$\cos(p) = -\frac{1}{2}$

$p = \frac{2}{3}\pi$

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18 **a** $O(V) = \int_0^{12} \sqrt{x}\,dx = \int_0^{12} x^{\frac{1}{2}}\,dx = \left[\frac{2}{3}x^{1\frac{1}{2}}\right]_0^{12} = \left[\frac{2}{3}x\sqrt{x}\right]_0^{12} = \frac{2}{3} \cdot 12\sqrt{12} - 0 = 8\sqrt{12} = 16\sqrt{3}$

b $O(V) = 18$ geeft $\int_0^{p} \sqrt{x}\,dx = 18$

$\left[\frac{2}{3}x\sqrt{x}\right]_0^{p} = 18$

$\frac{2}{3}p\sqrt{p} = 18$

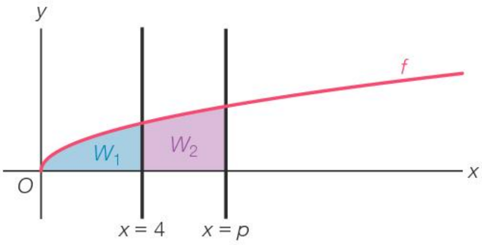
$p\sqrt{p} = 27$

$p^{1\frac{1}{2}} = 3^3$

$p = 3^2$

$p = 9$

c



$O(W_1) = O(W_2)$

$\int_0^{4} \sqrt{x}\,dx = \int_4^{p} \sqrt{x}\,dx$

$\left[\frac{2}{3}x\sqrt{x}\right]_0^{4} = \left[\frac{2}{3}x\sqrt{x}\right]_4^{p}$

$\frac{2}{3} \cdot 4\sqrt{4} - 0 = \frac{2}{3}p\sqrt{p} - \frac{2}{3} \cdot 4\sqrt{4}$

$\frac{2}{3}p\sqrt{p} = 2 \cdot \frac{2}{3} \cdot 4\sqrt{4}$

$p^{1\frac{1}{2}} = 16$

$p = 16^{\frac{2}{3}}$

$p = (2^4)^{\frac{2}{3}}$

$p = 2^{2\frac{2}{3}}$

$p = 4 \cdot \sqrt[3]{4}$

Bladzijde 106

19 **a** $O(V) = 5$

$\int_0^{p} e^x\,dx = 5$

$\left[e^x\right]_0^{p} = 5$

$e^p - e^0 = 5$

$e^p - 1 = 5$

$e^p = 6$

$p = \ln(6)$

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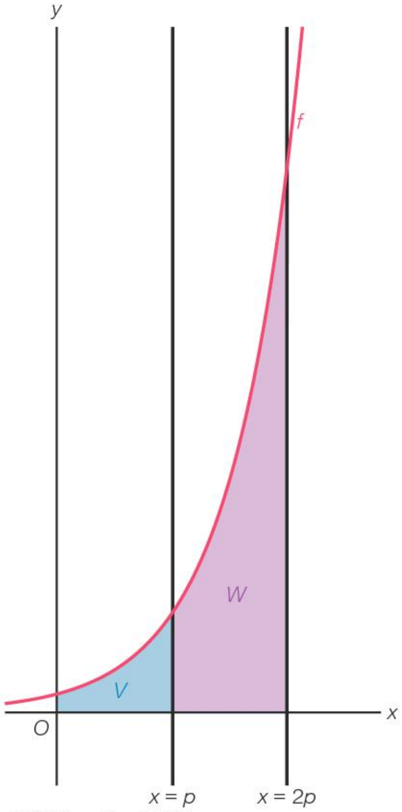
b



$O(W) = 2 \cdot O(V)$

$\int_p^{2p} e^x \, dx = 2\int_0^p e^x \, dx$

$[e^x]_p^{2p} = 2[e^x]_0^p$

$e^{2p} - e^p = 2(e^p - e^0)$

$e^{2p} - e^p = 2\,e^p - 2$

$(e^p)^2 - 3\,e^p + 2 = 0$

Stel $e^p = u$.

$u^2 - 3u + 2 = 0$

$(u-1)(u-2) = 0$

$u = 1 \vee u = 2$

$e^p = 1 \vee e^p = 2$

$p = 0 \quad \vee \quad p = \ln(2)$

vold. niet

Dus $p = \ln(2)$.

20 **a** $O(V) = \int_1^{e^2} \ln(x)\,dx = [x\ln(x) - x]_1^{e^2} = e^2\ln(e^2) - e^2 - (\ln(1) - 1) = 2\,e^2 - e^2 + 1 = e^2 + 1$

b $O(V) = 10$

$\int_1^p \ln(x)\,dx = 10$

$[x\ln(x) - x]_1^p = 10$

$p\ln(p) - p - (\ln(1) - 1) = 10$

$p\ln(p) - p + 1 = 10$

$p\ln(p) - p = 9$

Voer in $y_1 = x\ln(x) - x$ en $y_2 = 9$.

De optie snijpunt geeft $x \approx 8{,}174$.

Dus $p \approx 8{,}174$.

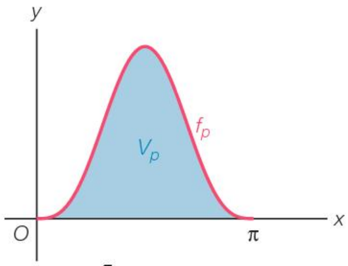
21 **a** $F_p(x) = \frac{1}{3}p\cos^3(x) - p\cos(x)$ geeft

$F_p'(x) = p\cos^2(x) \cdot -\sin(x) - p \cdot -\sin(x) = -p\cos^2(x)\sin(x) + p\sin(x)$
$= p\sin(x)(-\cos^2(x) + 1) = p\sin(x) \cdot \sin^2(x) = p\sin^3(x)$

Dus $F_p(x) = \frac{1}{3}p\cos^3(x) - p\cos(x)$ is een primitieve van $f_p(x) = p\sin^3(x)$.

b $f_p(x) = 0$ geeft $p\sin^3(x) = 0$
$\sin(x) = 0$
$x = 0 + k \cdot \pi$

x in $[0, \pi]$ geeft $x = 0 \vee x = \pi$



$O(V_p) = \int_0^{\pi} f_p(x)\,dx = \left[\frac{1}{3}p\cos^3(x) - p\cos(x)\right]_0^{\pi}$
$= \frac{1}{3}p \cdot (-1)^3 - p \cdot -1 - (\frac{1}{3}p \cdot 1^3 - p \cdot 1) = -\frac{1}{3}p + p - \frac{1}{3}p + p = 1\frac{1}{3}p$

$O(V_p) = 4$ geeft $1\frac{1}{3}p = 4$
$p = 3$

11.2 Oppervlakten

Bladzijde 108

22 **a** $F(x) = a(3x + 1)^6 + c$ geeft $F'(x) = a \cdot 6(3x + 1)^5 \cdot 3 = 18a(3x + 1)^5$

b $F'(x) = 18a(3x + 1)^5$ moet gelijk zijn aan $f(x) = (3x + 1)^5$, dus $18a = 1$ en dit geeft $a = \frac{1}{18}$.

Bladzijde 109

23 $\left[\frac{1}{a}F(ax + b) + c\right]' = \frac{1}{a} \cdot f(ax + b) \cdot a = f(ax + b)$

Dus $\frac{1}{a}F(ax + b) + c$ zijn de primitieven van $f(ax + b)$.

24 **a** $f(x) = (ax + b)^n$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{n+1}(ax + b)^{n+1} + c$

b $f(x) = g^{ax+b}$ geeft $F(x) = \frac{1}{a} \cdot \frac{g^{ax+b}}{\ln(g)} + c$

c $f(x) = \frac{1}{ax + b}$ geeft $F(x) = \frac{1}{a}\ln|ax + b| + c$

d $f(x) = \ln(ax + b)$ geeft $F(x) = \frac{1}{a}((ax + b)\ln(ax + b) - (ax + b)) + c$

e $f(x) = {}^g\log(ax + b)$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{\ln(g)}((ax + b)\ln(ax + b) - (ax + b)) + c$

f $f(x) = \cos(ax + b)$ geeft $F(x) = \frac{1}{a}\sin(ax + b) + c$

25 **a** $f(x) = (2x - 1)^6$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{7}(2x - 1)^7 + c = \frac{1}{14}(2x - 1)^7 + c$

b $f(x) = \frac{1}{(3x + 4)^3} = (3x + 4)^{-3}$ geeft $F(x) = \frac{1}{3} \cdot -\frac{1}{2}(3x + 4)^{-2} + c = -\frac{1}{6(3x + 4)^2} + c$

c $f(x) = 4\sqrt{3 - 2x} = 4(3 - 2x)^{\frac{1}{2}}$ geeft $F(x) = 4 \cdot -\frac{1}{2} \cdot \frac{2}{3}(3 - 2x)^{1\frac{1}{2}} + c = -1\frac{1}{3}(3 - 2x)\sqrt{3 - 2x} + c$

d $f(x) = \frac{2}{\sqrt{1 - x}} = 2(1 - x)^{-\frac{1}{2}}$ geeft $F(x) = 2 \cdot -2(1 - x)^{\frac{1}{2}} + c = -4\sqrt{1 - x} + c$

e $f(x) = \sin(2x + \frac{1}{3}\pi)$ geeft $F(x) = -\frac{1}{2}\cos(2x + \frac{1}{3}\pi) + c$

f $f(x) = 3\cos(\frac{1}{2}x - \frac{1}{6}\pi)$ geeft $F(x) = 3 \cdot 2\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c$

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26 **a** $f(x)=\dfrac{1}{x-1}$ geeft $F(x)=\ln|x-1|+c$

b $f(x)=(2x+1)\sqrt{2x+1}=(2x+1)^{1\frac{1}{2}}$ geeft

$F(x)=\frac{1}{2}\cdot\frac{2}{5}(2x+1)^{2\frac{1}{2}}+c=\frac{1}{5}(2x+1)^2\cdot\sqrt{2x+1}+c$

c $f(x)=\mathrm{e}^{4x-1}$ geeft $F(x)=\frac{1}{4}\mathrm{e}^{4x-1}+c$

d $f(x)=\ln(4x-1)$ geeft

$F(x)=\frac{1}{4}((4x-1)\ln(4x-1)-(4x-1))+c=(x-\frac{1}{4})\ln(4x-1)-(x-\frac{1}{4})+c$

e $f(x)=\dfrac{3}{2x-5}$ geeft $F(x)=3\cdot\frac{1}{2}\ln|2x-5|+c=1\frac{1}{2}\ln|2x-5|+c$

f $f(x)=2^{3x}$ geeft $F(x)=\frac{1}{3}\cdot\dfrac{2^{3x}}{\ln(2)}+c=\dfrac{2^{3x}}{3\ln(2)}+c$

g $f(x)=3^{2-5x}$ geeft $F(x)=-\frac{1}{5}\cdot\dfrac{3^{2-5x}}{\ln(3)}+c=-\dfrac{3^{2-5x}}{5\ln(3)}+c$

h $f(x)={}^2\log(5x+3)$ geeft

$F(x)=\frac{1}{5}\cdot\dfrac{1}{\ln(2)}\cdot((5x+3)\ln(5x+3)-(5x+3))+c=\dfrac{(5x+3)\ln(5x+3)-(5x+3)}{5\ln(2)}+c$

27 **a** $f(x)=\dfrac{3}{(2x-1)^4}=3(2x-1)^{-4}$ geeft $F(x)=3\cdot\frac{1}{2}\cdot-\frac{1}{3}(2x-1)^{-3}+c=-\dfrac{1}{2(2x-1)^3}+c$

b $f(x)=(4x+3)\sqrt{4x+3}=(4x+3)^{1\frac{1}{2}}$ geeft $F(x)=\frac{1}{4}\cdot\frac{2}{5}(4x+3)^{2\frac{1}{2}}+c=\frac{1}{10}(4x+3)^2\cdot\sqrt{4x+3}+c$

c $f(x)=2\sin(3x+\frac{1}{2}\pi)$ geeft $F(x)=2\cdot-\frac{1}{3}\cos(3x+\frac{1}{2}\pi)+c=-\frac{2}{3}\cos(3x+\frac{1}{2}\pi)+c$

d $f(x)=x^2-\ln(\frac{1}{2}x+1)$ geeft

$F(x)=\frac{1}{3}x^3-2((\frac{1}{2}x+1)\ln(\frac{1}{2}x+1)-(\frac{1}{2}x+1))+c=\frac{1}{3}x^3-(x+2)\ln(\frac{1}{2}x+1)+x+2+c$

e $f(x)=\dfrac{6}{3x+1}$ geeft $F(x)=6\cdot\frac{1}{3}\cdot\ln|3x+1|+c=2\ln|3x+1|+c$

f $f(x)=\mathrm{e}^{4x+1}-4^{\mathrm{e}x}$ geeft $F(x)=\frac{1}{4}\mathrm{e}^{4x+1}-\dfrac{1}{\mathrm{e}}\cdot\dfrac{4^{\mathrm{e}x}}{\ln(4)}+c=\frac{1}{4}\mathrm{e}^{4x+1}-\dfrac{4^{\mathrm{e}x}}{\mathrm{e}\ln(4)}+c$

g $f(x)=5\cdot\log(3x+2)$ geeft

$F(x)=5\cdot\frac{1}{3}\cdot\dfrac{1}{\ln(10)}\cdot((3x+2)\ln(3x+2)-(3x+2))+c=\dfrac{5((3x+2)\ln(3x+2)-(3x+2))}{3\ln(10)}+c$

h $f(x)=3x-\cos(3x+1)$ geeft $F(x)=1\frac{1}{2}x^2-\frac{1}{3}\sin(3x+1)+c$

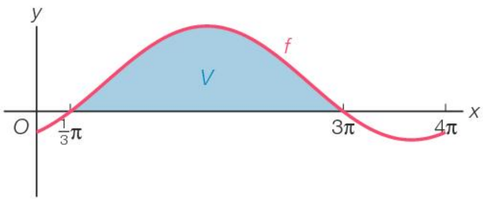
28 **a** $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}(2\sin(x-\tfrac{1}{6}\pi))\,\mathrm{d}x=\left[-2\cos(x-\tfrac{1}{6}\pi)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}=-2\cos(\tfrac{1}{6}\pi)+2\cos(0)=-\sqrt{3}+2$

b $\displaystyle\int_{0}^{\frac{1}{3}\pi}(2x+\cos(\tfrac{1}{2}x))\,\mathrm{d}x=\left[x^2+2\sin(\tfrac{1}{2}x)\right]_0^{\frac{1}{3}\pi}=(\tfrac{1}{3}\pi)^2+2\sin(\tfrac{1}{6}\pi)-(0+0)=\tfrac{1}{9}\pi^2+2\cdot\tfrac{1}{2}=\tfrac{1}{9}\pi^2+1$

29 $f(x) = 0$ geeft $1 + 2\cos(\tfrac{1}{2}x - \tfrac{5}{6}\pi) = 0$

$$2\cos(\tfrac{1}{2}x - \tfrac{5}{6}\pi) = -1$$
$$\cos(\tfrac{1}{2}x - \tfrac{5}{6}\pi) = -\tfrac{1}{2}$$
$$\tfrac{1}{2}x - \tfrac{5}{6}\pi = \tfrac{2}{3}\pi + k \cdot 2\pi \vee \tfrac{1}{2}x - \tfrac{5}{6}\pi = -\tfrac{2}{3}\pi + k \cdot 2\pi$$
$$\tfrac{1}{2}x = 1\tfrac{1}{2}\pi + k \cdot 2\pi \vee \tfrac{1}{2}x = \tfrac{1}{6}\pi + k \cdot 2\pi$$
$$x = 3\pi + k \cdot 4\pi \vee x = \tfrac{1}{3}\pi + k \cdot 4\pi$$

x in $[0, 4\pi]$ geeft $x = \tfrac{1}{3}\pi \vee x = 3\pi$

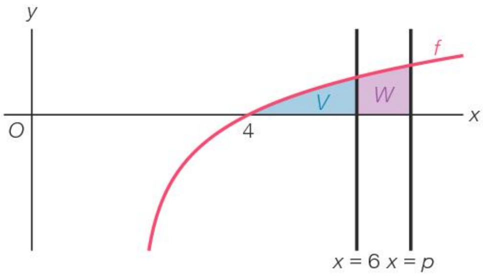


$$O(V) = \int_{\frac{1}{3}\pi}^{3\pi} (1 + 2\cos(\tfrac{1}{2}x - \tfrac{5}{6}\pi))\,dx = \left[x + 4\sin(\tfrac{1}{2}x - \tfrac{5}{6}\pi)\right]_{\frac{1}{3}\pi}^{3\pi} = 3\pi + 4\sin(\tfrac{2}{3}\pi) - (\tfrac{1}{3}\pi + 4\sin(-\tfrac{2}{3}\pi))$$
$$= 3\pi + 4 \cdot \tfrac{1}{2}\sqrt{3} - (\tfrac{1}{3}\pi + 4 \cdot -\tfrac{1}{2}\sqrt{3}) = 3\pi + 2\sqrt{3} - \tfrac{1}{3}\pi + 2\sqrt{3} = 2\tfrac{2}{3}\pi + 4\sqrt{3}$$

Bladzijde 110

30 $f(x) = 0$ geeft $\ln(\tfrac{1}{2}x - 1) = 0$

$$\tfrac{1}{2}x - 1 = 1$$
$$\tfrac{1}{2}x = 2$$
$$x = 4$$



$f(x) = \ln(\tfrac{1}{2}x - 1)$ geeft $F(x) = 2((\tfrac{1}{2}x - 1)\ln(\tfrac{1}{2}x - 1) - (\tfrac{1}{2}x - 1)) = (x - 2)\ln(\tfrac{1}{2}x - 1) - x + 2$

$$O(V) = \int_4^6 f(x)\,dx = \left[(x - 2)\ln(\tfrac{1}{2}x - 1) - x + 2\right]_4^6 = 4\ln(2) - 6 + 2 - (2\ln(1) - 4 + 2) = 4\ln(2) - 2$$

$$O(W) = \int_6^p f(x)\,dx = \left[(x - 2)\ln(\tfrac{1}{2}x - 1) - x + 2\right]_6^p = (p - 2)\ln(\tfrac{1}{2}p - 1) - p + 2 - (4\ln(2) - 4)$$
$$= (p - 2)\ln(\tfrac{1}{2}p - 1) - p + 6 - 4\ln(2)$$

Voer in $y_1 = 4\ln(2) - 2$ en $y_2 = (x - 2)\ln(\tfrac{1}{2}x - 1) - x + 6 - 4\ln(2)$.
De optie snijpunt geeft $x = 2{,}119...$ en $x = 6{,}960...$
Dus $p \approx 6{,}96$.

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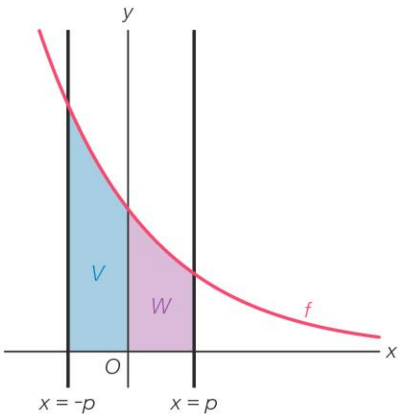
31



$O(V) = 2O(W)$

$\int_{-p}^{0} 2\,e^{1-\frac{1}{3}x}\,dx = 2\int_{0}^{p} 2\,e^{1-\frac{1}{3}x}\,dx$

$\left[2\cdot -3\,e^{1-\frac{1}{3}x}\right]_{-p}^{0} = 2\left[2\cdot -3\,e^{1-\frac{1}{3}x}\right]_{0}^{p}$

$\left[-6\,e^{1-\frac{1}{3}x}\right]_{-p}^{0} = 2\left[-6\,e^{1-\frac{1}{3}x}\right]_{0}^{p}$

$-6\,e^{1} + 6\,e^{1+\frac{1}{3}p} = 2(-6\,e^{1-\frac{1}{3}p} + 6\,e^{1})$

$-6e + 6e\cdot e^{\frac{1}{3}p} = -12e\cdot e^{-\frac{1}{3}p} + 12e$

$-1 + e^{\frac{1}{3}p} = -2\,e^{-\frac{1}{3}p} + 2$

Stel $e^{\frac{1}{3}p} = u$.

$-1 + u = -\frac{2}{u} + 2$

$-u + u^2 = -2 + 2u$

$u^2 - 3u + 2 = 0$

$(u-1)(u-2) = 0$

$u = 1 \vee u = 2$

$e^{\frac{1}{3}p} = 1 \vee e^{\frac{1}{3}p} = 2$

$\frac{1}{3}p = 0 \vee \frac{1}{3}p = \ln(2)$

$p = 0 \quad \vee \quad p = 3\ln(2)$

vold. niet

Dus $p = 3\ln(2)$.

32 **a** De regel gaat over functies van de vorm $f(ax+b)$ en g is van de vorm $f(ax^2+b)$.

b $G(x) = a(4x^2-1)^{1\frac{1}{2}}$ geeft $G'(x) = 1\frac{1}{2}a(4x^2-1)^{\frac{1}{2}}\cdot 8x = 12ax\sqrt{4x^2-1}$

Er bestaat geen waarde van a waarvoor $G'(x) = \sqrt{4x^2-1}$.

33 **a** Stel A is het gebied begrensd door de x-as, de lijnen $x = a$ en $x = b$ en de grafiek van f.

Dit geeft $O(A) = \int_{a}^{b} f(x)\,dx$.

Stel B is het gebied begrensd door de x-as, de lijnen $x = a$ en $x = b$ en de grafiek van g.

Dit geeft $O(B) = \int_{a}^{b} g(x)\,dx$.

Dus $O(V) = O(A) - O(B) = \int_{a}^{b} f(x)\,dx - \int_{a}^{b} g(x)\,dx$.

b $O(V) = \int_{a}^{b} f(x)\,dx - \int_{a}^{b} g(x)\,dx = \left[F(x)\right]_{a}^{b} - \left[G(x)\right]_{a}^{b} = F(b) - F(a) - (G(b) - G(a))$

$= F(b) - F(a) - G(b) + G(a) = F(b) - G(b) - F(a) + G(a)$

$= F(b) - G(b) - (F(a) - G(a))$

c $O(V) = F(b) - G(b) - (F(a) - G(a)) = \left[F(x) - G(x)\right]_{a}^{b} = \int_{a}^{b} (f(x) - g(x))\,dx$

Bladzijde 112

34 Verschuif de grafieken van f en g c omhoog waarbij c voldoende groot is om V geheel boven de x-as te krijgen. De beeldgrafieken zijn dan $y = f(x) + c$ en $y = g(x) + c$.

Dus $O(V) = \int_a^b (f(x) + c - (g(x) + c))\,dx = \int_a^b (f(x) + c - g(x) - c)\,dx = \int_a^b (f(x) - g(x))\,dx$.

35 **a** $O(V) = \int_0^{\pi} \sin(x)\,dx = [-\cos(x)]_0^{\pi} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$

$O(W) = -\int_{\pi}^{2\pi} \sin(x)\,dx = -[-\cos(x)]_{\pi}^{2\pi} = [\cos(x)]_{\pi}^{2\pi} = \cos(2\pi) - \cos(\pi) = 1 - -1 = 2$

b $\int_0^{2\pi} \sin(x)\,dx = [-\cos(x)]_0^{2\pi} = -\cos(2\pi) + \cos(0) = -1 + 1 = 0$

$\int_0^{2\pi} \sin(x)\,dx = \int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{2\pi} \sin(x)\,dx = O(V) + -O(W) = 2 - 2 = 0$

Bladzijde 113

36 $f(x) = g(x)$ geeft $\sin(x) = \cos(2x)$

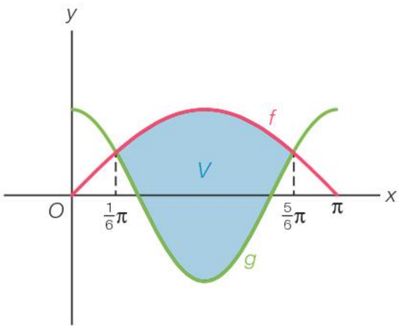
$\cos(x - \frac{1}{2}\pi) = \cos(2x)$

$x - \frac{1}{2}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x + k \cdot 2\pi$

$-x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, \pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$



$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (f(x) - g(x))\,dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin(x) - \cos(2x))\,dx = [-\cos(x) - \frac{1}{2}\sin(2x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$

$= -\cos(\frac{5}{6}\pi) - \frac{1}{2}\sin(1\frac{2}{3}\pi) - (-\cos(\frac{1}{6}\pi) - \frac{1}{2}\sin(\frac{1}{3}\pi)) = \frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{3})$

$= \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

37 **a** $x^3 = 2x$

$x = 0 \vee x^2 = 2$

$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2}$

$O(V) = \int_0^{\sqrt{2}} (2x - x^3)\,dx = [x^2 - \frac{1}{4}x^4]_0^{\sqrt{2}} = 2 - \frac{1}{4} \cdot 4 - 0 = 1$

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b $\int_0^p (2x - x^3)\,dx = \frac{1}{2}$ met $0 < p < \sqrt{2}$

$\left[x^2 - \frac{1}{4}x^4\right]_0^p = \frac{1}{2}$

$p^2 - \frac{1}{4}p^4 - 0 = \frac{1}{2}$

$4p^2 - p^4 = 2$

$p^4 - 4p^2 + 2 = 0$

Stel $p^2 = u$.

$u^2 - 4u + 2 = 0$

$(u-2)^2 - 4 + 2 = 0$

$(u-2)^2 = 2$

$u - 2 = \sqrt{2} \vee u - 2 = -\sqrt{2}$

$u = 2 + \sqrt{2} \vee u = 2 - \sqrt{2}$

$p^2 = 2 + \sqrt{2} \vee p^2 = 2 - \sqrt{2}$

vold. niet $\quad p = \sqrt{2 - \sqrt{2}} \vee p = -\sqrt{2 - \sqrt{2}}$

vold. niet

Dus $p = \sqrt{2 - \sqrt{2}}$.

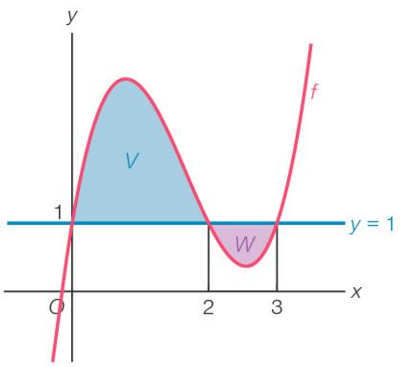
38 $f(x) = 1$ geeft $x^3 - 5x^2 + 6x + 1 = 1$

$x^3 - 5x^2 + 6x = 0$

$x(x^2 - 5x + 6) = 0$

$x(x-2)(x-3) = 0$

$x = 0 \vee x = 2 \vee x = 3$



$O(V) = \int_0^2 (f(x) - 1)\,dx = \int_0^2 (x^3 - 5x^2 + 6x)\,dx = \left[\frac{1}{4}x^4 - 1\frac{2}{3}x^3 + 3x^2\right]_0^2$

$= \frac{1}{4} \cdot 2^4 - 1\frac{2}{3} \cdot 2^3 + 3 \cdot 2^2 - 0 = 2\frac{2}{3}$

$O(W) = \int_2^3 (1 - f(x))\,dx = \int_2^3 (1 - (x^3 - 5x^2 + 6x + 1))\,dx = \int_2^3 (1 - x^3 + 5x^2 - 6x - 1)\,dx$

$= \int_2^3 (-x^3 + 5x^2 - 6x)\,dx = \left[-\frac{1}{4}x^4 + 1\frac{2}{3}x^3 - 3x^2\right]_2^3$

$= -\frac{1}{4} \cdot 3^4 + 1\frac{2}{3} \cdot 3^3 - 3 \cdot 3^2 - (-\frac{1}{4} \cdot 2^4 + 1\frac{2}{3} \cdot 2^3 - 3 \cdot 2^2) = \frac{5}{12}$

$\frac{O(V)}{O(W)} = \frac{2\frac{2}{3}}{\frac{5}{12}} = \frac{8}{3} \cdot \frac{12}{5} = \frac{32}{5}$

Dus $O(V)$ is $6\frac{2}{5}$ keer zo groot als $O(W)$.

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39 $f(x) = g(x)$ geeft $x - 2\sqrt{x} = -x$

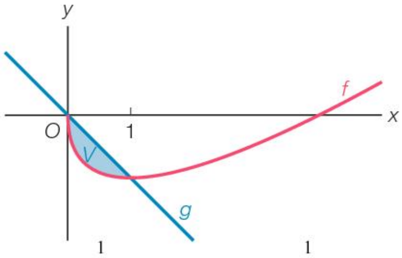
$2x = 2\sqrt{x}$

$x = \sqrt{x}$

kwadrateren geeft

$x^2 = x$

$x = 0 \vee x = 1$



$$O(V) = \int_0^1 (g(x) - f(x))\,dx = \int_0^1 (-x - (x - 2\sqrt{x}))\,dx = \int_0^1 (-2x + 2\sqrt{x})\,dx = \int_0^1 (-2x + 2x^{\frac{1}{2}})\,dx$$

$$= \left[-x^2 + 2 \cdot \tfrac{2}{3}x^{1\frac{1}{2}}\right]_0^1 = \left[-x^2 + 1\tfrac{1}{3}x\sqrt{x}\right]_0^1 = -1^2 + 1\tfrac{1}{3} \cdot 1\sqrt{1} - 0 = -1 + 1\tfrac{1}{3} = \tfrac{1}{3}$$

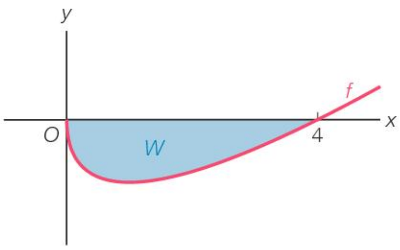
$f(x) = 0$ geeft $x - 2\sqrt{x} = 0$

$x = 2\sqrt{x}$

kwadrateren geeft

$x^2 = 4x$

$x = 0 \vee x = 4$



$$O(W) = -\int_0^4 f(x)\,dx = -\int_0^4 (x - 2\sqrt{x})\,dx = \int_0^4 (-x + 2x^{\frac{1}{2}})\,dx = \left[-\tfrac{1}{2}x^2 + 2 \cdot \tfrac{2}{3}x^{1\frac{1}{2}}\right]_0^4 = \left[-\tfrac{1}{2}x^2 + 1\tfrac{1}{3}x\sqrt{x}\right]_0^4$$

$$= -\tfrac{1}{2} \cdot 4^2 + 1\tfrac{1}{3} \cdot 4\sqrt{4} - 0 = -8 + 10\tfrac{2}{3} = 2\tfrac{2}{3}$$

$8 \cdot O(V) = 8 \cdot \tfrac{1}{3} = 2\tfrac{2}{3} = O(W)$

40 **a** $f(x) = -1\tfrac{1}{2}$ geeft $\dfrac{x^2 + x + 1}{x} = -1\tfrac{1}{2}$

$x^2 + x + 1 = -1\tfrac{1}{2}x$

$x^2 + 2\tfrac{1}{2}x + 1 = 0$

$(x + \tfrac{1}{2})(x + 2) = 0$

$x = -\tfrac{1}{2} \vee x = -2$

$$O(V) = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2 + x + 1}{x} - -1\tfrac{1}{2}\right)dx = \int_{-2}^{-\frac{1}{2}} \left(x + 1 + \frac{1}{x} + 1\tfrac{1}{2}\right)dx = \int_{-2}^{-\frac{1}{2}} \left(x + \frac{1}{x} + 2\tfrac{1}{2}\right)dx$$

$$= \left[\tfrac{1}{2}x^2 + \ln|x| + 2\tfrac{1}{2}x\right]_{-2}^{-\frac{1}{2}} = \tfrac{1}{8} + \ln(\tfrac{1}{2}) - 1\tfrac{1}{4} - (2 + \ln(2) - 5) = \tfrac{1}{8} + \ln(\tfrac{1}{2}) - 1\tfrac{1}{4} - 2 - \ln(2) + 5$$

$$= 1\tfrac{7}{8} + \ln(\tfrac{1}{2}) - \ln(2) = 1\tfrac{7}{8} + \ln(\tfrac{1}{4})$$

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b $O(W) = 2$ geeft $\int_1^p \left(\frac{x^2+x+1}{x} - (x+1)\right) dx = 2$

$\int_1^p \left(x + 1 + \frac{1}{x} - (x+1)\right) dx = 2$

$\int_1^p \frac{1}{x} dx = 2$

$\left[\ln|x|\right]_1^p = 2$

$\ln|p| - \ln(1) = 2$

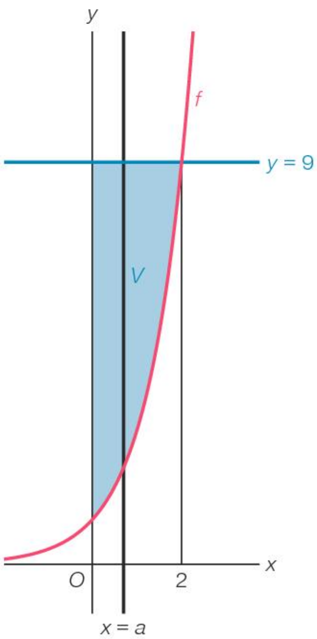
$\ln(p) = 2$

$p = e^2$

Bladzijde 114

41 $f(x) = 9$ geeft $3^x = 9$

$x = 2$



$O(V) = \int_0^2 (9 - 3^x)\,dx = \left[9x - \frac{3^x}{\ln(3)}\right]_0^2 = 18 - \frac{9}{\ln(3)} - \left(0 - \frac{1}{\ln(3)}\right) = 18 - \frac{8}{\ln(3)}$

$O(W) = \int_0^a (9 - 3^x)\,dx = \left[9x - \frac{3^x}{\ln(3)}\right]_0^a = 9a - \frac{3^a}{\ln(3)} - \left(0 - \frac{1}{\ln(3)}\right) = 9a + \frac{1 - 3^a}{\ln(3)}$

Los op $9a + \frac{1 - 3^a}{\ln(3)} = 9 - \frac{4}{\ln(3)}$.

Voer in $y_1 = 9x + \frac{1 - 3^x}{\ln(3)}$ en $y_2 = 9 - \frac{4}{\ln(3)}$.

De optie snijpunt geeft $x = 0{,}716...$ en $x = 2{,}875...$

Dus $a \approx 0{,}72$.

42 $f(x) = 8$ geeft $\frac{8}{x^2} = 8$

$x^2 = 1$

$x = 1 \vee x = -1$

vold. niet

$$O(V) = 1 \cdot 8 + \int_1^8 \frac{8}{x^2}\,dx = 8 + \int_1^8 8x^{-2}\,dx = 8 + \left[-8x^{-1}\right]_1^8 = 8 + \left[-\frac{8}{x}\right]_1^8 = 8 - \frac{8}{8} + \frac{8}{1} = 15$$

$$O(V_1) = \tfrac{2}{3}O(V) \text{ geeft } 8 + \int_1^a \frac{8}{x^2}\,dx = \tfrac{2}{3} \cdot 15$$

$$8 + \int_1^a 8x^{-2}\,dx = 10$$

$$\int_1^a 8x^{-2}\,dx = 2$$

$$\left[-\frac{8}{x}\right]_1^a = 2$$

$$-\frac{8}{a} + \frac{8}{1} = 2$$

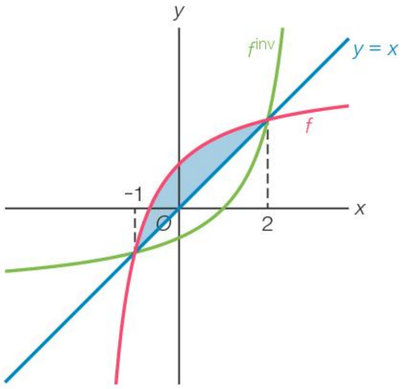
$$-\frac{8}{a} = -6$$

$$-6a = -8$$

$$a = 1\tfrac{1}{3}$$

Bladzijde 115

43



De snijpunten van de grafiek f en de grafiek van f^{inv} liggen op de lijn $y = x$.

$f(x) = x$ geeft $3 - \frac{4}{x+2} = x$

$3(x + 2) - 4 = x(x + 2)$

$3x + 6 - 4 = x^2 + 2x$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1 \vee x = 2$

$$O(V) = 2 \cdot \int_{-1}^{2} (f(x) - x)\,dx = 2 \cdot \int_{-1}^{2} \left(3 - \frac{4}{x+2} - x\right)dx = 2 \cdot \left[3x - 4\ln|x + 2| - \tfrac{1}{2}x^2\right]_{-1}^{2}$$

$$= 2(6 - 4\ln(4) - 2 - 2(-3 - 4\ln(1) - \tfrac{1}{2}) = 8 - 8\ln(4) + 7 = 15 - 8\ln(4)$$

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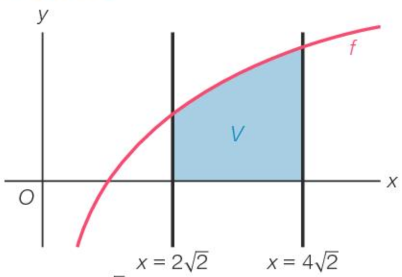
44 $P(p, 2\,\mathrm{e}^{0,2p})$
$O_{\text{rechthoek}} = p \cdot 2\,\mathrm{e}^{0,2p}$
$O(V) = O(W)$
$O(W) = \frac{1}{2} O_{\text{rechthoek}}$
$$\int_0^p (2\,\mathrm{e}^{0,2x})\,\mathrm{d}x = p \cdot \mathrm{e}^{0,2p}$$
$$\left[10\,\mathrm{e}^{0,2x}\right]_0^p = p \cdot \mathrm{e}^{0,2p}$$
$$10\,\mathrm{e}^{0,2p} - 10 = p \cdot \mathrm{e}^{0,2p}$$
Voer in $y_1 = 10\,\mathrm{e}^{0,2x} - 10$ en $y_2 = x \cdot \mathrm{e}^{0,2x}$.
De optie snijpunt geeft $x = 7,968...$
Dus $p \approx 7,97$.

45 **a** $F_p(x) = -2p\sqrt{1-x}$ geeft $F_p{}'(x) = -2p \cdot \frac{1}{2\sqrt{1-x}} \cdot -1 = \frac{p}{\sqrt{1-x}}$
Dus $F_p(x) = -2p\sqrt{1-x}$ is een primitieve van $f_p(x) = \frac{p}{\sqrt{1-x}}$.

b $O(V_q) = \int_q^0 (f_6(x) - f_2(x))\,\mathrm{d}x = \int_q^0 \left(\frac{6}{\sqrt{1-x}} - \frac{2}{\sqrt{1-x}}\right)\mathrm{d}x = \int_q^0 \left(\frac{4}{\sqrt{1-x}}\right)\mathrm{d}x$
$= \left[-8\sqrt{1-x}\right]_q^0 = -8\sqrt{1} + 8\sqrt{1-q} = -8 + 8\sqrt{1-q}$
$O(V_q) = 12$ geeft $-8 + 8\sqrt{1-q} = 12$
$8\sqrt{1-q} = 20$
$\sqrt{1-q} = 2\frac{1}{2}$
$1 - q = 6\frac{1}{4}$
$q = -5\frac{1}{4}$

Bladzijde 116

46 **a**



$O(V) = \int_{2\sqrt{2}}^{4\sqrt{2}} \ln(x)\,\mathrm{d}x = \left[x\ln(x) - x\right]_{2\sqrt{2}}^{4\sqrt{2}} = 4\sqrt{2} \cdot \ln(4\sqrt{2}) - 4\sqrt{2} - (2\sqrt{2} \cdot \ln(2\sqrt{2}) - 2\sqrt{2})$
$= 4\sqrt{2} \cdot 2\frac{1}{2}\ln(2) - 4\sqrt{2} - 2\sqrt{2} \cdot 1\frac{1}{2}\ln(2) + 2\sqrt{2} = 10\sqrt{2} \cdot \ln(2) - 2\sqrt{2} - 3\sqrt{2} \cdot \ln(2)$
$= 7\sqrt{2} \cdot \ln(2) - 2\sqrt{2} = \sqrt{2} \cdot (7\ln(2) - 2)$

b $L = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + (f'(x))^2}\,\mathrm{d}x = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + \frac{1}{x^2}}\,\mathrm{d}x = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{\frac{x^2+1}{x^2}}\,\mathrm{d}x = \int_{2\sqrt{2}}^{4\sqrt{5}} \frac{\sqrt{x^2+1}}{x}\,\mathrm{d}x$
$= \left[\sqrt{x^2+1} - \ln\left(\frac{1+\sqrt{x^2+1}}{x}\right)\right]_{2\sqrt{2}}^{4\sqrt{5}} = \sqrt{80+1} - \ln\left(\frac{1+\sqrt{80+1}}{4\sqrt{5}}\right) - \left(\sqrt{8+1} - \ln\left(\frac{1+\sqrt{8+1}}{2\sqrt{2}}\right)\right)$
$= 9 - \ln\left(\frac{10}{4\sqrt{5}}\right) - \left(3 - \ln\left(\frac{4}{2\sqrt{2}}\right)\right) = 6 - \ln(\frac{1}{2}\sqrt{5}) + \ln(\sqrt{2}) = 6 + \ln\left(\frac{\sqrt{2}}{\frac{1}{2}\sqrt{5}}\right) = 6 + \ln(\frac{2}{5}\sqrt{10})$

c $4\sqrt{5} - 2\sqrt{2} + \ln(4\sqrt{5}) + 6 + \ln(\frac{2}{5}\sqrt{10}) + \ln(2\sqrt{2}) = 15,581...$
Dus de omtrek van W is ongeveer 15,58.

47 $G(x)=\sqrt{x^2+1}-\ln\left(\frac{1+\sqrt{x^2+1}}{x}\right)$ geeft

$$G'(x)=\frac{1}{2\sqrt{x^2+1}}\cdot 2x-\frac{1}{\frac{1+\sqrt{x^2+1}}{x}}\cdot\frac{x\cdot\frac{1}{2\sqrt{x^2+1}}\cdot 2x-(1+\sqrt{x^2+1})\cdot 1}{x^2}$$

$$=\frac{x}{\sqrt{x^2+1}}-\frac{x}{1+\sqrt{x^2+1}}\cdot\left(\frac{1}{\sqrt{x^2+1}}-\frac{1+\sqrt{x^2+1}}{x^2}\right)=\frac{x}{\sqrt{x^2+1}}-\frac{x}{1+\sqrt{x^2+1}}\cdot\frac{x^2-\sqrt{x^2+1}-x^2-1}{x^2\sqrt{x^2+1}}$$

$$=\frac{x}{\sqrt{x^2+1}}-\frac{x}{1+\sqrt{x^2+1}}\cdot\frac{-1-\sqrt{x^2+1}}{x^2\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}+\frac{1}{x\sqrt{x^2+1}}=\frac{x^2+1}{x\sqrt{x^2+1}}=\frac{\sqrt{x^2+1}}{x}$$

Dus $G'(x)=g(x)$ oftewel G is een primitieve van g.

11.3 Inhouden

Bladzijde 118

48 **a** Het groene lichaam is te benaderen door de cilinder met straal $f(x)$ en hoogte h.
Dus de inhoud is ongeveer $\pi\cdot(f(x))^2\cdot h$.

b $B'(x)=\lim\limits_{h\to 0}\frac{B(x+h)-B(x)}{h}=\lim\limits_{h\to 0}\frac{\pi\cdot(f(x))^2\cdot h}{h}=\lim\limits_{h\to 0}\pi\cdot(f(x))^2=\pi\cdot(f(x))^2$
Dus $B(x)$ is een primitieve van $\pi\cdot(f(x))^2$.

Bladzijde 119

49 $f(x)=x$ geeft $2-\sqrt{x}=x$
$2-x=\sqrt{x}$
kwadrateren geeft
$4-4x+x^2=x$
$x^2-5x+4=0$
$(x-1)(x-4)=0$
$x=1\vee x=4$
vold.niet

$f(x)=0$ geeft $2-\sqrt{x}=0$, oftewel $x=4$.

$$I(K)=\pi\int_0^1 x^2\,\mathrm{d}x+\pi\int_1^4(2-\sqrt{x})^2\,\mathrm{d}x=\pi\left[\tfrac{1}{3}x^3\right]_0^1+\pi\left[4x-\tfrac{8}{3}x\sqrt{x}+\tfrac{1}{2}x^2\right]_1^4$$

$$=\tfrac{1}{3}\pi-0+\pi(4\cdot 4-\tfrac{8}{3}\cdot 4\cdot 2+\tfrac{1}{2}\cdot 4^2-(4\cdot 1-\tfrac{8}{3}\cdot 1\cdot 1+\tfrac{1}{2}\cdot 1^2))=\tfrac{1}{3}\pi+\tfrac{5}{6}\pi=1\tfrac{1}{6}\pi$$

50 $g(x)=2x$ geeft $3-\sqrt{x}=2x$
$-2x+3=\sqrt{x}$
kwadrateren geeft
$4x^2-12x+9=x$
$4x^2-13x+9=0$
$D=(-13)^2-4\cdot 4\cdot 9=25$
$x=\frac{13+5}{8}=2\frac{1}{4}\vee x=\frac{13-5}{8}=1$
vold. niet

$g(x)=0$ geeft $3-\sqrt{x}=0$, oftewel $x=9$.

$$I(L)=\pi\int_0^1(2x)^2\,\mathrm{d}x+\pi\int_1^9(3-\sqrt{x})^2\,\mathrm{d}x=\pi\int_0^1 4x^2\,\mathrm{d}x+\pi\int_1^9(9-6\sqrt{x}+x)\,\mathrm{d}x$$

$$=\pi\left[\tfrac{4}{3}x^3\right]_0^1+\pi\left[9x-4x\sqrt{x}+\tfrac{1}{2}x^2\right]_1^9=\pi(\tfrac{4}{3}\cdot 1^3-0)+\pi(81-108+40\tfrac{1}{2}-(9-4+\tfrac{1}{2}))$$

$$=\tfrac{4}{3}\pi+8\pi=9\tfrac{1}{3}\pi$$

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Bladzijde 120

51 **a** $I(K) = \pi\int_0^1 (e^x)^2\,dx = \pi\int_0^1 e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_0^1 = \pi(\frac{1}{2}e^2 - \frac{1}{2})$

$I(L_2) = \pi\int_1^2 (e^x)^2\,dx = \pi\int_1^2 e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_1^2 = \pi(\frac{1}{2}e^4 - \frac{1}{2}e^2)$

$\frac{I(L_2)}{I(K)} = \frac{\pi(\frac{1}{2}e^4 - \frac{1}{2}e^2)}{\pi(\frac{1}{2}e^2 - \frac{1}{2})} = \frac{\frac{1}{2}\pi e^2(e^2 - 1)}{\frac{1}{2}\pi(e^2 - 1)} = e^2$

Dus $I(L_2)$ is e^2 keer zo groot als $I(K)$.

b $I(L_p) = \pi\int_1^p (e^x)^2\,dx = \pi\int_1^p e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_1^p = \pi(\frac{1}{2}e^{2p} - \frac{1}{2}e^2)$

$\frac{I(L_p)}{I(K)} = \frac{\pi(\frac{1}{2}e^{2p} - \frac{1}{2}e^2)}{\pi(\frac{1}{2}e^2 - \frac{1}{2})} = \frac{\frac{1}{2}\pi(e^{2p} - e^2)}{\frac{1}{2}\pi(e^2 - 1)} = \frac{e^{2p} - e^2}{e^2 - 1}$

$\frac{I(L_p)}{I(K)} = 10$ geeft $\frac{e^{2p} - e^2}{e^2 - 1} = 10$

$e^{2p} - e^2 = 10e^2 - 10$

$e^{2p} = 11e^2 - 10$

$2p = \ln(11e^2 - 10)$

$p = \frac{1}{2}\ln(11e^2 - 10) = 2{,}1333...$

Dus $p \approx 2{,}133$.

52 **a** $g(x) = \frac{3}{x}$

b $I(M) = \pi\int_1^3 \left(\frac{3}{x}\right)^2 dx = \pi\int_1^3 9x^{-2}\,dx = \pi\left[-9x^{-1}\right]_1^3 = \pi\left[-\frac{9}{x}\right]_1^3 = \pi\left(-\frac{9}{3} + \frac{9}{1}\right) = 6\pi$

Wentelen van V om de lijn $y = 2$ levert hetzelfde lichaam als wentelen van W om de x-as omdat V en de lijn $y = 2$ beide 2 omlaag zijn geschoven.

53 **a** $O(V) = \int_1^5 f(x)\,dx = \int_1^5 \left(3 + \frac{4}{2x - 1}\right)dx = \left[3x + 4 \cdot \frac{1}{2}\ln|2x - 1|\right]_1^5 = \left[3x + 2\ln|2x - 1|\right]_1^5$

$= 15 + 2\ln(9) - (3 + 2\ln(1)) = 12 + 2\ln(9)$

b De grafiek van f wentelen om de lijn $y = 3$ komt neer op het wentelen van de grafiek van $g(x) = f(x) - 3 = \frac{4}{2x - 1}$ wentelen om de x-as.

$I(K) = \pi\int_1^5 (g(x))^2\,dx = \pi\int_1^5 \left(\frac{4}{2x - 1}\right)^2 dx = \pi\int_1^5 \frac{16}{(2x - 1)^2}\,dx = \pi\int_1^5 16(2x - 1)^{-2}\,dx$

$= \pi\left[16 \cdot \frac{1}{2} \cdot -(2x - 1)^{-1}\right]_1^5 = \pi\left[\frac{-8}{2x - 1}\right]_1^5 = \pi(-\frac{8}{9} + 8) = 7\frac{1}{9}\pi$

54 $I(L_1 + L_2) = \pi\int_2^8 (\sqrt{x-2})^2\,dx = \pi\int_2^8 (x-2)\,dx = \pi\left[\frac{1}{2}x^2 - 2x\right]_2^8 = \pi(32-16) - \pi(2-4) = 18\pi$

$I(L_1) = \frac{1}{2} \cdot 18\pi$ geeft $\pi\int_2^a (\sqrt{x-2})^2\,dx = 9\pi$

$$\pi\int_2^a (x-2)\,dx = 9\pi$$
$$\int_2^a (x-2)\,dx = 9$$
$$\left[\tfrac{1}{2}x^2 - 2x\right]_2^a = 9$$
$$(\tfrac{1}{2}a^2 - 2a) - (2-4) = 9$$
$$\tfrac{1}{2}a^2 - 2a + 2 = 9$$
$$a^2 - 4a - 14 = 0$$
$$(a-2)^2 - 4 - 14 = 0$$
$$(a-2)^2 = 18$$
$$a - 2 = \sqrt{18} \vee a - 2 = -\sqrt{18}$$
$$a = 2 + 3\sqrt{2} \vee a = 2 - 3\sqrt{2}$$

vold. niet

Dus $a = 2 + 3\sqrt{2}$.

55 **a** $O(V_p) = 2e$ geeft $\int_0^p e^{\frac{1}{2}x+1}\,dx = 2e$

$$\left[2\,e^{\frac{1}{2}x+1}\right]_0^p = 2e$$
$$2\,e^{\frac{1}{2}p+1} - 2e^1 = 2e$$
$$2e \cdot e^{\frac{1}{2}p} = 4e$$
$$e^{\frac{1}{2}p} = 2$$
$$\tfrac{1}{2}p = \ln(2)$$
$$p = 2\ln(2)$$

b $I(L_p) = 9\pi\,e^2$ geeft $\pi\int_0^p (e^{\frac{1}{2}x+1})^2\,dx = 9\pi e^2$

$$\int_0^p e^{x+2}\,dx = 9e^2$$
$$\left[e^{x+2}\right]_0^p = 9e^2$$
$$e^{p+2} - e^2 = 9e^2$$
$$e^p \cdot e^2 = 10e^2$$
$$e^p = 10$$
$$p = \ln(10)$$

56 **a** $I(L) = 8\frac{2}{3}\pi - 2\frac{1}{6}\pi = 6\frac{1}{2}\pi$

b $\pi\int_1^3 (f(x) - g(x))^2\,dx = \pi\int_1^3 (x - \frac{1}{2}x)^2\,dx = \pi\int_1^3 (\frac{1}{2}x)^2\,dx = \pi\int_1^3 \frac{1}{4}x^2\,dx =$

$\pi\left[\frac{1}{12}x^3\right]_1^3 = \pi(\frac{1}{12} \cdot 3^3 - \frac{1}{12} \cdot 1^3) = 2\frac{1}{6}\pi$

$\pi\int_1^3 ((f(x))^2 - (g(x))^2\,dx = \pi\int_1^3 (x^2 - (\frac{1}{2}x)^2)\,dx = \pi\int_1^3 (x^2 - \frac{1}{4}x^2)\,dx = \pi\int_1^3 \frac{3}{4}x^2\,dx =$

$\pi\left[\frac{1}{4}x^3\right]_1^3 = \pi(\frac{1}{4} \cdot 3^3 - \frac{1}{4} \cdot 1^3) = 6\frac{1}{2}\pi$

Dus Irma heeft gelijk.

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Bladzijde 122

57 $f(x)=g(x)$ geeft $\sqrt{x}=\frac{1}{2}x$
kwadrateren geeft
$x=\frac{1}{4}x^2$
$4x=x^2$
$x=0 \vee x=4$

$$I(L)=\pi\int_0^4((f(x))^2-(g(x))^2)\,dx=\pi\int_0^4(x-\tfrac{1}{4}x^2)\,dx=\pi\left[\tfrac{1}{2}x^2-\tfrac{1}{12}x^3\right]_0^4=\pi(8-5\tfrac{1}{3})=2\tfrac{2}{3}\pi$$

Bladzijde 123

58 $f(x)=g(x)$ geeft $e^{\frac{1}{2}x}=2e-e^{\frac{1}{2}x}$
$2e^{\frac{1}{2}x}=2e$
$\frac{1}{2}x=1$
$x=2$

$$I(L)=\pi\int_0^2((2\,e-e^{\frac{1}{2}x})^2-(e^{\frac{1}{2}x})^2)\,dx=\pi\int_0^2(4e^2-2\cdot 2e\cdot e^{\frac{1}{2}x}+e^x-e^x)\,dx=\pi\int_0^2(4e^2-4\,e^{\frac{1}{2}x+1})\,dx$$
$$=\pi\left[4e^2x-8\,e^{\frac{1}{2}x+1}\right]_0^2=\pi(8e^2-8e^2-(0-8e))=8\pi e$$

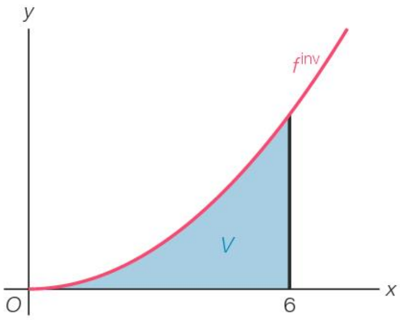
59 **a** $f(x)=10$ geeft $\dfrac{10}{x\sqrt{x}}=10$
$x\sqrt{x}=1$
$x=1$

$$O(V)=\int_1^4\left(10-\frac{10}{x\sqrt{x}}\right)dx=\int_1^4(10-10x^{-1\frac{1}{2}})\,dx=\left[10x-10\cdot{-2}x^{-\frac{1}{2}}\right]_1^4=\left[10x+\frac{20}{\sqrt{x}}\right]_1^4$$
$$=40+10-(10+20)=20$$

b $$I(L)=\pi\int_1^4(10^2-(f(x))^2)\,dx=\pi\int_1^4\left(100-\frac{100}{x^3}\right)dx=\pi\int_1^4(100-100x^{-3})\,dx$$
$$=\pi\left[100x+50x^{-2}\right]_1^4=\pi\left[100x+\frac{50}{x^2}\right]_1^4=\pi\left(400+\frac{50}{16}-(100+50)\right)=253\tfrac{1}{8}\pi$$

60 **a** Voor f geldt $y=3\sqrt{x}$, dus voor f^{inv} geldt $x=3\sqrt{y}$
$x^2=9y$ met $x\geq 0$
$y=\frac{1}{9}x^2$ met $x\geq 0$

Dus $f^{\text{inv}}(x)=\frac{1}{9}x^2$ met $x\geq 0$.



$$I(L)=\pi\int_0^6\tfrac{1}{81}x^4\,dx=\pi\left[\tfrac{1}{405}x^5\right]_0^6=\pi(\tfrac{1}{405}\cdot 6^5-0)=19\tfrac{1}{5}\pi$$

De gebruikte grenzen zijn 0 en 6.

b $y=3\sqrt{x}$
$y^2=9x$
$x=\frac{1}{9}y^2$
$x^2=\frac{1}{81}y^4$

c Jasmijn moet de grenzen 0 en 6 gebruiken, want y loopt van 0 tot 6.

d $I(L) = \pi\int_0^6 \frac{1}{81}x^4\,dx$ is hetzelfde als bij onderdeel a.

Bladzijde 125

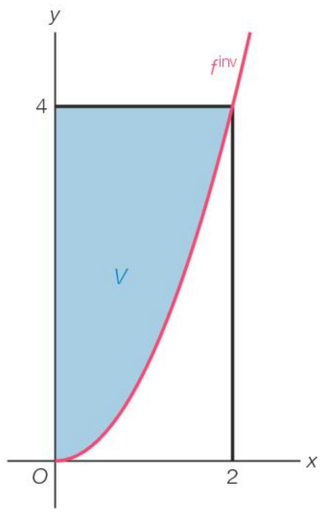
61 **a** Gebruik ‘buitenste’ inhoud min ‘binnenste’ inhoud.

Je krijgt dan $I(L) = \pi\int_0^2 (4^2 - x^2)\,dy = \pi\int_0^2 (16 - y^4)\,dy = \pi\left[16y - \frac{1}{5}y^5\right]_0^2 = \pi(32 - \frac{32}{5}) = 25\frac{3}{5}\pi$.

b Voor f geldt $y = \sqrt{x}$, dus voor f^{inv} geldt $x = \sqrt{y}$

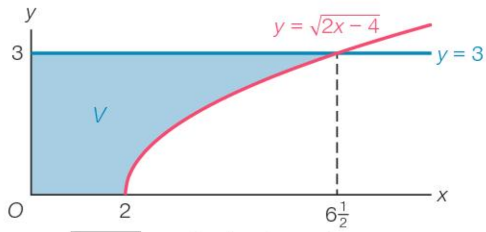
$y = x^2$ met $x \geq 0$

Dus $f^{\text{inv}}(x) = x^2$ met $x \geq 0$.



$I(L) = \pi\int_0^2 (4^2 - (f^{\text{inv}}(x))^2)\,dx = \pi\int_0^2 (16 - x^4)\,dx = \pi\left[16x - \frac{1}{5}x^5\right]_0^2 = \pi(32 - \frac{32}{5}) = 25\frac{3}{5}\pi$

62 **a**



$y = \sqrt{2x - 4}$ geeft $y^2 = 2x - 4$

$2x - 4 = y^2$

$2x = y^2 + 4$

$x = \frac{1}{2}y^2 + 2$

$x^2 = (\frac{1}{2}y^2 + 2)^2$

$x^2 = \frac{1}{4}y^4 + 2y^2 + 4$

$I(L) = \pi\int_0^3 x^2\,dy = \pi\int_0^3 (\frac{1}{4}y^4 + 2y^2 + 4)\,dy = \pi\left[\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y\right]_0^3 = \pi(\frac{1}{20} \cdot 3^5 + \frac{2}{3} \cdot 3^3 + 4 \cdot 3 - 0)$

$= \pi(12\frac{3}{20} + 18 + 12) = 42\frac{3}{20}\pi$

b $y = 3$ geeft $\sqrt{2x - 4} = 3$

$2x - 4 = 9$

$2x = 13$

$x = 6\frac{1}{2}$

$I(M) = I(\text{cilinder}) - \pi\int_2^{6\frac{1}{2}} (\sqrt{2x-4})^2\,dx = \pi \cdot 3^2 \cdot 6\frac{1}{2} - \pi\int_2^{6\frac{1}{2}} (2x - 4)\,dx = 58\frac{1}{2}\pi - \pi\left[x^2 - 4x\right]_2^{6\frac{1}{2}}$

$= 58\frac{1}{2}\pi - \pi((6\frac{1}{2})^2 - 4 \cdot 6\frac{1}{2} - (2^2 - 4 \cdot 2)) = 58\frac{1}{2}\pi - 20\frac{1}{4}\pi = 38\frac{1}{4}\pi$

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63 **a** $I(L_4) = \pi\int_0^4 x^2\,dy = \pi\int_0^4 y\,dy = \pi\left[\tfrac{1}{2}y^2\right]_0^4 = \pi(8 - 0) = 8\pi$

b $P(p, q)$ ligt op de parabool, dus $q = p^2$.

$I(L_p) = I(M_q)$ geeft $\pi\int_0^p y^2\,dx = \pi\int_0^q x^2\,dy$

$\int_0^p x^4\,dx = \int_0^{p^2} y\,dy$

$\left[\tfrac{1}{5}x^5\right]_0^p = \left[\tfrac{1}{2}y^2\right]_0^{p^2}$

$\tfrac{1}{5}p^5 - 0 = \tfrac{1}{2}p^4 - 0$

$2p^5 = 5p^4$

$p^4 = 0 \vee 2p = 5$

$p = 0 \vee p = 2\tfrac{1}{2}$

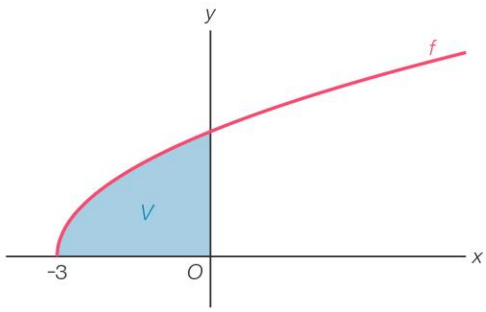
$p = 0$ voldoet niet en $p = 2\tfrac{1}{2}$ geeft $q = 6\tfrac{1}{4}$.

Dus voor $p = 2\tfrac{1}{2}$ en $q = 6\tfrac{1}{4}$.

64 **a** $f(x) = 0$ geeft $2x + 6 = 0$

$2x = -6$

$x = -3$



$I(L) = \pi\int_{-3}^{0} (\sqrt{2x + 6})^2\,dx = \pi\int_{-3}^{0} (2x + 6)\,dx = \pi\left[x^2 + 6x\right]_{-3}^{0} = \pi(0 - (9 - 18)) = 9\pi$

b $f(0) = \sqrt{6}$

$y = \sqrt{2x + 6}$ geeft $y^2 = 2x + 6$

$2x + 6 = y^2$

$2x = y^2 - 6$

$x = \tfrac{1}{2}y^2 - 3$

$x^2 = (\tfrac{1}{2}y^2 - 3)^2$

$x^2 = \tfrac{1}{4}y^4 - 3y^2 + 9$

$I(M) = \pi\int_0^{\sqrt{6}} x^2\,dy = \pi\int_0^{\sqrt{6}} (\tfrac{1}{4}y^4 - 3y^2 + 9)\,dy = \pi\left[\tfrac{1}{20}y^5 - y^3 + 9y\right]_0^{\sqrt{6}}$

$= \pi(\tfrac{1}{20} \cdot (\sqrt{6})^5 - (\sqrt{6})^3 + 9 \cdot \sqrt{6} - 0) = \pi(\tfrac{1}{20} \cdot 36\sqrt{6} - 6\sqrt{6} + 9\sqrt{6}) = 4\tfrac{4}{5}\pi\sqrt{6}$

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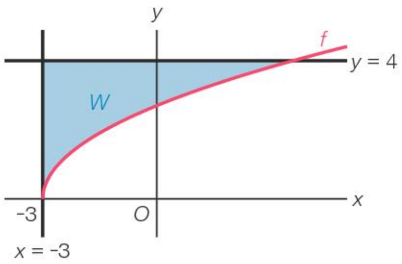
c



$y = \sqrt{2x+6}$
$\downarrow$ translatie (3, 0)
$y = \sqrt{2(x-3)+6}$
oftewel $y = \sqrt{2x}$
$y = \sqrt{2x}$ geeft $y^2 = 2x$
$2x = y^2$
$x = \frac{1}{2}y^2$
$x^2 = \frac{1}{4}y^4$

$$I(N) = \pi\int_0^4 x^2\,dy = \pi\int_0^4 \tfrac{1}{4}y^4\,dy = \pi\left[\tfrac{1}{20}y^5\right]_0^4 = \pi(\tfrac{1}{20}\cdot 4^5 - 0) = 51\tfrac{1}{5}\pi$$

Bladzijde 126

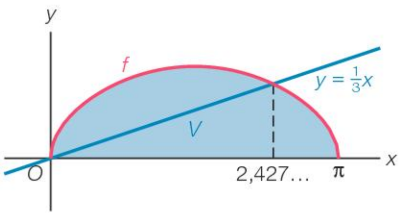
65 **a** $I(L) = \pi\int_{-4}^{4}(x^2+4)\,dx = \pi\left[\frac{1}{3}x^3+4x\right]_{-4}^{4} = \pi(\frac{64}{3}+16-(-\frac{64}{3}-16)) = 74\frac{2}{3}\pi$

b $O(V) = \int_{-4}^{4}\sqrt{x^2+4}\,dx$ en van $\sqrt{x^2+4}$ is geen primitieve te vinden met de regels die tot nu toe zijn geweest.

66 Voer in $y_1 = \sqrt{x^2+4}$.

De optie integraal geeft $O(V) = \int_{-4}^{4}\sqrt{x^2+4}\,dx \approx 23{,}66$.

67 Voer in $y_1 = \sqrt{\sin(x)}$ en $y_2 = \frac{1}{3}x$.
De optie snijpunt geeft $x = 0$ en $x = 2{,}427...$



De optie integraal geeft $\int_0^{2{,}427...}(\sqrt{\sin(x)} - \frac{1}{3}x)\,dx = 1{,}019...$ en $\int_0^{\pi}\sqrt{\sin(x)}\,dx = 2{,}396...$

1,019... is niet gelijk aan de helft van 2,396..., dus de lijn $y = \frac{1}{3}x$ verdeelt het vlakdeel V niet in twee delen met gelijke oppervlakte.

Bladzijde 127

68 **a** $f(x) = 0$ geeft $2\ln(x) - \ln^2(x) = 0$
$\ln(x)(2-\ln(x)) = 0$
$\ln(x) = 0 \vee \ln(x) = 2$
$x = 1 \vee x = e^2$
Dus de nulpunten zijn 1 en e^2.

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b $f(x) = 2\ln(x) - \ln^2(x)$ geeft $f'(x) = \frac{2}{x} - 2\ln(x) \cdot \frac{1}{x} = \frac{2 - 2\ln(x)}{x}$

$f'(x) = 0$ geeft $2 - 2\ln(x) = 0$

$\ln(x) = 1$

$x = \mathrm{e}$

$f(\mathrm{e}) = 2 \cdot 1 - 1^2 = 1$, dus de top is $(\mathrm{e}, 1)$.

c $F(x) = 4x\ln(x) - x\ln^2(x) - 4x$ geeft

$F'(x) = 4 \cdot \ln(x) + 4x \cdot \frac{1}{x} - \left(1 \cdot \ln^2(x) + x \cdot 2\ln(x) \cdot \frac{1}{x}\right) - 4 = 4\ln(x) + 4 - \ln^2(x) - 2\ln(x) - 4$

$= 2\ln(x) - \ln^2(x)$

Dus $F(x) = 4x\ln(x) - x\ln^2(x) - 4x$ is een primitieve van $f(x) = 2\ln(x) - \ln^2(x)$.

d $O(V) = \int_1^{\mathrm{e}^2} (2\ln(x) - \ln^2(x))\,\mathrm{d}x = \left[4x\ln(x) - x\ln^2(x) - 4x\right]_1^{\mathrm{e}^2}$

$= 4\mathrm{e}^2 \cdot \ln(\mathrm{e}^2) - \mathrm{e}^2 \cdot (\ln(\mathrm{e}^2))^2 - 4\mathrm{e}^2 - (4\ln(1) - (\ln(1))^2 - 4) = 8\mathrm{e}^2 - 4\mathrm{e}^2 - 4\mathrm{e}^2 - (0 - 0 - 4) = 4$

e De optie integraal geeft $\pi\int_1^{\mathrm{e}^2} (2\ln(x) - \ln^2(x))^2\,\mathrm{d}x = 9{,}7780...$

Dus $I(L) \approx 9{,}778$.

11.4 Toepassingen van integralen

Bladzijde 129

69 $I_{\text{bol}} = 2 \cdot \pi\int_0^r y^2\,\mathrm{d}x = 2\pi\int_0^r (r^2 - x^2)\,\mathrm{d}x = 2\pi\left[r^2x - \tfrac{1}{3}x^3\right]_0^r = 2\pi(r^3 - \tfrac{1}{3}r^3 - 0) = 2\pi \cdot \tfrac{2}{3}r^3 = \tfrac{4}{3}\pi r^3$

Bladzijde 130

70 **a** $I(T) = \pi\int_{\frac{1}{4}r}^{\frac{1}{2}r} (r^2 - x^2)\,\mathrm{d}x = \pi\left[r^2x - \tfrac{1}{3}x^3\right]_{\frac{1}{4}r}^{\frac{1}{2}r} = \pi(r^2 \cdot \tfrac{1}{2}r - \tfrac{1}{3} \cdot (\tfrac{1}{2}r)^3 - (r^2 \cdot \tfrac{1}{4}r - \tfrac{1}{3} \cdot (\tfrac{1}{4}r)^3)$

$= \pi(\tfrac{1}{2}r^3 - \tfrac{1}{24}r^3 - \tfrac{1}{4}r^3 + \tfrac{1}{192}r^3) = \tfrac{41}{192}\pi r^3$

$\frac{I(T)}{I(S)} = \frac{\frac{41}{192}\pi r^3}{\frac{29}{192}\pi r^3} = \frac{41}{29} = 1\frac{12}{29}$

Dus de inhoud van bolschijf T is $1\frac{12}{29}$ keer zo groot als de inhoud van bolschijf S.

b $I_p = 24\pi$ geeft $\tfrac{1}{3} \cdot \pi \cdot (\tfrac{2}{3} \cdot 6)^2 \cdot 6 - \tfrac{1}{3} \cdot \pi \cdot (\tfrac{2}{3}p)^2 \cdot p = 24\pi$

$32 - \tfrac{4}{27}p^3 = 24$

$-\tfrac{4}{27}p^3 = -8$

$p^3 = 54$

$p = \sqrt[3]{54}$

Bladzijde 131

71 $I = \pi\int_{\frac{1}{3}r}^{r} y^2\,\mathrm{d}x = \pi\int_{\frac{1}{3}r}^{r} (r^2 - x^2)\,\mathrm{d}x = \pi\left[r^2x - \tfrac{1}{3}x^3\right]_{\frac{1}{3}r}^{r} = \pi(r^3 - \tfrac{1}{3}r^3 - (r^2 \cdot \tfrac{1}{3}r - \tfrac{1}{3} \cdot (\tfrac{1}{3}r)^3)$

$= \pi(r^3 - \tfrac{1}{3}r^3 - \tfrac{1}{3}r^3 + \tfrac{1}{81}r^3) = \tfrac{28}{81}\pi r^3$

72 $I(S_p) = \pi\int_{-3}^{p} y^2\,dx = \pi\int_{-3}^{p}(36 - x^2)\,dx = \pi\left[36x - \frac{1}{3}x^3\right]_{-3}^{p} = \pi(36p - \frac{1}{3}p^3 - (36 \cdot -3 - \frac{1}{3} \cdot (-3)^3)) = \pi(36p - \frac{1}{3}p^3 + 99)$

$I_{bol} = \frac{4}{3}\pi r^3 = \frac{4}{3} \cdot \pi \cdot 6^3 = 288\pi$

$I(S_p) = \frac{1}{2} \cdot 288\pi$ geeft $36p - \frac{1}{3}p^3 + 99 = 144$

$36p - \frac{1}{3}p^3 = 45$

Voer in $y_1 = 36x - \frac{1}{3}x^3$ en $y_2 = 45$.
De optie snijpunt geeft $x = 1{,}268...$ en $x = 9{,}699....$
Dus $p \approx 1{,}27$.

73 $\text{rc}_{OA} = \dfrac{4-0}{3-0} = \frac{4}{3}$

$\left.\begin{array}{l} k \perp OA, \text{ dus } \text{rc}_k \cdot \text{rc}_{OA} = -1 \\ \text{rc}_{OA} = \frac{4}{3} \end{array}\right\} \begin{array}{l} \text{rc}_k \cdot \frac{4}{3} = -1 \\ \text{rc}_k = -\frac{3}{4} \end{array}$

$\left.\begin{array}{l} y = -\frac{3}{4}x + b \\ \text{door } A(3, 4) \end{array}\right\} -\frac{3}{4} \cdot 3 + b = 4$

$-2\frac{1}{4} + b = 4$

$b = 6\frac{1}{4}$

Dus $k: y = -\frac{3}{4}x + 6\frac{1}{4}$.

k snijden met de x-as geeft $-\frac{3}{4}x + 6\frac{1}{4} = 0$

$-\frac{3}{4}x = -6\frac{1}{4}$

$x = 8\frac{1}{3}$

$I(L) = \pi\int_{3}^{8\frac{1}{3}}(-\frac{3}{4}x + 6\frac{1}{4})^2\,dx - \pi\int_{3}^{5}(25 - x^2)\,dx = \pi\left[-\frac{4}{3} \cdot \frac{1}{3} \cdot (-\frac{3}{4}x + 6\frac{1}{4})^3\right]_{3}^{8\frac{1}{3}} - \pi\left[25x - \frac{1}{3}x^3\right]_{3}^{5}$

$= \pi\left[-\frac{4}{9}(-\frac{3}{4}x + 6\frac{1}{4})^3\right]_{3}^{8\frac{1}{3}} - \pi\left[25x - \frac{1}{3}x^3\right]_{3}^{5}$

$= \pi(-\frac{4}{9}(-\frac{3}{4} \cdot \frac{25}{3} + 6\frac{1}{4})^3 + \frac{4}{9}(-\frac{3}{4} \cdot 3 + 6\frac{1}{4})^3) - \pi(25 \cdot 5 - \frac{1}{3} \cdot 5^3 - (25 \cdot 3 - \frac{1}{3} \cdot 3^3))$

$= \pi(-\frac{4}{9} \cdot 0^3 + \frac{4}{9} \cdot 4^3) - \pi(125 - \frac{1}{3} \cdot 5^3 - (75 - \frac{1}{3} \cdot 3^3)) = \pi(0 + \frac{4}{9} \cdot 64) - \pi(83\frac{1}{3} - 66) = 11\frac{1}{9}\pi$

74 **a** De gemiddelde snelheid gedurende de eerste zes seconden is $\dfrac{0 + 10}{2} = 5$ m/s.

Er wordt $5 \cdot 6 = 30$ m afgelegd gedurende deze zes seconden.

b De afgelegde afstand is een primitieve van de snelheid. Dus de afstand gedurende de eerste zes seconden is $\int_0^6 v(t)\,dt$. Ook is $O(\triangle OAB) = \int_0^6 v(t)\,dt$.

Dus de oppervlakte van driehoek OAB geeft de afgelegde afstand gedurende de eerste zes seconden.

c De afgelegde afstand op het interval [0, 15] is O(gehele driehoek) $= \frac{1}{2} \cdot 15 \cdot 10 = 75$ m.

Bladzijde 133

75 Stel $s(0) = c$. Dan geldt $s(t) = -\frac{1}{12}t^3 + 2\frac{1}{2}t^2 + 4t + c$ en dit geeft $s(10) = 206\frac{2}{3} + c$ meter.

In de eerste 10 seconden wordt dan afgelegd $s(10) - s(0) = 206\frac{2}{3} + c - c = 206\frac{2}{3}$ meter.
Je krijgt dus hetzelfde antwoord.

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76 Stel $a(t) = mt + n$.

$m = \dfrac{a(20) - a(0)}{20 - 0} = \dfrac{5 - 0}{20} = \frac{1}{4}$

$a(t) = \frac{1}{4}t + n$ en $a(0) = 0$ geeft $a(t) = \frac{1}{4}t$.

$a(t) = \frac{1}{4}t$ en $v(0) = 1$ geeft $v(t) = \frac{1}{8}t^2 + 1$

$v(t) = \frac{1}{8}t^2 + 1$ en $s(0) = 0$ geeft $s(t) = \frac{1}{24}t^3 + t$

De afgelegde afstand gedurende deze 20 seconden is $s(20) = \frac{1}{24} \cdot 20^3 + 20 = 353\frac{1}{3}$ meter.

77 **a** $a(t) = -t^2 + 6t$ en $v(0) = 0$ geeft $v(t) = -\frac{1}{3}t^3 + 3t^2$

$v(6) = -\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2 = 36$

Dus de snelheid op $t = 6$ is 36 m/s.

b $v(t) = -\frac{1}{3}t^3 + 3t^2$ en $s(0) = 0$ geeft $s(t) = -\frac{1}{12}t^4 + t^3$

De afgelegde afstand gedurende deze 6 seconden is $s(6) = -\frac{1}{12} \cdot 6^4 + 6^3 = 108$ meter.

c $s(6) = 108$ en $v(6) = 36$ geeft $s(10) = 108 + 4 \cdot 36 = 252$

Dus op $t = 10$ is in totaal 252 meter afgelegd.

d $500 - 108 = 392$

392 meter met een snelheid van 36 m/s duurt $\dfrac{392}{36} = 10\frac{8}{9}$ seconden.

Dus op $t = 6 + 10\frac{8}{9} = 16\frac{8}{9}$ is in totaal 500 meter afgelegd.

78 **a** 54 km/uur = 15 m/s

De constante versnelling is a.

$v(t) = at + b$ en $v(0) = 15$ geeft $v(t) = at + 15$.

$v(t) = at + 15$ en $s(0) = 0$ geeft $s(t) = \frac{1}{2}at^2 + 15t$.

Noem de remtijd t_r, dan geldt $v(t_r) = 0$ en $s(t_r) = 0{,}75$.

$v(t_r) = 0$ geeft $at_r + 15 = 0$

$at_r = -15$

$s(t_r) = 0{,}75$ geeft $\frac{1}{2}at_r^2 + 15t_r = 0{,}75$ oftewel $\frac{1}{2}at_r \cdot t_r + 15t_r = 0{,}75$.

Substitutie van $at_r = -15$ in $\frac{1}{2}at_r \cdot t_r + 15t_r = 0{,}75$ geeft $-7{,}5t_r + 15t_r = 0{,}75$

$7{,}5t_r = 0{,}75$

$t_r = 0{,}1$

De botsing duurt 0,1 seconde.

Alternatieve uitwerking

Het rekenwerk wordt eenvoudiger als het probleem 'achterstevoren' wordt aangepakt.

Noem de remtijd t_r en neem $v(0) = 0$, $s(0) = 0$, $v(t_r) = 15$ en $s(t_r) = 0{,}75$.

Dit geeft $v(t) = at$ en $s(t) = \frac{1}{2}at^2$. Dan geldt

$$\left.\begin{array}{l} at_r = 15 \\ \frac{1}{2}at_r^2 = 0{,}75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r = 0{,}75 \end{array}\right\} \begin{array}{l} 7{,}5t_r = 0{,}75 \\ t_r = 0{,}1 \end{array}$$

De botsing duurt 0,1 seconde.

b

$$\left.\begin{array}{l} at_r = -15 \\ t_r = 0{,}1 \end{array}\right\} \begin{array}{l} a \cdot 0{,}1 = -15 \\ a = -150 \end{array}$$

De versnelling is -150 m/s^2.

Dit is 15 keer zo groot als de versnelling van de zwaartekracht.

Bladzijde 134

79 **a** $v(t) = -0{,}2t^2 + 4t$ en $s(0) = 0$ geeft $s(t) = -\frac{2}{30}t^3 + 2t^2$

$s(10) = -\frac{2}{30} \cdot 10^3 + 2 \cdot 10^2 = 133\frac{1}{3}$

$v(t) = 0{,}8t^2 - 24t + 180$ geeft $s(t) = \frac{4}{15}t^3 - 12t^2 + 180t + c$

$s(10) = 133\frac{1}{3}$ geeft $\frac{4}{15} \cdot 10^3 - 12 \cdot 10^2 + 180 \cdot 10 + c = 133\frac{1}{3}$

$866\frac{2}{3} + c = 133\frac{1}{3}$

$c = -733\frac{1}{3}$

Dus $s(t) = \frac{4}{15}t^3 - 12t^2 + 180t - 733\frac{1}{3}$.

$s(15) = \frac{4}{15} \cdot 15^3 - 12 \cdot 15^2 + 180 \cdot 15 - 733\frac{1}{3} = 166\frac{2}{3}$

De totale afstand die de auto gedurende deze 15 seconden aflegt is $166\frac{2}{3}$ meter.

b gemiddelde snelheid $= \dfrac{166\frac{2}{3}}{15} = 11\frac{1}{9}$ m/s

Voer in $y_1 = -0{,}2x^2 + 4x$, $y_2 = 0{,}8x^2 - 24x + 180$ en $y_3 = 11\frac{1}{9}$.

De optie snijpunt met y_1 en y_3 geeft $x = 3{,}333...$

De optie snijpunt met y_2 en y_3 geeft $x = 11{,}273...$

Dus op $t \approx 3{,}33$ en $t \approx 11{,}27$ is de snelheid gelijk aan de gemiddelde snelheid.

c Voer in $y_4 = -\frac{2}{30}x^3 + 2x^2$ en $y_5 = 100$.

De optie snijpunt met y_4 en y_5 geeft $x = 8{,}317...$

Dus na ongeveer 8,3 seconde heeft de auto 100 meter afgelegd.

80 **a** $a(t) = -4\,e^{-0{,}1t}$

$\left.\begin{array}{l} v(t) = 40\,e^{-0{,}1t} + c \\ v(3) = 32 \end{array}\right\} \begin{array}{l} 40\,e^{-0{,}3} + c = 32 \\ 29{,}632... + c = 32 \\ c = 2{,}367... \end{array}$

$v(t) = 40\,e^{-0{,}1t} + 2{,}367...$

$v(0) = 40 + 2{,}367... = 42{,}367...$

De snelheid is 42,4 m/s.

b $v(t) = 40\,e^{-0{,}1t} + 2{,}367...$

$\left.\begin{array}{l} s(t) = -400\,e^{-0{,}1t} + 2{,}367...t + c \\ s(0) = 0 \end{array}\right\} \begin{array}{l} -400 + 0 + c = 0 \\ c = 400 \end{array}$

Dus $s(t) = -400\,e^{-0{,}1t} + 2{,}367...t + 400$.

Voer in $y_1 = -400\,e^{-0{,}1x} + 2{,}367...x + 400$ en $y_2 = 800$.

De optie snijpunt geeft $x = 168{,}970...$

$v(168{,}970...) = 2{,}367...$

De snelheid waarmee de parachutiste op de grond komt is ongeveer 2,4 m/s.

Bladzijde 135

81 **a** $a(t) = 5\,e^{0{,}012t}$ geeft

$\left.\begin{array}{l} v(t) = 416\frac{2}{3}e^{0{,}012t} + c \\ v(0) = 0 \end{array}\right\} \begin{array}{l} 416\frac{2}{3} + c = 0 \\ c = -416\frac{2}{3} \end{array}$

Dus $v(t) = 416\frac{2}{3}e^{0{,}012t} - 416\frac{2}{3}$.

$v(t) = 416\frac{2}{3}e^{0{,}012t} - 416\frac{2}{3}$ geeft

$\left.\begin{array}{l} s(t) = 34\,722\frac{2}{9}e^{0{,}012t} - 416\frac{2}{3}t + c \\ s(0) = 0 \end{array}\right\} \begin{array}{l} 34\,722\frac{2}{9} + c = 0 \\ c = -34\,722\frac{2}{9} \end{array}$

Dus $s(t) = 34\,722\frac{2}{9}e^{0{,}012t} - 416\frac{2}{3}t - 34722\frac{2}{9}$.

$v(170) = 416\frac{2}{3}e^{0{,}012 \cdot 170} - 416\frac{2}{3} = 2787{,}7...$ m/s $\approx 10\,036$ km/uur

$s(170) = 34\,722\frac{2}{9}e^{0{,}012 \cdot 170} - 416\frac{2}{3} \cdot 170 - 34\,722\frac{2}{9} = 161479{,}4...$ m ≈ 161 km

Dus de snelheid na 170 seconden is 10 036 km/uur en de Apollo heeft dan 161 km afgelegd.

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b $a(t) = 4\,e^{0{,}0045(t-170)}$ geeft

$$\left.\begin{array}{l} v(t) = 888\tfrac{8}{9}\,e^{0{,}0045(t-170)} + c \\ v(170) = 2787{,}7... \end{array}\right\} \begin{array}{l} 888\tfrac{8}{9} + c = 2787{,}7... \\ c = 1898{,}8... \end{array}$$

Dus $v(t) = 888\tfrac{8}{9}\,e^{0{,}0045(t-170)} + 1898{,}8...$

$v(t) = 888\tfrac{8}{9}\,e^{0{,}0045(t-170)} + 1898{,}8...$ geeft

$$\left.\begin{array}{l} s(t) = 197\,530{,}8...\,e^{0{,}0045(t-170)} + 1898{,}8...t + c \\ s(170) = 161\,479{,}4... \end{array}\right\} \begin{array}{l} 197\,530{,}8... + 1898{,}8... \cdot 170 + c = 161\,479{,}4... \\ 520\,337{,}9... + c = 161\,479{,}4... \\ c = -358\,858{,}4... \end{array}$$

Dus $s(t) = 197\,530{,}8...e^{0{,}0045(t-170)} + 1898{,}8...t - 358\,858{,}4...$

$v(560) = 888\tfrac{8}{9}\,e^{0{,}0045(560-170)} + 1898{,}8... = 7039{,}7...$ m/s $\approx 25\,342$ km/uur

$s(560) = 197\,530{,}8...\,e^{0{,}0045(560-170)} + 1898{,}8... \cdot 560 - 358\,858{,}4... = 1\,846\,915{,}3...$ m ≈ 1847 km

Dus op $t = 560$ is de snelheid 25 342 km/uur en de afgelegde afstand 1847 km.

Diagnostische toets

Bladzijde 138

1 **a** $F(x) = x^3\,e^x$ geeft $F'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = (x^3 + 3x^2)\,e^x$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

b $G(x) = \tfrac{1}{2}x^2 - x + 2\ln(x+1)$ geeft $G'(x) = x - 1 + 2 \cdot \dfrac{1}{x+1} = \dfrac{(x-1)(x+1)+2}{x+1}$

Dus $G'(x) = g(x)$ oftewel G is een primitieve van g.

2 **a** $f(x) = \dfrac{2x+6}{x^2} = 2 \cdot \dfrac{1}{x} + 6x^{-2}$ geeft $F(x) = 2\ln|x| - 6x^{-1} + c = 2\ln|x| - \dfrac{6}{x} + c$

b $f(x) = 3 \cdot 2^x$ geeft $F(x) = \dfrac{3}{\ln(2)} \cdot 2^x + c$

c $f(x) = 6\,e^x + x^2$ geeft $F(x) = 6\,e^x + \tfrac{1}{3}x^3 + c$

d $f(x) = \ln(x^5) = 5\ln(x)$ geeft $F(x) = 5(x\ln(x) - x) + c = 5x\ln(x) - 5x + c$

e $f(x) = {}^2\log(4x)$ geeft $F(x) = \tfrac{1}{4} \cdot \dfrac{1}{\ln(2)}(4x\ln(4x) - 4x) + c = \dfrac{1}{\ln(2)} \cdot (x\ln(4x) - x) + c$

f $f(x) = 3\sin(x) + 2\cos(x)$ geeft $F(x) = -3\cos(x) + 2\sin(x) + c$

3 $f(x) = 4x - \ln(x)$ geeft $F(x) = 2x^2 - (x\ln(x) - x) + c = 2x^2 + x - x\ln(x) + c$

$F(1) = 7$ geeft $2 \cdot 1^2 + 1 - 1 \cdot \ln(1) + c = 7$

$2 + 1 - 0 + c = 7$

$c = 4$

Dus $F(x) = 2x^2 + x - x\ln(x) + 4$.

4 **a** $O(V) = \displaystyle\int_0^2 \frac{2}{x+1}\,dx = \left[2\ln|x+1|\right]_0^2 = 2\ln(3) - 2\ln(1) = 2\ln(3)$

b $\displaystyle\int_0^p \frac{2}{x+1}\,dx = \left[2\ln|x+1|\right]_0^p = 2\ln(p+1) - 2\ln(1) = 2\ln(p+1)$

$\displaystyle\int_0^p \frac{2}{x+1}\,dx = \tfrac{1}{2}O(V)$ geeft $2\ln(p+1) = \ln(3)$

$\ln(p+1) = \tfrac{1}{2}\ln(3)$

$\ln(p+1) = \ln(\sqrt{3})$

$p + 1 = \sqrt{3}$

$p = -1 + \sqrt{3}$

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5 **a** $f(x) = (2x+6)^5 + \dfrac{10}{(3x-1)^2} = (2x+6)^5 + 10(3x-1)^{-2}$ geeft

$F(x) = \frac{1}{2} \cdot \frac{1}{6}(2x+6)^6 + 10 \cdot -\frac{1}{3}(3x-1)^{-1} + c = \frac{1}{12}(2x+6)^6 - \dfrac{10}{3(3x-1)} + c$

b $f(x) = (5x+2)^2 \cdot \sqrt{5x+2} = (5x+2)^{2\frac{1}{2}}$ geeft

$F(x) = \frac{1}{5} \cdot \frac{2}{7}(5x+2)^{3\frac{1}{2}} + c = \frac{2}{35}(5x+2)^3 \cdot \sqrt{5x+2} + c$

c $f(x) = 4\,\mathrm{e}^{2x+3}$ geeft $F(x) = 4 \cdot \frac{1}{2}\,\mathrm{e}^{2x+3} + c = 2\,\mathrm{e}^{2x+3} + c$

d $f(x) = 8 \cdot 2^{2x-1}$ geeft $F(x) = 8 \cdot \frac{1}{2} \cdot \dfrac{1}{\ln(2)} \cdot 2^{2x-1} + c = \dfrac{4}{\ln(2)} \cdot 2^{2x-1} + c$

e $f(x) = \ln(2x+3)$ geeft

$F(x) = \frac{1}{2}((2x+3)\ln(2x+3) - (2x+3)) + c = (x + 1\frac{1}{2})\ln(2x+3) - (x + 1\frac{1}{2}) + c$

f $f(x) = \dfrac{6}{2x+5}$ geeft $F(x) = 6 \cdot \frac{1}{2} \cdot \ln|2x+5| + c = 3\ln|2x+5| + c$

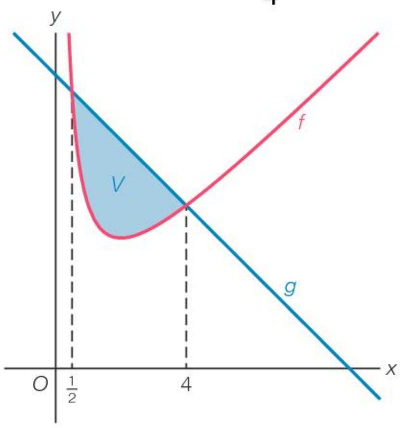
6 **a** $f(x) = g(x)$ geeft $\dfrac{x^2+4}{x} = -x + 9$

$x^2 + 4 = -x^2 + 9x$

$2x^2 - 9x + 4 = 0$

$D = (-9)^2 - 4 \cdot 2 \cdot 4 = 49$

$x = \dfrac{9+7}{4} = 4 \vee x = \dfrac{9-7}{4} = \frac{1}{2}$

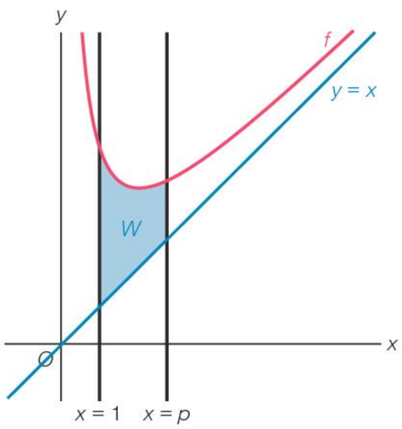


$O(V) = \displaystyle\int_{\frac{1}{2}}^{4} \left(-x + 9 - \frac{x^2+4}{x}\right)\mathrm{d}x = \int_{\frac{1}{2}}^{4} \left(-x + 9 - x - \frac{4}{x}\right)\mathrm{d}x = \int_{\frac{1}{2}}^{4} \left(-2x + 9 - \frac{4}{x}\right)\mathrm{d}x$

$= \left[-x^2 + 9x - 4\ln|x|\right]_{\frac{1}{2}}^{4} = -16 + 36 - 4\ln(4) - (-\frac{1}{4} + 4\frac{1}{2} - 4\ln(\frac{1}{2})) = 20 - 4\ln(4) - 4\frac{1}{4} + 4\ln(\frac{1}{2})$

$= 15\frac{3}{4} - 8\ln(2) - 4\ln(2) = 15\frac{3}{4} - 12\ln(2)$

b



$O(W) = \displaystyle\int_{1}^{p} \left(\frac{x^2+4}{x} - x\right)\mathrm{d}x = \int_{1}^{p} \frac{4}{x}\mathrm{d}x = \left[4\ln|x|\right]_{1}^{p} = 4\ln(p) - 0 = 4\ln(p)$

$O(W) = 3$ geeft $4\ln(p) = 3$

$\ln(p) = \frac{3}{4}$

$p = \mathrm{e}^{\frac{3}{4}} = \sqrt[4]{\mathrm{e}^3}$

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Bladzijde 139

7 $f(x) = 0$ geeft $(2x+1)\sqrt{2x+1} = 0$

$$(2x+1)^{1\frac{1}{2}} = 0$$
$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\tfrac{1}{2}$$

$$I(K) = \pi\int_{-\frac{1}{2}}^{0}(2x+1)^3\,\mathrm{d}x = \pi\left[\tfrac{1}{2}\cdot\tfrac{1}{4}(2x+1)^4\right]_{-\frac{1}{2}}^{0} = \pi(\tfrac{1}{8} - 0) = \tfrac{1}{8}\pi$$

$$I(L_p) = \pi\int_{0}^{p}(2x+1)^3\,\mathrm{d}x = \pi\left[\tfrac{1}{2}\cdot\tfrac{1}{4}(2x+1)^4\right]_{0}^{p} = \pi(\tfrac{1}{8}(2p+1)^4 - \tfrac{1}{8}\cdot 1^4) = \tfrac{1}{8}\pi((2p+1)^4 - 1)$$

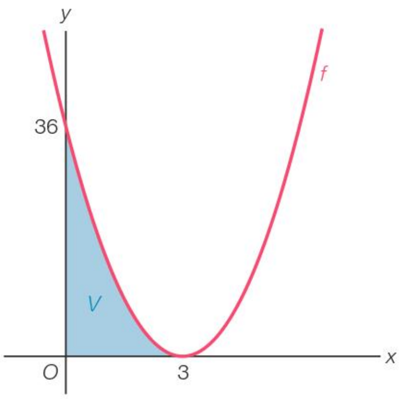
$I(L_p) = 624 \cdot I(K) = 624 \cdot \tfrac{1}{8}\pi = 78\pi$ geeft $\tfrac{1}{8}\pi((2p+1)^4 - 1) = 78\pi$

$$(2p+1)^4 - 1 = 624$$
$$(2p+1)^4 = 625$$
$$2p + 1 = 5 \vee 2p + 1 = -5$$
$$2p = 4 \vee 2p = -6$$
$$p = 2 \vee p = -3$$

vold. niet

Dus $p = 2$.

8 $f(0) = (-6)^2 = 36$



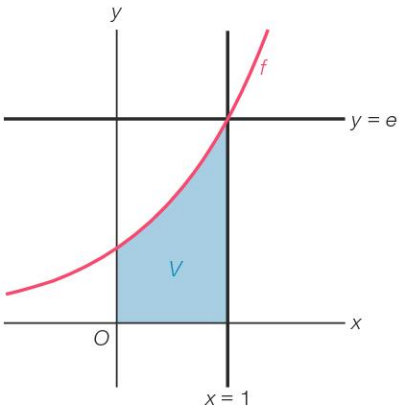
$y = (2x-6)^2$ en $0 \le x \le 3$ geeft $2x - 6 = -\sqrt{y}$

$$2x = -\sqrt{y} + 6$$
$$x = -\tfrac{1}{2}\sqrt{y} + 3$$
$$x^2 = (-\tfrac{1}{2}\sqrt{y} + 3)^2$$
$$x^2 = \tfrac{1}{4}y - 3\sqrt{y} + 9$$

$$I(L) = \pi\int_{0}^{36} x^2\,\mathrm{d}y = \pi\int_{0}^{36}(\tfrac{1}{4}y - 3y^{\frac{1}{2}} + 9)\,\mathrm{d}y = \pi\left[\tfrac{1}{8}y^2 - 3\cdot\tfrac{2}{3}y^{1\frac{1}{2}} + 9y\right]_{0}^{36} = \pi\left[\tfrac{1}{8}y^2 - 2y\sqrt{y} + 9y\right]_{0}^{36}$$

$$= \pi(162 - 432 + 324 - 0) = 54\pi$$

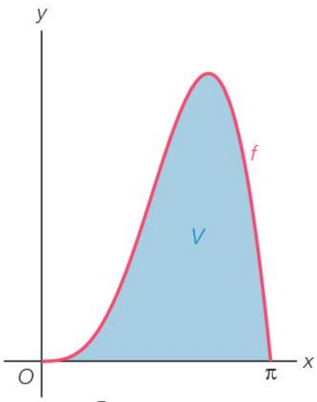
9



$f(x) = e^x$
$\downarrow$ translatie $(0, -e)$
$y = e^x - e$
Wentelen om de lijn $y = e$ geeft dezelfde inhoud als het vlakdeel ingesloten door de y-as, de lijn $x = 1$, de lijn $y = -e$ en de grafiek van $y = e^x - e$ wentelen om de x-as.

$$I(L) = \pi\int_0^1 ((-e)^2 - (e^x - e)^2)\,dx = \pi\int_0^1 (e^2 - (e^{2x} - 2e \cdot e^x + e^2))\,dx = \pi\int_0^1 (-e^{2x} + 2e \cdot e^x)\,dx$$
$$= \pi\left[-\tfrac{1}{2}e^{2x} + 2e \cdot e^x\right]_0^1 = \pi(-\tfrac{1}{2}e^2 + 2e^2 - (-\tfrac{1}{2} + 2e)) = (1\tfrac{1}{2}e^2 - 2e + \tfrac{1}{2})\pi \approx 19{,}31$$

10 $f(x) = 0$ geeft $x^2 \sin(x) = 0$
$x^2 = 0 \vee \sin(x) = 0$
$x = 0 \vee x = k \cdot \pi$
$x = 0 \vee x = \pi$



$O(V) = \int_0^\pi x^2 \sin(x)\,dx$

De optie integraal geeft $\int_0^\pi x^2 \sin(x)\,dx = 5{,}8696...$

Dus $O(V) \approx 5{,}870$.

11 OA: $y = 2x$

$$I(L) = \pi\int_0^1 (2x)^2\,dx + \pi\int_1^{\sqrt{5}} (5 - x^2)\,dx = \pi\int_0^1 4x^2\,dx + \pi\int_1^{\sqrt{5}} (5 - x^2)\,dx = \pi\left[\tfrac{4}{3}x^3\right]_0^1 + \pi\left[5x - \tfrac{1}{3}x^3\right]_1^{\sqrt{5}}$$
$$= \pi(\tfrac{4}{3} \cdot 1^3 - \tfrac{4}{3} \cdot 0^3) + \pi(5\sqrt{5} - \tfrac{1}{3}(\sqrt{5})^3 - (5 \cdot 1 - \tfrac{1}{3} \cdot 1^3)) = 1\tfrac{1}{3}\pi + \pi(5\sqrt{5} - \tfrac{1}{3} \cdot 5\sqrt{5} - 4\tfrac{2}{3})$$
$$= \pi(1\tfrac{1}{3} + 3\tfrac{1}{3}\sqrt{5} - 4\tfrac{2}{3}) = \pi(3\tfrac{1}{3}\sqrt{5} - 3\tfrac{1}{3}) = 3\tfrac{1}{3}\pi(\sqrt{5} - 1) \approx 12{,}94$$

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12 **a** Stel $a(t) = mt + n$ met $m = \dfrac{a(60) - a(0)}{60} = \dfrac{68 - 8}{60} = 1$
$a(0) = 8$
$\Big\}\ a(t) = t + 8$

$a(t) = t + 8$ en $v(0) = 0$ geeft $v(t) = \frac{1}{2}t^2 + 8t$
$v(t) = \frac{1}{2}t^2 + 8t$ en $s(0) = 0$ geeft $s(t) = \frac{1}{6}t^3 + 4t^2$
$s(60) = \frac{1}{6} \cdot 60^3 + 4 \cdot 60^2 = 50\,400$
Dus de raket bereikt in deze 60 seconden een hoogte van 50,4 km.

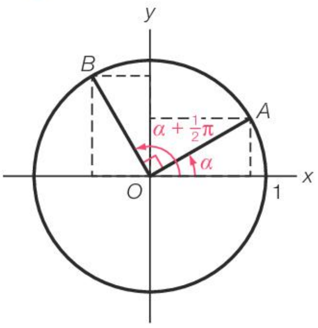
b Vanaf $t = 60$ is $a(t) = -10$, dus $v(t) = -10t + c$.
$v(60) = 2280$ geeft $-600 + c_1 = 2280$, dus $c_1 = 2880$ en $v(t) = -10t + 2880$.
$s(t) = -5t^2 + 2880t + c_2$
$s(60) = 50\,400$ geeft $-5 \cdot 60^2 + 2880 \cdot 60 + c_2 = 50\,400$
$-18\,000 + 172\,800 + c_2 = 50\,400$
$c_2 = -104\,400$
Dus $s(t) = -5t^2 + 2880t - 104\,400$.
$s(288) = -5 \cdot 288^2 + 2880 \cdot 288 - 104\,400 = 310\,320$
Dus de maximale hoogte is 310,32 km.

12 Goniometrische formules

Voorkennis Goniometrische formules herleiden

Bladzijde 143

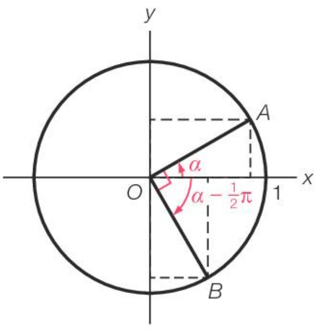
1 **a**



$\sin(\alpha + \frac{1}{2}\pi) = y_B = x_A = \cos(\alpha)$ oftewel $\sin(A + \frac{1}{2}\pi) = \cos(A)$.
$\cos(\alpha + \frac{1}{2}\pi) = x_B = -y_A = -\sin(\alpha)$ oftewel $\cos(A + \frac{1}{2}\pi) = -\sin(A)$.

b $\sin(A) = -\cos(A + \frac{1}{2}\pi) = \cos(A + \frac{1}{2}\pi + \pi) = \cos(A + 1\frac{1}{2}\pi) = \cos(A + 1\frac{1}{2}\pi - 2\pi) = \cos(A - \frac{1}{2}\pi)$

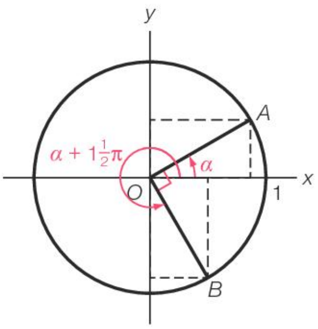
2 **a**



$y = \sin(\alpha - \frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha)$, dus $y = -\cos(\alpha)$.

b $y = \cos(\alpha - \frac{1}{2}\pi) = x_B = y_A = \sin(\alpha)$, dus $y = \sin(\alpha)$.

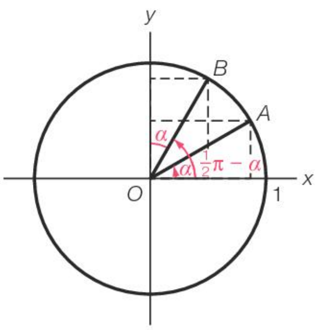
c



$y = \sin(\alpha + 1\frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha)$, dus $y = -\cos(\alpha)$.

d $y = \cos(\alpha + 1\frac{1}{2}\pi) = x_B = y_A = \sin(\alpha)$, dus $y = \sin(\alpha)$.

e



$y = \sin(\frac{1}{2}\pi - \alpha) = y_B = x_A = \cos(\alpha)$, dus $y = \cos(\alpha)$.

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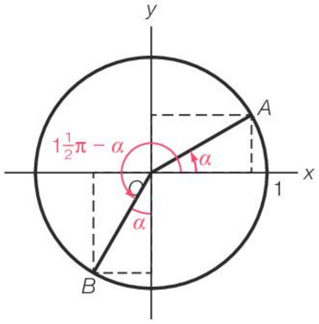
f $y = \cos(\frac{1}{2}\pi - \alpha) = x_B = y_A = \sin(\alpha)$, dus $y = \sin(\alpha)$.

g



$y = \sin(1\frac{1}{2}\pi - \alpha) = y_B = -x_A = -\cos(\alpha)$, dus $y = -\cos(\alpha)$.

h $y = \cos(1\frac{1}{2}\pi - \alpha) = x_B = -y_A = -\sin(\alpha)$, dus $y = -\sin(\alpha)$.

Bladzijde 144

3 **a** $y = \sin(2x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(2x - \frac{5}{6}\pi)$

b $y = -\sin(3x - \frac{1}{4}\pi) = \sin(3x - \frac{1}{4}\pi + \pi) = \sin(3x + \frac{3}{4}\pi) = \cos(3x + \frac{3}{4}\pi - \frac{1}{2}\pi) = \cos(3x + \frac{1}{4}\pi)$

c $y = \cos(\frac{1}{2}x + \frac{1}{4}\pi) = \sin(\frac{1}{2}x + \frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(\frac{1}{2}x + \frac{3}{4}\pi)$

d $y = -\cos(2(x - \frac{1}{6}\pi)) = \cos(2x - \frac{1}{3}\pi + \pi) = \cos(2x + \frac{2}{3}\pi) = \sin(2x + \frac{2}{3}\pi + \frac{1}{2}\pi) = \sin(2x + 1\frac{1}{6}\pi)$

12.1 Goniometrische formules bij vergelijkingen en herleidingen

Bladzijde 145

1 **a** $\sin(2x) + \sin(x) = 0$

$\sin(2x) = -\sin(x)$

$\sin(2x) = \sin(x + \pi)$

b $2\sin^2(x) = 3\cos(x)$

$2(1 - \cos^2(x)) = 3\cos(x)$

$2 - 2\cos^2(x) = 3\cos(x)$

$2\cos^2(x) + 3\cos(x) - 2 = 0$

Bladzijde 147

2 $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$

$\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$

$2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$

$x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$

3 **a** $\sin(x + \frac{1}{2}\pi) = \cos(2x)$

$\cos(x) = \cos(2x)$

$x = 2x + k \cdot 2\pi \vee x = -2x + k \cdot 2\pi$

$-x = k \cdot 2\pi \vee 3x = k \cdot 2\pi$

$x = k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = 2\pi$

b $\sin(3x) = -\cos(x)$

$\cos(3x - \frac{1}{2}\pi) = \cos(x + \pi)$

$3x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -(x + \pi) + k \cdot 2\pi$

$2x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$

$x = \frac{3}{4}\pi + k \cdot \pi \vee 4x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{3}{4}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{3}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = \frac{3}{8}\pi \vee x = \frac{7}{8}\pi \vee x = 1\frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi$

c $\sin^2(x) + \frac{1}{2}\cos(x) = 1$

$1 - \cos^2(x) + \frac{1}{2}\cos(x) - 1 = 0$

$-\cos^2(x) + \frac{1}{2}\cos(x) = 0$

$-\cos(x)(\cos(x) - \frac{1}{2}) = 0$

$\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$

$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$

d $\sin(\frac{1}{2}x) + \sqrt{3} \cdot \cos(\frac{1}{2}x) = 0$

$\sin(\frac{1}{2}x) = -\sqrt{3} \cdot \cos(\frac{1}{2}x)$

$\frac{\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)} = -\sqrt{3}$

$\tan(\frac{1}{2}x) = -\sqrt{3}$

$\frac{1}{2}x = -\frac{1}{3}\pi + k \cdot \pi$

$x = -\frac{2}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{3}\pi$

4 **a** $\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$

$\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$

$2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -(\frac{1}{2}\pi t - \frac{1}{2}\pi) + k \cdot 2\pi$

$1\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$

$1\frac{1}{2}t = -\frac{1}{2} + k \cdot 2 \vee 2\frac{1}{2}\pi t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = -\frac{1}{3} + k \cdot 1\frac{1}{3} \vee 2\frac{1}{2}t = \frac{1}{2} + k \cdot 2$

$t = -\frac{1}{3} + k \cdot 1\frac{1}{3} \vee t = \frac{1}{5} + k \cdot \frac{4}{5}$

t in $[0, 3]$ geeft $t = 1 \vee t = 2\frac{1}{3} \vee t = \frac{1}{5} \vee t = 1\frac{4}{5} \vee t = 2\frac{3}{5}$

b $\sin(\frac{1}{6}\pi t) = -\cos(\pi t)$

$\cos(\frac{1}{6}\pi t - \frac{1}{2}\pi) = \cos(\pi t + \pi)$

$\frac{1}{6}\pi t - \frac{1}{2}\pi = \pi t + \pi + k \cdot 2\pi \vee \frac{1}{6}\pi t - \frac{1}{2}\pi = -\pi t - \pi + k \cdot 2\pi$

$\frac{1}{6}t - \frac{1}{2} = t + 1 + k \cdot 2 \vee \frac{1}{6}t - \frac{1}{2} = -t - 1 + k \cdot 2$

$-\frac{5}{6}t = 1\frac{1}{2} + k \cdot 2 \vee 1\frac{1}{6}t = -\frac{1}{2} + k \cdot 2$

$t = -1\frac{4}{5} + k \cdot 2\frac{2}{5} \vee t = -\frac{3}{7} + k \cdot 1\frac{5}{7}$

t in $[0, 3]$ geeft $t = \frac{3}{5} \vee t = 3 \vee t = 1\frac{2}{7}$

5 **a** $2\sin^2(x) + \cos^2(x) + \cos(x) = 0$

$2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) = 0$

$2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = 0$

$\cos^2(x) - \cos(x) - 2 = 0$

$(\cos(x) + 1)(\cos(x) - 2) = 0$

$\cos(x) = -1 \vee \cos(x) = 2$

$x = \pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \pi$

b $\sin(2x - \frac{1}{3}\pi) = -\cos(\frac{1}{2}x)$

$\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(\frac{1}{2}x + \pi)$

$\cos(2x - \frac{5}{6}\pi) = \cos(\frac{1}{2}x + \pi)$

$2x - \frac{5}{6}\pi = \frac{1}{2}x + \pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -\frac{1}{2}x - \pi + k \cdot 2\pi$

$1\frac{1}{2}x = 1\frac{5}{6}\pi + k \cdot 2\pi \vee 2\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = 1\frac{2}{9}\pi + k \cdot 1\frac{1}{3}\pi \vee x = -\frac{1}{15}\pi + k \cdot \frac{4}{5}\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{2}{9}\pi \vee x = \frac{11}{15}\pi \vee x = 1\frac{8}{15}\pi$

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c $\cos(\frac{1}{4}\pi t) = -\sin(\frac{2}{5}\pi t)$

$\sin(\frac{1}{4}\pi t + \frac{1}{2}\pi) = \sin(\frac{2}{5}\pi t + \pi)$

$\frac{1}{4}\pi t + \frac{1}{2}\pi = \frac{2}{5}\pi t + \pi + k \cdot 2\pi \vee \frac{1}{4}\pi t + \frac{1}{2}\pi = \pi - (\frac{2}{5}\pi t + \pi) + k \cdot 2\pi$

$\frac{1}{4}t + \frac{1}{2} = \frac{2}{5}t + 1 + k \cdot 2 \vee \frac{1}{4}t + \frac{1}{2} = 1 - (\frac{2}{5}t + 1) + k \cdot 2$

$-\frac{3}{20}t = \frac{1}{2} + k \cdot 2 \vee \frac{13}{20}t = -\frac{1}{2} + k \cdot 2$

$t = -\frac{10}{3} + k \cdot \frac{40}{3} \vee t = -\frac{10}{13} + k \cdot \frac{40}{13}$

t in $[0, 10]$ geeft $t = 10 \vee t = 2\frac{4}{13} \vee t = 5\frac{5}{13} \vee t = 8\frac{6}{13}$

d $\cos(\frac{1}{3}\pi x) = \sqrt{3} \cdot \sin(\frac{1}{3}\pi x)$

$\frac{\sin(\frac{1}{3}\pi x)}{\cos(\frac{1}{3}\pi x)} = \frac{1}{\sqrt{3}}$

$\tan(\frac{1}{3}\pi x) = \frac{1}{3}\sqrt{3}$

$\frac{1}{3}\pi x = \frac{1}{6}\pi + k \cdot \pi$

$\frac{1}{3}x = \frac{1}{6} + k \cdot 1$

$x = \frac{1}{2} + k \cdot 3$

x in $[0, 10]$ geeft $x = \frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 6\frac{1}{2} \vee x = 9\frac{1}{2}$

6 **a** $2\sin(x) = \sin(x)$

$\sin(x) = 0$

b $\sin(2x) = \sin(x)$

$2x = x + k \cdot 2\pi \vee 2x = \pi - x + k \cdot 2\pi$

c –

d –

e $\sin(2x) = \sin(x + \frac{1}{3}\pi)$

$2x = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$

f –

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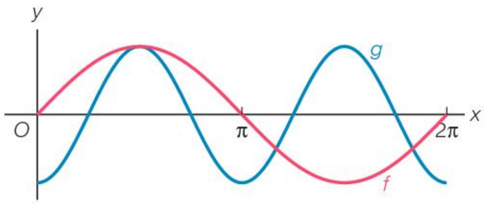
7 **a**



b $f(x) = -\frac{1}{2}\sqrt{2}$ geeft $\sin(x) = -\frac{1}{2}\sqrt{2}$

$x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \pi - -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$

c $g(x) = \frac{1}{2}$ geeft $-\cos(2x) = \frac{1}{2}$

$\cos(2x) = -\frac{1}{2}$

$2x = \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = -\frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot \pi \vee x = -\frac{1}{3}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi$

d $f(x) = g(x)$ geeft $\sin(x) = -\cos(2x)$

$\cos(x - \frac{1}{2}\pi) = \cos(2x + \pi)$

$x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -(2x + \pi) + k \cdot 2\pi$

$-x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$

$x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$

$f(x) \leq g(x)$ geeft $x = \frac{1}{2}\pi \vee 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

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8 $f(x) = g(x)$ geeft $\sin(2x - \frac{1}{3}\pi) = -\cos(x - \frac{1}{6}\pi)$

$$\cos(2x - \tfrac{5}{6}\pi) = \cos(x + \tfrac{5}{6}\pi)$$

$$2x - \tfrac{5}{6}\pi = x + \tfrac{5}{6}\pi + k \cdot 2\pi \vee 2x - \tfrac{5}{6}\pi = -x - \tfrac{5}{6}\pi + k \cdot 2\pi$$

$$x = \tfrac{5}{3}\pi + k \cdot 2\pi \vee 3x = k \cdot 2\pi$$

$$x = 1\tfrac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot \tfrac{2}{3}\pi$$

x in $[0, 2\pi]$ geeft $x = 1\frac{2}{3}\pi \vee x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \pi$

$$O(V) = \int_0^{\frac{2}{3}\pi} (f(x) - g(x))\,\mathrm{d}x = \int_0^{\frac{2}{3}\pi} (\sin(2x - \tfrac{1}{3}\pi) + \cos(x - \tfrac{1}{6}\pi))\,\mathrm{d}x = \left[-\tfrac{1}{2}\cos(2x - \tfrac{1}{3}\pi) + \sin(x - \tfrac{1}{6}\pi)\right]_0^{\frac{2}{3}\pi}$$

$$= -\tfrac{1}{2}\cos(\pi) + \sin(\tfrac{1}{2}\pi) - (-\tfrac{1}{2}\cos(-\tfrac{1}{3}\pi) + \sin(-\tfrac{1}{6}\pi)) = \tfrac{1}{2} + 1 - (-\tfrac{1}{4} - \tfrac{1}{2}) = 2\tfrac{1}{4}$$

$$O(W) = \int_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi} (f(x) - g(x))\,\mathrm{d}x = \int_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi} (\sin(2x - \tfrac{1}{3}\pi) + \cos(x - \tfrac{1}{6}\pi))\,\mathrm{d}x = \left[-\tfrac{1}{2}\cos(2x - \tfrac{1}{3}\pi) + \sin(x - \tfrac{1}{6}\pi)\right]_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi}$$

$$= -\tfrac{1}{2}\cos(3\pi) + \sin(1\tfrac{1}{2}\pi) - (-\tfrac{1}{2}\cos(2\tfrac{1}{3}\pi) + \sin(1\tfrac{1}{6}\pi)) = \tfrac{1}{2} - 1 - (-\tfrac{1}{4} - \tfrac{1}{2}) = \tfrac{1}{4}$$

De oppervlakte van V is $\dfrac{2\frac{1}{4}}{\frac{1}{4}} = 9$ keer zo groot als de oppervlakte van W.

9 $f(x) = \frac{1}{2}$ geeft $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$

$$2x - \tfrac{1}{3}\pi = \tfrac{1}{6}\pi + k \cdot 2\pi \vee 2x - \tfrac{1}{3}\pi = \tfrac{5}{6}\pi + k \cdot 2\pi$$

$$2x = \tfrac{1}{2}\pi + k \cdot 2\pi \vee 2x = 1\tfrac{1}{6}\pi + k \cdot 2\pi$$

$$x = \tfrac{1}{4}\pi + k \cdot \pi \vee x = \tfrac{7}{12}\pi + k \cdot \pi$$

Het is duidelijk dat het kleinste deel het deel boven de lijn $y = \frac{1}{2}$ is.

$$\text{Dus } O = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (f(x) - \tfrac{1}{2})\,\mathrm{d}x = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (\sin(2x - \tfrac{1}{3}\pi) - \tfrac{1}{2})\,\mathrm{d}x = \left[-\tfrac{1}{2}\cos(2x - \tfrac{1}{3}\pi) - \tfrac{1}{2}x\right]_{\frac{1}{4}\pi}^{\frac{7}{12}\pi}$$

$$= -\tfrac{1}{2}\cos(\tfrac{5}{6}\pi) - \tfrac{7}{24}\pi - (-\tfrac{1}{2}\cos(\tfrac{1}{6}\pi) - \tfrac{1}{8}\pi) = -\tfrac{1}{2} \cdot -\tfrac{1}{2}\sqrt{3} - \tfrac{7}{24}\pi + \tfrac{1}{2} \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{8}\pi$$

$$= \tfrac{1}{4}\sqrt{3} + \tfrac{1}{4}\sqrt{3} - \tfrac{4}{24}\pi = \tfrac{1}{2}\sqrt{3} - \tfrac{1}{6}\pi$$

10 Er geldt $\vec{a} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}$ en $\angle(\vec{a}, \vec{b}) = \alpha - \beta$.

$$\cos(\alpha - \beta) = \cos(\angle(\vec{a}, \vec{b})) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}}{1 \cdot 1} = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

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11 **a** $\cos(t - u) = \cos(t)\cos(u) + \sin(t)\sin(u)$

u vervangen door $-u$ geeft

$\cos(t + u) = \cos(t)\cos(-u) + \sin(t)\sin(-u)$

$\cos(t + u) = \cos(t)\cos(u) + \sin(t) \cdot -\sin(u)$

$\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$

b $\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$

u vervangen door $u - \frac{1}{2}\pi$ geeft

$\cos(t + u - \frac{1}{2}\pi) = \cos(t)\cos(u - \frac{1}{2}\pi) - \sin(t)\sin(u - \frac{1}{2}\pi)$

$\sin(t + u) = \cos(t)\sin(u) - \sin(t) \cdot -\cos(u)$

$\sin(t + u) = \cos(t)\sin(u) + \sin(t)\cos(u)$

$\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

c $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

u vervangen door $-u$ geeft

$\sin(t - u) = \sin(t)\cos(-u) + \cos(t)\sin(-u)$

$\sin(t - u) = \sin(t)\cos(u) + \cos(t) \cdot -\sin(u)$

$\sin(t - u) = \sin(t)\cos(u) - \cos(t)\sin(u)$

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12 **a** $\sin(t+u)=\sin(t)\cos(u)+\cos(t)\sin(u)$
$t=A$ en $u=A$ geeft
$\sin(A+A)=\sin(A)\cos(A)+\cos(A)\sin(A)$
$\sin(2A)=2\sin(A)\cos(A)$

$\cos(t+u)=\cos(t)\cos(u)-\sin(t)\sin(u)$
$t=A$ en $u=A$ geeft
$\cos(A+A)=\cos(A)\cos(A)-\sin(A)\sin(A)$
$\cos(2A)=\cos^2(A)-\sin^2(A)$

b $\cos(2A)=\cos^2(A)-\sin^2(A)$
$\cos(2A)=\cos^2(A)-(1-\cos^2(A))$
$\cos(2A)=\cos^2(A)-1+\cos^2(A)$
$\cos(2A)=2\cos^2(A)-1$

$\cos(2A)=\cos^2(A)-\sin^2(A)$
$\cos(2A)=1-\sin^2(A)-\sin^2(A)$
$\cos(2A)=1-2\sin^2(A)$

13 **a** $\cos(2A)=2\cos^2(A)-1$
$2\cos^2(A)=1+\cos(2A)$
$\cos^2(A)=\frac{1}{2}+\frac{1}{2}\cos(2A)$

b $\cos(2A)=1-2\sin^2(A)$
$2\sin^2(A)=1-\cos(2A)$
$\sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A)$

14 **a** $\sin(x-\frac{1}{4}\pi)=\sin(x)\cos(\frac{1}{4}\pi)-\cos(x)\sin(\frac{1}{4}\pi)=\sin(x)\cdot\frac{1}{2}\sqrt{2}-\cos(x)\cdot\frac{1}{2}\sqrt{2}=\frac{1}{2}\sqrt{2}(\sin(x)-\cos(x))$

b $\sin(\frac{1}{12}\pi)=\sin(\frac{1}{3}\pi-\frac{1}{4}\pi)=\sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi)-\cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi)=\frac{1}{2}\sqrt{3}\cdot\frac{1}{2}\sqrt{2}-\frac{1}{2}\cdot\frac{1}{2}\sqrt{2}$
$=\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}=\frac{1}{4}(\sqrt{6}-\sqrt{2})$

c $\sin(x+\frac{1}{4}\pi)=\sin(x)\cos(\frac{1}{4}\pi)+\cos(x)\sin(\frac{1}{4}\pi)=\sin(x)\cdot\frac{1}{2}\sqrt{2}+\cos(x)\cdot\frac{1}{2}\sqrt{2}=\frac{1}{2}\sqrt{2}(\sin(x)+\cos(x))$

d $\sin(\frac{7}{12}\pi)=\sin(\frac{1}{3}\pi+\frac{1}{4}\pi)=\sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi)+\cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi)=\frac{1}{2}\sqrt{3}\cdot\frac{1}{2}\sqrt{2}+\frac{1}{2}\cdot\frac{1}{2}\sqrt{2}$
$=\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}=\frac{1}{4}(\sqrt{6}+\sqrt{2})$

15 **a** $\cos(\frac{1}{12}\pi)=\cos(\frac{1}{3}\pi-\frac{1}{4}\pi)=\cos(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi)+\sin(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi)=\frac{1}{2}\cdot\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{3}\cdot\frac{1}{2}\sqrt{2}=\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}$

b $\sin^2(\frac{1}{8}\pi)=\frac{1}{2}-\frac{1}{2}\cos(\frac{1}{4}\pi)=\frac{1}{2}-\frac{1}{2}\cdot\frac{1}{2}\sqrt{2}=\frac{1}{2}-\frac{1}{4}\sqrt{2}$
$\sin(\frac{1}{8}\pi)=\sqrt{\frac{1}{2}-\frac{1}{4}\sqrt{2}}\vee\sin(\frac{1}{8}\pi)=-\sqrt{\frac{1}{2}-\frac{1}{4}\sqrt{2}}$
vold. niet, want $\sin(\frac{1}{8}\pi)$ is positief

16 **a** $\cos(4x)\cos(x)+\sin(4x)\sin(x)=\cos(4x-x)=\cos(3x)$

b $\sin(1\frac{1}{2}x)\cos(x)-\cos(1\frac{1}{2}x)\sin(x)=\sin(1\frac{1}{2}x-x)=\sin(\frac{1}{2}x)$

c $\cos(\frac{3}{4}x)\cos(1\frac{1}{4}x)-\sin(\frac{3}{4}x)\sin(1\frac{1}{4}x)=\cos(\frac{3}{4}x+1\frac{1}{4}x)=\cos(2x)$

d $\sin(1\frac{1}{3}x)\cos(1\frac{2}{3}x)+\cos(1\frac{1}{3}x)\sin(1\frac{2}{3}x)=\sin(1\frac{1}{3}x+1\frac{2}{3}x)=\sin(3x)$

17 $\sin(x)(\sin(4x)-\sin(2x))+\cos(x)(\cos(4x)+\cos(2x))=$
$\sin(x)\sin(4x)-\sin(x)\sin(2x)+\cos(x)\cos(4x)+\cos(x)\cos(2x)=$
$\cos(4x)\cos(x)+\sin(4x)\sin(x)+\cos(x)\cos(2x)-\sin(x)\sin(2x)=$
$\cos(4x-x)+\cos(x+2x)=\cos(3x)+\cos(3x)=2\cos(3x)$

18 **a** $2\sin(x)\cos(x)=\sin(x-1)$
$\sin(2x)=\sin(x-1)$
$2x=x-1+k\cdot 2\pi\vee 2x=\pi-(x-1)+k\cdot 2\pi$
$x=-1+k\cdot 2\pi\vee 2x=\pi-x+1+k\cdot 2\pi$
$x=-1+k\cdot 2\pi\vee 3x=\pi+1+k\cdot 2\pi$
$x=-1+k\cdot 2\pi\vee x=\frac{1}{3}\pi+\frac{1}{3}+k\cdot\frac{2}{3}\pi$

b Gebruik $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$.

$\cos^2(2x) = \cos(4x) + \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2}\cos(4x) = \cos(4x) + \frac{1}{2}$

$\frac{1}{2}\cos(4x) = \cos(4x)$

$\cos(4x) = 0$

$4x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$

c Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$.

$\sin^2(\frac{1}{2}x) = \cos(x) + 1\frac{1}{4}$

$\frac{1}{2} - \frac{1}{2}\cos(x) = \cos(x) + 1\frac{1}{4}$

$-1\frac{1}{2}\cos(x) = \frac{3}{4}$

$\cos(x) = -\frac{1}{2}$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

d $(\sin(x) + \cos(x))^2 = 1\frac{1}{2}$

$\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = 1\frac{1}{2}$

$\sin^2(x) + \cos^2(x) + \sin(2x) = 1\frac{1}{2}$

$1 + \sin(2x) = 1\frac{1}{2}$

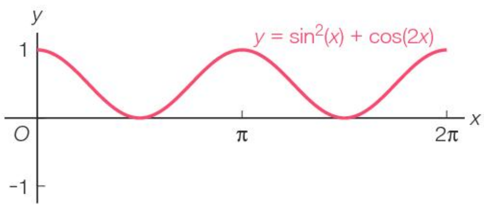
$\sin(2x) = \frac{1}{2}$

$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{12}\pi + k \cdot \pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi$

19 **a**



De evenwichtsstand is $\frac{1+0}{2} = \frac{1}{2}$, dus $a = \frac{1}{2}$.

De amplitude is $1 - \frac{1}{2} = \frac{1}{2}$, dus $b = \frac{1}{2}$.

De periode is π, dus $c = \frac{2\pi}{\pi} = 2$.

Een hoogste punt is $(0, 1)$, dus $y = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

b Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$.

$y = \sin^2(x) + \cos(2x)$

$= \frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(2x)$

$= \frac{1}{2} + \frac{1}{2}\cos(2x)$

20 $\sin(3x) = \sin(2x + x)$

$= \sin(2x)\cos(x) + \cos(2x)\sin(x)$

$= 2\sin(x)\cos(x)\cos(x) + (1 - 2\sin^2(x)) \cdot \sin(x)$

$= 2\sin(x)\cos^2(x) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x)(1 - \sin^2(x)) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$

$= 3\sin(x) - 4\sin^3(x)$

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Bladzijde 151

21 **a** Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$.

$y = 1 - \cos(x) - \sin^2(\frac{1}{2}x)$

$= 1 - \cos(x) - (\frac{1}{2} - \frac{1}{2}\cos(x))$

$= 1 - \cos(x) - \frac{1}{2} + \frac{1}{2}\cos(x)$

$= \frac{1}{2} - \frac{1}{2}\cos(x)$

b $\cos(3x) = \cos(2x + x)$

$= \cos(2x)\cos(x) - \sin(2x)\sin(x)$

$= (2\cos^2(x) - 1) \cdot \cos(x) - 2\sin(x)\cos(x)\sin(x)$

$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$

$= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x)) \cdot \cos(x)$

$= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$

$= 4\cos^3(x) - 3\cos(x)$

22 **a** $-\cos(A) = \cos(A + \pi)$

b $\sin(A) = \cos(A - \frac{1}{2}\pi)$

c $\cos(2A) = 1 - 2\sin^2(A)$ oftewel $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

d $\cos(2A) = 2\cos^2(A) - 1$ oftewel $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$

e $\cos(A) = \sin(A + \frac{1}{2}\pi)$

12.2 Goniometrische formules bij symmetrie en primitiveren

Bladzijde 153

23 $x_B = -x_A$ en $y_B = y_A$

$x_C = -x_A$ en $y_C = -y_A$

Bladzijde 155

24 **a** In voorbeeld a worden twee functiewaarden apart berekend die achteraf gelijk blijken te zijn.

In voorbeeld b wordt in één keer de som van twee functiewaarden berekend. Bij die berekening vallen verschillende termen tegen elkaar weg.

b $f(\frac{1}{4}\pi - p) + f(\frac{1}{4}\pi + p) =$

$\sin(\frac{1}{4}\pi - p) - \cos(\frac{1}{4}\pi - p) + 1\frac{1}{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + 1\frac{1}{2} =$

$\cos(\frac{1}{4}\pi + p) - \sin(\frac{1}{4}\pi + p) + 1\frac{1}{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + 1\frac{1}{2} = 3$

Er geldt $f(\frac{1}{4}\pi - p) + f(\frac{1}{4} + p) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A(\frac{1}{4}\pi, 1\frac{1}{2})$.

25 **a** $f(-p) + f(p) = -p\cos(-p) + p\cos(p) = -p\cos(p) + p\cos(p) = 0$

Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in de oorsprong.

b $g(-p) = -p\sin(-p) = p\sin(p)$

$g(p) = p\sin(p)$

Er geldt $g(-p) = g(p)$, dus de grafiek van g is symmetrisch in de y-as.

Bladzijde 156

26 **a** $f(-p) + f(p) = \cos^2(-p)\sin(-p) + \cos^2(p)\sin(p) = \cos^2(p) \cdot -\sin(p) + \cos^2(p)\sin(p) = 0$

Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in O.

b $f(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi - p)\sin(\frac{1}{2}\pi - p)$

$= (\cos(\frac{1}{2}\pi)\cos(p) + \sin(\frac{1}{2}\pi)\sin(p))^2 (\sin(\frac{1}{2}\pi)\cos(p) - \cos(\frac{1}{2}\pi)\sin(p))$

$= (0 \cdot \cos(p) + 1 \cdot \sin(p))^2 (1 \cdot \cos(p) - 0 \cdot \sin(p)) = \sin^2(p)\cos(p)$

$f(\frac{1}{2}\pi + p) = \cos^2(\frac{1}{2}\pi + p)\sin(\frac{1}{2}\pi + p)$

$= (\cos(\frac{1}{2}\pi)\cos(p) - \sin(\frac{1}{2}\pi)\sin(p))^2 (\sin(\frac{1}{2}\pi)\cos(p) + \cos(\frac{1}{2}\pi)\sin(p))$

$= (0 \cdot \cos(p) - 1 \cdot \sin(p))^2 (1 \cdot \cos(p) + 0 \cdot \sin(p)) = \sin^2(p)\cos(p)$

Er geldt $f(\frac{1}{2}\pi - p) = f(\frac{1}{2}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.

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27 **a** $f(\frac{1}{4}\pi - p) = \cos(\frac{1}{4}\pi - p) + \sin(\frac{1}{4}\pi - p) + 1$

$= \cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) + 1$

$= \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$

$f(\frac{1}{4}\pi + p) = \cos(\frac{1}{4}\pi + p) + \sin(\frac{1}{4}\pi + p) + 1$

$= \cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) + 1$

$= \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$

Er geldt $f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{4}\pi$.

b $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = \cos(\frac{3}{4}\pi - p) + \sin(\frac{3}{4}\pi - p) + 1 + \cos(\frac{3}{4}\pi + p) + \sin(\frac{3}{4}\pi + p) + 1$

$= \cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) + 1$

$+ \cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) + 1$

$= 2\cos(\frac{3}{4}\pi)\cos(p) + 2\sin(\frac{3}{4}\pi)\cos(p) + 2 = 2 \cdot -\frac{1}{2}\sqrt{2}\cos(p) + 2 \cdot \frac{1}{2}\sqrt{2}\cos(p) + 2 = 2$

Er geldt $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A(\frac{3}{4}\pi, 1)$.

28 $f(\frac{1}{6}\pi - p) = \sin(\frac{1}{6}\pi - p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi - p)$

$= \sin(\frac{1}{6}\pi)\cos(p) - \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) + \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$

$= \frac{1}{2}\cos(p) - \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) + \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$

$f(\frac{1}{6}\pi + p) = \sin(\frac{1}{6}\pi + p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi + p)$

$= \sin(\frac{1}{6}\pi)\cos(p) + \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) - \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$

$= \frac{1}{2}\cos(p) + \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) - \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$

$f(\frac{1}{6}\pi - p) = f(\frac{1}{6}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{6}\pi$.

29 **a** $F(x) = \frac{1}{3}\sin^3(x)$ geeft $F'(x) = \sin^2(x) \cdot \cos(x)$ en dit is niet $f(x)$,
dus $y = \frac{1}{3}\sin^3(x)$ is geen primitieve van f.

b $\cos(2A) = 1 - 2\sin^2(A)$

$2\sin^2(A) = 1 - \cos(2A)$

$\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

Dus $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$.

c $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c$

Bladzijde 157

30 **a** $f(x) = \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c$

b $g(x) = \sin^2(3x) = \frac{1}{2} - \frac{1}{2}\cos(6x)$ geeft $G(x) = \frac{1}{2}x - \frac{1}{12}\sin(6x) + c$

c $h(x) = \sin(\frac{1}{2}x)\cos(\frac{1}{2}x) = \frac{1}{2}\sin(x)$ geeft $H(x) = -\frac{1}{2}\cos(x) + c$

31 **a** $\int_0^{\frac{1}{6}\pi} \sin(2x)\cos(2x)\,dx = \int_0^{\frac{1}{6}\pi} \frac{1}{2}\sin(4x)\,dx = \left[-\frac{1}{8}\cos(4x)\right]_0^{\frac{1}{6}\pi}$

$= -\frac{1}{8}\cos(\frac{2}{3}\pi) + \frac{1}{8}\cos(0) = -\frac{1}{8} \cdot -\frac{1}{2} + \frac{1}{8} \cdot 1 = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$

b $\int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{2}\sin^2(x))\,dx = \int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{4} + \frac{1}{4}\cos(2x))\,dx = \int_{\frac{1}{3}\pi}^{\pi} (1\frac{3}{4} + \frac{1}{4}\cos(2x))\,dx$

$= \left[1\frac{3}{4}x + \frac{1}{8}\sin(2x)\right]_{\frac{1}{3}\pi}^{\pi} = 1\frac{3}{4}\pi + \frac{1}{8}\sin(2\pi) - (1\frac{3}{4} \cdot \frac{1}{3}\pi + \frac{1}{8}\sin(\frac{2}{3}\pi))$

$= 1\frac{3}{4}\pi + 0 - (\frac{7}{12}\pi + \frac{1}{8} \cdot \frac{1}{2}\sqrt{3}) = 1\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$

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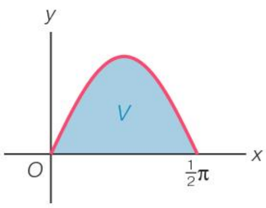
32



$$I(L) = \pi\int_0^{\frac{1}{2}\pi} (f(x))^2\,dx = \pi\int_0^{\frac{1}{2}\pi} \sin^2(2x)\,dx = \pi\int_0^{\frac{1}{2}\pi} (\tfrac{1}{2} - \tfrac{1}{2}\cos(4x))\,dx = \pi\left[(\tfrac{1}{2}x - \tfrac{1}{8}\sin(4x))\right]_0^{\frac{1}{2}\pi}$$
$$= \pi(\tfrac{1}{4}\pi - \tfrac{1}{8}\sin(2\pi) - (0 - \tfrac{1}{8}\sin(0)) = \tfrac{1}{4}\pi^2$$

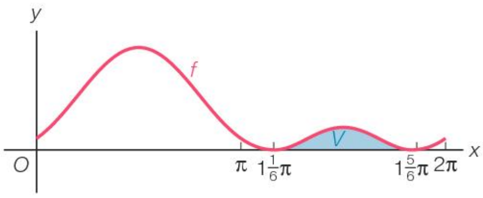
33 $f(x) = 0$ geeft $\sin^2(x) + \sin(x) + \frac{1}{4} = 0$

$(\sin(x) + \frac{1}{2})^2 = 0$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$



$$O(V) = \int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} (\sin^2(x) + \sin(x) + \tfrac{1}{4})\,dx = \int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} (\tfrac{1}{2} - \tfrac{1}{2}\cos(2x) + \sin(x) + \tfrac{1}{4})\,dx$$
$$= \int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} (\tfrac{3}{4} - \tfrac{1}{2}\cos(2x) + \sin(x))\,dx = \left[\tfrac{3}{4}x - \tfrac{1}{4}\sin(2x) - \cos(x)\right]_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi}$$
$$= \tfrac{11}{8}\pi - \tfrac{1}{4}\sin(3\tfrac{2}{3}\pi) - \cos(1\tfrac{5}{6}\pi) - (\tfrac{7}{8}\pi - \tfrac{1}{4}\sin(2\tfrac{1}{3}\pi) - \cos(1\tfrac{1}{6}\pi))$$
$$= \tfrac{11}{8}\pi - \tfrac{1}{4} \cdot -\tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\sqrt{3} - (\tfrac{7}{8}\pi - \tfrac{1}{4} \cdot \tfrac{1}{2}\sqrt{3} - -\tfrac{1}{2}\sqrt{3})$$
$$= \tfrac{11}{8}\pi + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{2}\sqrt{3} - \tfrac{7}{8}\pi + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{2}\sqrt{3}$$
$$= \tfrac{1}{2}\pi - \tfrac{3}{4}\sqrt{3}$$

34 **a** $f(x) = 2\sin^2(x) + \sin(x) - 1$ geeft $f'(x) = 4\sin(x)\cos(x) + \cos(x)$

$f'(x) = 0$ geeft $4\sin(x)\cos(x) + \cos(x) = 0$

$\cos(x)(4\sin(x) + 1) = 0$

$\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$

$\cos(x) = 0$ geeft $x = \frac{1}{2}\pi + k \cdot \pi$, dus $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$.

$f(\frac{1}{2}\pi) = 2\sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) - 1 = 2 \cdot 1^2 + 1 - 1 = 2$

$f(1\frac{1}{2}\pi) = 2\sin^2(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) - 1 = 2 \cdot (-1)^2 + -1 - 1 = 0$

$\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} - 1 = -1\frac{1}{8}$

Dus $B_f = [-1\frac{1}{8}, 2]$.

b $f(x) = 0$ geeft $2\sin^2(x) + \sin(x) - 1 = 0$

$$\text{Stel } \sin(x) = u$$
$$2u^2 + u - 1 = 0$$
$$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$$
$$u = \frac{-1+3}{4} = \tfrac{1}{2} \vee u = \frac{-1-3}{4} = -1$$
$$\sin(x) = \tfrac{1}{2} \vee \sin(x) = -1$$
$$x = \tfrac{1}{6}\pi + k \cdot 2\pi \vee x = \tfrac{5}{6}\pi + k \cdot 2\pi \vee x = 1\tfrac{1}{2}\pi + k \cdot 2\pi$$

x in $[0, 2\pi]$ geeft $x = \tfrac{1}{6}\pi \vee x = \tfrac{5}{6}\pi \vee x = 1\tfrac{1}{2}\pi$

$$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin^2(x) + \sin(x) - 1)\,dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (1 - \cos(2x) + \sin(x) - 1)\,dx$$
$$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (-\cos(2x) + \sin(x))\,dx = \left[-\tfrac{1}{2}\sin(2x) - \cos(x)\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$$
$$= -\tfrac{1}{2}\sin(\tfrac{5}{3}\pi) - \cos(\tfrac{5}{6}\pi) - (-\tfrac{1}{2}\sin(\tfrac{1}{3}\pi) - \cos(\tfrac{1}{6}\pi))$$
$$= -\tfrac{1}{2} \cdot -\tfrac{1}{2}\sqrt{3} - -\tfrac{1}{2}\sqrt{3} - (-\tfrac{1}{2} \cdot \tfrac{1}{2}\sqrt{3} - \tfrac{1}{2}\sqrt{3})$$
$$= \tfrac{1}{4}\sqrt{3} + \tfrac{1}{2}\sqrt{3} + \tfrac{1}{4}\sqrt{3} + \tfrac{1}{2}\sqrt{3} = 1\tfrac{1}{2}\sqrt{3}$$

35 Domein $[0, 1\tfrac{1}{2}\pi]$ en $p > 0$, dus $f_p(x) = p\sin(\tfrac{1}{3}x)\cos(\tfrac{1}{3}x) \geq 0$.

$f_p(x) = 0$ geeft $p\sin(\tfrac{1}{3}x)\cos(\tfrac{1}{3}x) = 0$

$$\sin(\tfrac{1}{3}x) = 0 \vee \cos(\tfrac{1}{3}x) = 0$$
$$\tfrac{1}{3}x = k \cdot \pi \vee \tfrac{1}{3}x = \tfrac{1}{2}\pi + k \cdot \pi$$
$$x = k \cdot 3\pi \vee x = 1\tfrac{1}{2}\pi + k \cdot 3\pi$$

x in $[0, 1\tfrac{1}{2}\pi]$ geeft $x = 0 \vee x = 1\tfrac{1}{2}\pi$

$$O(V_p) = \int_0^{1\frac{1}{2}\pi} p\sin(\tfrac{1}{3}x)\cos(\tfrac{1}{3}x)\,dx = \int_0^{1\frac{1}{2}\pi} \tfrac{1}{2}p\sin(\tfrac{2}{3}x)\,dx = \left[\tfrac{1}{2}p \cdot -\tfrac{3}{2}\cos(\tfrac{2}{3}x)\right]_0^{1\frac{1}{2}\pi} = -\tfrac{3}{4}p\cos(\pi) + \tfrac{3}{4}p\cos(0)$$
$$= \tfrac{3}{4}p + \tfrac{3}{4}p = 1\tfrac{1}{2}p$$

$O(V_p) = 10$ geeft $1\tfrac{1}{2}p = 10$

$$p = 6\tfrac{2}{3}$$

12.3 Eenparige cirkelbewegingen en harmonische trillingen

Bladzijde 159

36 P bevindt zich voor het eerst weer in $(1, 0)$ als t gelijk is aan 2π.
Dus na 2π seconden.

Bladzijde 160

37 **a** De formules $\sin^2(A) + \cos^2(A) = 1$ met $A = 1\tfrac{1}{2}t$ en met $A = t$
en $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ met $A = 1\tfrac{1}{2}t$ en $B = t$.

b Voor $t = \tfrac{1}{6}\pi$ is $PQ = \sqrt{2 - 2\cos(\tfrac{1}{12}\pi)} \approx 0{,}26$.

c $PQ = 1$ geeft $\sqrt{2 - 2\cos(\tfrac{1}{2}t)} = 1$

$$2 - 2\cos(\tfrac{1}{2}t) = 1$$
$$-2\cos(\tfrac{1}{2}t) = -1$$
$$\cos(\tfrac{1}{2}t) = \tfrac{1}{2}$$
$$\tfrac{1}{2}t = \tfrac{1}{3}\pi + k \cdot 2\pi \vee \tfrac{1}{2}t = -\tfrac{1}{3}\pi + k \cdot 2\pi$$
$$t = \tfrac{2}{3}\pi + k \cdot 4\pi \vee t = -\tfrac{2}{3}\pi + k \cdot 4\pi$$

t in $[0, 4\pi]$ geeft $t = \tfrac{2}{3}\pi \vee t = 3\tfrac{1}{3}\pi$

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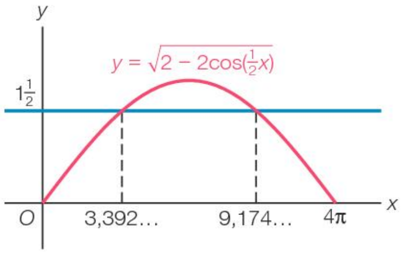
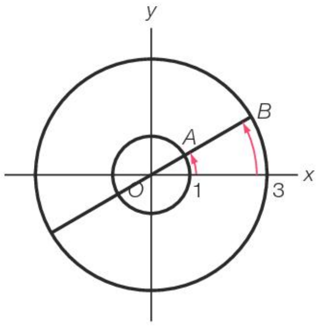
d Voer in $y_1 = \sqrt{2 - 2\cos(\frac{1}{2}x)}$ en $y_2 = 1\frac{1}{2}$.
De optie snijpunt geeft $x = 3{,}392...$ en $x = 9{,}174...$



$PQ > 1\frac{1}{2}$ geeft $3{,}39 < t < 9{,}17$

38 **a** $t = \frac{1}{4}\pi$ geeft $A(\cos(\frac{1}{2}\pi), \sin(\frac{1}{2}\pi)) = A(0, 1)$ en $B(3\cos(\frac{1}{4}\pi), 3\sin(\frac{1}{4}\pi)) = B(1\frac{1}{2}\sqrt{2}, 1\frac{1}{2}\sqrt{2})$
$AB = \sqrt{(1\frac{1}{2}\sqrt{2} - 0)^2 + (1\frac{1}{2}\sqrt{2} - 1)^2} = \sqrt{4\frac{1}{2} + 4\frac{1}{2} - 3\sqrt{2} + 1} = \sqrt{10 - 3\sqrt{2}}$

b

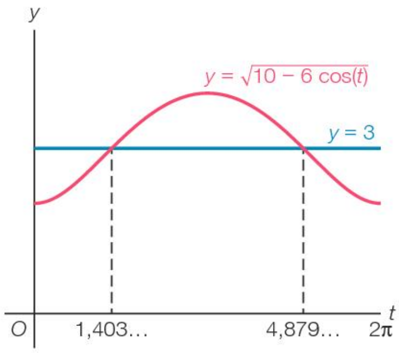


De kleinste afstand tussen A en B is 2 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B.
Dit geeft $2t = t + k \cdot 2\pi$
$t = k \cdot 2\pi$
$t = 0 \vee t = 2\pi$
De grootste afstand tussen A en B is 4 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B plus π.
Dit geeft $2t = t + \pi + k \cdot 2\pi$
$t = \pi + k \cdot 2\pi$
$t = \pi$

c $AB^2 = (3\cos(t) - \cos(2t))^2 + (3\sin(t) - \sin(2t))^2$
$= 9\cos^2(t) - 6\cos(t)\cos(2t) + \cos^2(2t) + 9\sin^2(t) - 6\sin(t)\sin(2t) + \sin^2(2t)$
$= 9 + 1 - 6(\cos(2t)\cos(t) + \sin(2t)\sin(t)) = 10 - 6\cos(2t - t)$
$= 10 - 6\cos(t)$
Dus $AB = \sqrt{10 - 6\cos(t)}$.

d $\sqrt{10 - 6\cos(t)} = \sqrt{7}$
$10 - 6\cos(t) = 7$
$6\cos(t) = 3$
$\cos(t) = \frac{1}{2}$
$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$
$0 \leq t \leq 2\pi$ geeft $t = \frac{1}{3}\pi \vee t = 1\frac{2}{3}\pi$

e Voer in $y_1 = \sqrt{10 - 6\cos(x)}$ en $y_2 = 3$.
De optie snijpunt geeft $x = 1{,}403...$ en $x = 4{,}879...$



$AB < 3$ geeft $0 \leq t < 1{,}40 \vee 4{,}88 < t \leq 2\pi$

Bladzijde 161

39 **a** $y = \frac{1}{2}\sqrt{2}$ geeft op de eenheidscirkel punten met richtingshoek $\frac{1}{4}\pi$ en met richtingshoek $\frac{3}{4}\pi$.
Tussen deze twee punten ligt precies een kwart van de eenheidscirkel. De lengte van de baan is dus $\frac{1}{4} \cdot 2\pi \cdot 1 = \frac{1}{2}\pi$.

b Voor A geldt $\sin(2t) = \frac{2}{3}\cos^2(2t)$

$$\sin(2t) = \tfrac{2}{3}(1 - \sin^2(2t))$$
$$\sin(2t) = \tfrac{2}{3} - \tfrac{2}{3}\sin^2(2t)$$
$$1\tfrac{1}{2}\sin(2t) = 1 - \sin^2(2t)$$
$$\sin^2(2t) + 1\tfrac{1}{2}\sin(2t) - 1 = 0$$
$$(\sin(2t) - \tfrac{1}{2})(\sin(2t) + 2) = 0$$
$$\sin(2t) = \tfrac{1}{2} \vee \sin(2t) = -2$$
$$2t = \tfrac{1}{6}\pi + k \cdot 2\pi \vee 2t = \tfrac{5}{6}\pi + k \cdot 2\pi$$
$$t = \tfrac{1}{12}\pi + k \cdot \pi \vee t = \tfrac{5}{12}\pi + k \cdot \pi$$

$P(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi))$ oftewel $P(\frac{1}{2}\sqrt{3}, \frac{1}{2})$
$Q(\cos(\frac{5}{6}\pi), \sin(\frac{5}{6}\pi))$ oftewel $Q(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$

c Voor B geldt $3\sin(t) = \frac{2}{3} \cdot (3\cos(t))^2$

$$3\sin(t) = 6\cos^2(t)$$
$$\sin(t) = 2\cos^2(t)$$

Voer in $y_1 = \sin(x)$ en $y_2 = 2\cos^2(x)$.
De optie snijpunt geeft $x = 0{,}895...$ en $x = 2{,}245...$
Dus $t = 0{,}895...$ en $t = 2{,}245...$
$R(3\cos(0{,}895...), 3\sin(0{,}895...)$ oftewel $R(1{,}874; 2{,}342)$
$S(3\cos(2{,}245...), 3\sin(2{,}245...)$ oftewel $S(-1{,}874; 2{,}342)$

40 **a** Substitutie van $x = 2\cos(t)$ en $y = 2\sin(t)$ in $y = x + 1$ geeft $2\sin(t) = 2\cos(t) + 1$.
Voer in $y_1 = 2\sin(x)$ en $y_2 = 2\cos(x) + 1$.
De optie snijpunt geeft $x \approx 1{,}15$ en $y \approx 1{,}82$ en $x \approx 3{,}57$ en $y \approx -0{,}82$.
Voor $t \approx 1{,}15$ is $y \approx 1{,}82$ en $x = y - 1 \approx 0{,}82$.
Voor $t \approx 3{,}57$ is $y \approx -0{,}82$ en $x = y - 1 \approx -1{,}82$.
Dit geeft de snijpunten $(0{,}82; 1{,}82)$ en $(-1{,}82; -0{,}82)$.

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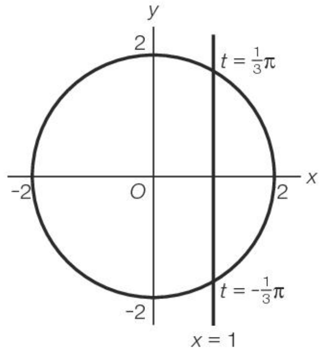
b $x = 1$ geeft $2\cos(t) = 1$
$\cos(t) = \frac{1}{2}$
$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$



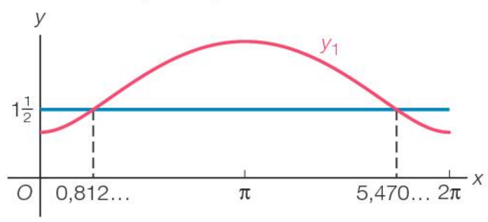
De baan ligt rechts van de lijn $x = 1$ indien $2\cos(t) > 1$, dus als $\cos(t) > \frac{1}{2}$
en dit geeft $-\frac{1}{3}\pi < t < \frac{1}{3}\pi$.
De baan is de cirkel met middelpunt (0, 0) en straal 2.
Dus ligt $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ deel van de cirkel rechts van de lijn $x = 1$.
De lengte van dit deel is $\frac{1}{3} \cdot 2\pi \cdot 2 = 1\frac{1}{3}\pi$.

c $P(2\cos(t), 2\sin(t))$ en $Q(\cos(2t), \sin(2t))$ geeft

$$\begin{aligned} PQ^2 &= (2\cos(t) - \cos(2t))^2 + (2\sin(t) - \sin(2t))^2 \\ &= 4\cos^2(t) - 4\cos(t)\cos(2t) + \cos^2(2t) + 4\sin^2(t) - 4\sin(t)\sin(2t) + \sin^2(2t) \\ &= 4\cos^2(t) + 4\sin^2(t) + \cos^2(2t) + \sin^2(2t) - 4\cos(t)\cos(2t) - 4\sin(t)\sin(2t) \\ &= 4 + 1 - 4(\cos(2t)\cos(t) + \sin(2t)\sin(t)) \\ &= 5 - 4\cos(2t - t) \\ &= 5 - 4\cos(t) \end{aligned}$$

Dus $PQ = \sqrt{5 - 4\cos(t)}$.

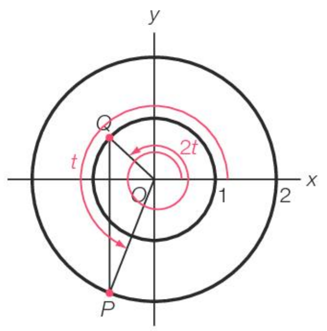
d Voer in $y_1 = \sqrt{5 - 4\cos(x)}$ en $y_2 = 1{,}5$.
De optie snijpunt geeft $x = 0{,}812...$ en $x = 5{,}470...$



Dus gedurende $5{,}470... - 0{,}812... \approx 4{,}66$ seconde per rondgang.

e $x_P = x_Q$ geeft $2\cos(t) = \cos(2t)$
Voer in $y_1 = 2\cos(x)$ en $y_2 = \cos(2x)$.
De optie snijpunt geeft $x = 1{,}9455...$ en $x = 4{,}3376...$
Voor de getekende situatie geldt $t \approx 1{,}946$.

f Voor de andere situatie geldt $t \approx 4{,}338$.



41 $\left.\begin{array}{l} y_{P'} = y_P \\ y_P = 3\sin(\frac{2}{5}\pi t) \end{array}\right\} y_{P'} = 3\sin(\frac{2}{5}\pi t)$

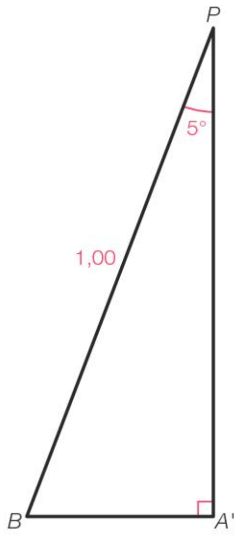
42 **a** De frequentie is $\frac{50\pi}{2\pi} = 25$ Hz.

b Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ cm.
Er zijn 25 trillingen per seconde, dus per kwartier zijn er $15 \cdot 60 \cdot 25 = 22\,500$ trillingen.
De afgelegde afstand in één kwartier is $22\,500 \cdot 20 = 450\,000$ cm $= 4{,}5$ km.

43 amplitude = 10 geeft $b = 10$
frequentie = 3 Hz geeft $c = 2\pi \cdot 3 = 6\pi$
Dus $u = 10\sin(6\pi t)$.

44 **a** De omtrek van de cirkel met middelpunt P en straal 1,00 m is $2\pi \cdot 1 = 2\pi$ m.
De lengte van boog BC is $\frac{10}{360} \cdot 2\pi \approx 0{,}1745$ m.

b



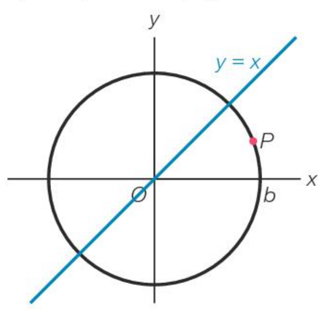
$\sin(5°) = \frac{A'B}{1}$, dus $A'B = \sin(5°)$.
$BC = 2A'B = 2\sin(5°) \approx 0{,}1743$ m

c $b = A'B = \sin(5°) \approx 0{,}09$
$$c = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} = \sqrt{\frac{9{,}81}{1}} \approx 3{,}13$$

d $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9{,}81}} \approx 2{,}01$ seconde, dus de klok geeft ongeveer 1 tik per seconde.

45 **a** Door de figuur een kwartslag te draaien, zie je dat de projectie van P op de x-as op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y-as.
Dus P'' voert een harmonische trilling uit.
$x_{P''} = x_P = b\cos(ct)$

b



Door de figuur een achtste slag te draaien, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y-as.
Dus de projectie van P op de lijn $y = x$ voert een harmonische trilling uit.

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12.4 Bewegingsvergelijkingen met goniometrische formules

Bladzijde 165

46 **a** Snijden met de x-as, dus $y = 0$.

$y = 0$ geeft $\sin(4t) = 0$

$4t = k \cdot \pi$

$t = k \cdot \frac{1}{4}\pi$

t in $[0, 2\pi]$ geeft $t = 0$, $t = \frac{1}{4}\pi$, $t = \frac{1}{2}\pi$, $t = \frac{3}{4}\pi$, $t = \pi$, $t = 1\frac{1}{4}\pi$, $t = 1\frac{1}{2}\pi$, $t = 1\frac{3}{4}\pi$ en $t = 2\pi$

Het punt P gaat door de oorsprong voor $t = 0$, $t = \pi$ en $t = 2\pi$

en snijdt de x-as in $(-1, 0)$ voor $t = 1\frac{1}{2}\pi$ en in $(1, 0)$ voor $t = \frac{1}{2}\pi$.

$t = \frac{1}{4}\pi$ en $t = \frac{3}{4}\pi$ geven $x = \sin(\frac{1}{4}\pi) = \sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$, dus $x_B = \frac{1}{2}\sqrt{2}$.

$t = 1\frac{1}{4}\pi$ en $t = 1\frac{3}{4}\pi$ geven $x = \sin(1\frac{1}{4}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$, dus $x_A = -\frac{1}{2}\sqrt{2}$.

b C is een top, dus $y_C = 1$.

$y = 1$ geeft $\sin(4t) = 1$

$4t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$

$t = \frac{1}{8}\pi$ geeft $x = \sin(\frac{1}{8}\pi) = 0{,}3826...$, dus $x_C \approx 0{,}383$.

Bladzijde 166

47 **a** $x = \sin(2t)$ en $y = \sin(t)$ substitueren in $y = -x + 1$ geeft $\sin(t) = -\sin(2t) + 1$.

Voer in $y_1 = \sin(x)$ en $y_2 = -\sin(2x) + 1$.

De optie snijpunt geeft $x = 0{,}3552...$ en $x = 1{,}5707...$

$t = 1{,}5707...$ geeft $\sin(2t) = 0$ en $\sin(t) = 1$, dus het punt $(0, 1)$.

$t = 0{,}3552...$ geeft $\sin(2t) = 0{,}6521...$ en $\sin(t) = 0{,}3478...$, dus het punt $S(0{,}652; 0{,}348)$.

b De baansnelheid in O is $v(0) = \sqrt{(2\cos(0))^2 + (\cos(0))^2} = \sqrt{(2 \cdot 1)^2 + 1^2} = \sqrt{5}$.

c $\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2\cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\tan(\varphi) = \frac{1}{2}$, dus $\varphi \approx 26{,}6°$.

Bladzijde 167

48 **a** $y = \frac{1}{2}$ geeft $\sin(t) = \frac{1}{2}$

$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$

t in $[0, 2\pi]$ geeft $t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$

$x(\frac{1}{6}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$, dus $x_D = \frac{1}{2}\sqrt{3}$.

$x(\frac{5}{6}\pi) = \sin(1\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}$, dus $x_C = -\frac{1}{2}\sqrt{3}$.

$CD = x_D - x_C = \frac{1}{2}\sqrt{3} - -\frac{1}{2}\sqrt{3} = \sqrt{3}$

b De baansnelheid in D is

$v(\frac{1}{6}\pi) = \sqrt{(2\cos(\frac{1}{3}\pi))^2 + (\cos(\frac{1}{6}\pi))^2} = \sqrt{(2 \cdot \frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{1 + \frac{3}{4}} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}$.

c $\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix}$ en $\vec{r}_{y=\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\cos(\alpha) = \frac{\left|\begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|} = \frac{\left|1 \cdot 1 + \frac{1}{2}\sqrt{3} \cdot 0\right|}{\sqrt{1^2 + (\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{1}{\sqrt{1\frac{3}{4}}}$$

Dus $\alpha \approx 40{,}9°$.

d $x(t) = 0$ geeft $\sin(2t) = 0$

$2t = k \cdot \pi$

$t = k \cdot \frac{1}{2}\pi$

$y(t) = 0$ geeft $\sin(t) = 0$

$t = k \cdot \pi$

Dus de baan gaat in $[0, 2\pi]$ door de oorsprong voor $t = 0$, $t = \pi$ en $t = 2\pi$.

$$\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2\cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ en } \vec{v}(\pi) = \begin{pmatrix} 2\cos(2\pi) \\ \cos(\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\cos(\beta) = \frac{\left|\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -1 \end{pmatrix}\right|} = \frac{|2 \cdot 2 + 1 \cdot -1|}{\sqrt{2^2 + 1^2} \cdot \sqrt{2^2 + (-1)^2}} = \frac{|4 - 1|}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5}$$

Dus $\beta \approx 53{,}1°$.

49 **a** $y = 1$ geeft $\sin(3t) = 1$

$3t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

$t = \frac{5}{6}\pi$ geeft $A(\sin(1\frac{2}{3}\pi), \sin(2\frac{1}{2}\pi)) = A(-\frac{1}{2}\sqrt{3}, 1)$

$x = -1$ geeft $\sin(2t) = -1$

$2t = 1\frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{3}{4}\pi + k \cdot \pi$

$t = \frac{3}{4}\pi$ geeft $B(\sin(1\frac{1}{2}\pi), \sin(2\frac{1}{4}\pi)) = B(-1, \frac{1}{2}\sqrt{2})$

b $x(t) = \sin(2t)$ en $y(t) = \sin(3t)$ substitueren in $y = x$ geeft

$\sin(3t) = \sin(2t)$

$3t = 2t + k \cdot 2\pi \vee 3t = \pi - 2t + k \cdot 2\pi$

$t = k \cdot 2\pi \vee 5t = \pi + k \cdot 2\pi$

$t = k \cdot 2\pi \vee t = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$

t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{5}\pi \vee t = \frac{3}{5}\pi \vee t = \pi \vee t = 1\frac{2}{5}\pi \vee t = 1\frac{4}{5}\pi \vee t = 2\pi$

Bij de oorsprong horen $t = 0$, $t = \pi$ en $t = 2\pi$.

Bij de andere punten horen $t = \frac{1}{5}\pi$, $t = \frac{3}{5}\pi$, $t = 1\frac{2}{5}\pi$ en $t = 1\frac{4}{5}\pi$.

c $x(t) = \frac{1}{2}$ geeft $\sin(2t) = \frac{1}{2}$

$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$

$t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12}\pi \vee t = 1\frac{1}{12}\pi \vee t = \frac{5}{12}\pi \vee t = 1\frac{5}{12}\pi$

$y(t) = \frac{1}{2}\sqrt{2}$ geeft $\sin(3t) = \frac{1}{2}\sqrt{2}$

$3t = \frac{1}{4}\pi + k \cdot 2\pi \vee 3t = \frac{3}{4}\pi + k \cdot 2\pi$

$t = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi \vee t = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{5}{12}\pi \vee t = \frac{1}{4}\pi \vee t = \frac{11}{12}\pi \vee t = 1\frac{7}{12}\pi$

Dus voor $t = \frac{1}{12}\pi$ en $t = 1\frac{5}{12}\pi$ passeert P het punt $(\frac{1}{2}, \frac{1}{2}\sqrt{2})$,

dus de baan snijdt zichzelf in $C(\frac{1}{2}, \frac{1}{2}\sqrt{2})$.

d $x(t) = \sin(2t)$ geeft $x'(t) = 2\cos(2t)$

$y(t) = \sin(3t)$ geeft $y'(t) = 3\cos(3t)$

P is voor het eerst in C op $t = \frac{1}{12}\pi$.

De baansnelheid in C is dan

$$v(\tfrac{1}{12}\pi) = \sqrt{(2\cos(\tfrac{1}{6}\pi))^2 + (3\cos(\tfrac{1}{4}\pi))^2} = \sqrt{(2 \cdot \tfrac{1}{2}\sqrt{3})^2 + (3 \cdot \tfrac{1}{2}\sqrt{2})^2} = \sqrt{3 + 4\tfrac{1}{2}} = \sqrt{7\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{30}.$$

e $$\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2\cos(0) \\ 3\cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ en } \vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(2\pi) \\ 3\cos(3\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left|\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}\right|}{\left|\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -3 \end{pmatrix}\right|} = \frac{|4 - 9|}{\sqrt{2^2 + 3^2} \cdot \sqrt{2^2 + (-3)^2}} = \frac{|-5|}{\sqrt{13} \cdot \sqrt{13}} = \frac{5}{13}$$

Dus $\varphi \approx 67°$.

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50 **a** $x = 1$ geeft $\sin(t - \frac{1}{6}\pi) = 1$
$$t - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$
$$t = \frac{2}{3}\pi + k \cdot 2\pi$$
t in $[0, 2\pi]$ geeft $t = \frac{2}{3}\pi$
$x(\frac{2}{3}\pi) = \sin(\frac{1}{2}\pi) = 1$ en $y(\frac{2}{3}\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$, dus $A(1, -\frac{1}{2}\sqrt{3})$.

b $x = \frac{1}{2}$ geeft $\sin(t - \frac{1}{6}\pi) = \frac{1}{2}$
$$t - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee t - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$
$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = \pi + k \cdot 2\pi$$
t in $[0, 2\pi]$ geeft $t = \frac{1}{3}\pi \vee t = \pi$
$y(\frac{1}{3}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3}$ geeft $B(\frac{1}{2}, \frac{1}{2}\sqrt{3})$
$y(\pi) = \sin(2\pi) = 0$ geeft $C(\frac{1}{2}, 0)$
Dus $BC = \frac{1}{2}\sqrt{3}$.

c $x(t) = \sin(t - \frac{1}{6}\pi)$ en $y(t) = \sin(2t)$ substitueren in $y = x$ geeft
$\sin(2t) = \sin(t - \frac{1}{6}\pi)$
$2t = t - \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - (t - \frac{1}{6}\pi) + k \cdot 2\pi$
$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - t + \frac{1}{6}\pi + k \cdot 2\pi$
$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 3t = 1\frac{1}{6}\pi + k \cdot 2\pi$
$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$
t in $[0, 2\pi]$ geeft $t = 1\frac{5}{6}\pi \vee t = \frac{7}{18}\pi \vee t = 1\frac{1}{18}\pi \vee t = 1\frac{13}{18}\pi$

d $x(t) = 0$ geeft $\sin(t - \frac{1}{6}\pi) = 0$
$$t - \frac{1}{6}\pi = k \cdot \pi$$
$$t = \frac{1}{6}\pi + k \cdot \pi$$
Dus de baan snijdt zichzelf voor $t = \frac{1}{6}\pi$ en voor $t = 1\frac{1}{6}\pi$ in $(0, \frac{1}{2}\sqrt{3})$.
$x(t) = \sin(t - \frac{1}{6}\pi)$ geeft $x'(t) = \cos(t - \frac{1}{6}\pi)$
$y(t) = \sin(2t)$ geeft $y'(t) = 2\cos(2t)$
$$\vec{v}(\tfrac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(0) \\ 2\cos(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ en } \vec{v}(1\tfrac{1}{6}\pi) = \begin{pmatrix} x'(1\frac{1}{6}\pi) \\ y'(1\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(\pi) \\ 2\cos(2\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0$, dus de hoek waaronder de baan zichzelf snijdt is 90°.

e De baansnelheid op $t = \frac{1}{3}\pi$ is
$$v(\tfrac{1}{3}\pi) = \sqrt{(\cos(\tfrac{1}{6}\pi))^2 + (2\cos(\tfrac{2}{3}\pi))^2} = \sqrt{(\tfrac{1}{2}\sqrt{3})^2 + (2 \cdot -\tfrac{1}{2})^2} = \sqrt{\tfrac{3}{4} + 1} = \sqrt{\tfrac{7}{4}} = \tfrac{1}{2}\sqrt{7}.$$

Bladzijde 168

51 **a** $x = 0$ geeft $\sin(2t) = 0$
$$2t = k \cdot \pi$$
$$t = k \cdot \frac{1}{2}\pi$$
t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{2}\pi \vee t = \pi \vee t = 1\frac{1}{2}\pi \vee t = 2\pi$
$y(0) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$
$y(\frac{1}{2}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$
$y(\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$
$y(1\frac{1}{2}\pi) = \sin(1\frac{5}{6}\pi) = -\frac{1}{2}$
$y(2\pi) = \sin(2\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$
Dus $A(0, \frac{1}{2}\sqrt{3})$, $B(0, \frac{1}{2})$, $C(0, -\frac{1}{2})$ en $D(0, -\frac{1}{2}\sqrt{3})$.

b $x = -\frac{1}{2}$ geeft $\sin(2t) = -\frac{1}{2}$

$$2t = -\tfrac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi + \tfrac{1}{6}\pi + k \cdot 2\pi$$
$$t = -\tfrac{1}{12}\pi + k \cdot \pi \vee t = \tfrac{7}{12}\pi + k \cdot \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{11}{12}\pi \vee t = 1\frac{7}{12}\pi \vee t = 1\frac{11}{12}\pi$

$y(\frac{7}{12}\pi) = \sin(\frac{11}{12}\pi) \approx 0{,}26$

$y(\frac{11}{12}\pi) = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

$y(1\frac{7}{12}\pi) = \sin(1\frac{11}{12}\pi) \approx -0{,}26$

$y(1\frac{11}{12}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$

Dus $E(-\frac{1}{2}, \frac{1}{2}\sqrt{2})$ en $H(-\frac{1}{2}, -\frac{1}{2}\sqrt{2})$ en dit geeft $EH = \frac{1}{2}\sqrt{2} - -\frac{1}{2}\sqrt{2} = \sqrt{2}$.

c $x(t) = \sin(2t)$ geeft $x'(t) = 2\cos(2t)$

$y(t) = \sin(t + \frac{1}{3}\pi)$ geeft $y'(t) = \cos(t + \frac{1}{3}\pi)$

$\vec{v}(\frac{1}{2}\pi) = \begin{pmatrix} x'(\frac{1}{2}\pi) \\ y'(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix}$ en $\vec{r}_{y\text{-as}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\cos(\varphi) = \frac{\left|\begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|} = \frac{\left|-\frac{1}{2}\sqrt{3}\right|}{\sqrt{(-2)^2 + (-\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4 + \frac{3}{4}}} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4\frac{3}{4}}}$$

Dus $\varphi \approx 67°$.

d De baansnelheid in E is

$v(1\frac{11}{12}\pi) = \sqrt{(2\cos(3\frac{5}{6}\pi))^2 + (\cos(2\frac{1}{4}\pi))^2} = \sqrt{4 \cdot (\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{3 + \frac{1}{2}} = \frac{1}{2}\sqrt{14}$.

52 **a** $x(a) = \sin(2a)$ en $y(a) = \sin(a + \frac{1}{3}\pi)$, dus $T(\sin(2a), \sin(a + \frac{1}{3}\pi))$.

$x(a + \pi) = \sin(2(a + \pi)) = \sin(2a + 2\pi) = \sin(2a)$ en $y(a + \pi) = \sin(a + \pi + \frac{1}{3}\pi) = \sin(a + 1\frac{1}{3}\pi)$, dus $U(\sin(2a), \sin(a + 1\frac{1}{3}\pi))$.

$TU = |y_T - y_U| = |\sin(a + \frac{1}{3}\pi) - \sin(a + 1\frac{1}{3}\pi)| = |\sin(a + \frac{1}{3}\pi) - -\sin(a + \frac{1}{3}\pi)| = |2\sin(a + \frac{1}{3}\pi)|$

b $|2\sin(a + \frac{1}{3}\pi)| = 1$

$2\sin(a + \frac{1}{3}\pi) = 1 \vee 2\sin(a + \frac{1}{3}\pi) = -1$

$\sin(a + \frac{1}{3}\pi) = \frac{1}{2} \vee \sin(a + \frac{1}{3}\pi) = -\frac{1}{2}$

$a + \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$

$a = -\frac{1}{6}\pi + k \cdot 2\pi \vee a = \frac{1}{2}\pi + k \cdot 2\pi \vee a = -\frac{1}{2}\pi + k \cdot 2\pi \vee a = \frac{5}{6}\pi + k \cdot 2\pi$

$0 \le a \le 2\pi$ en $0 \le a + \pi \le 2\pi$ oftewel $0 \le a \le \pi$ geeft $a = \frac{1}{2}\pi \vee a = \frac{5}{6}\pi$.

53 **a**

t	$\frac{1}{4}\pi$	$\frac{3}{4}\pi$	$1\frac{1}{4}\pi$
x	1	0	-1
y	1	-1	1

b $\left.\begin{array}{l} y = px^2 + q \\ \text{door } (0, -1) \end{array}\right\} \begin{array}{l} p \cdot 0^2 + q = -1 \\ q = -1 \end{array}$

$\left.\begin{array}{l} y = px^2 - 1 \\ \text{door } (1, 1) \end{array}\right\} \begin{array}{l} p \cdot 1^2 - 1 = 1 \\ p = 2 \end{array}$

Dus $p = 2$ en $q = -1$.

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Bladzijde 170

54 Substitutie van $x = \sin(t - \frac{1}{4}\pi)$ en $y = \sin(2t)$ in $y = -2x^2 + 1$ geeft

$\sin(2t) = -2\sin^2(t - \frac{1}{4}\pi) + 1$

$\sin(2t) = 1 - 2\sin^2(t - \frac{1}{4}\pi)$

$\sin(2t) = \cos(2(t - \frac{1}{4}\pi))$

$\sin(2t) = \cos(2t - \frac{1}{2}\pi)$

$\sin(2t) = \sin(2t - \frac{1}{2}\pi + \frac{1}{2}\pi)$

$\sin(2t) = \sin(2t)$

$\left.\begin{array}{l}\text{Dit klopt voor elke } t.\\ x = \sin(t - \frac{1}{4}\pi)\\ -1 \leq \sin(t - \frac{1}{4}\pi) \leq 1\end{array}\right\}$ Bij de parametervoorstelling hoort de formule $y = -2x^2 + 1$ met $-1 \leq x \leq 1$.

55 **a** $-2 \leq 2\sin(t) \leq 2$, dus $-2 \leq x \leq 2$.
$-1 \leq \sin(2t - \frac{1}{2}\pi) \leq 1$, dus $-1 \leq y \leq 1$.
Dus de keerpunten zijn $(-2, 1)$ en $(2, 1)$.

b Stel K: $y = ax^2 + b$.
$x = 0$ geeft $t = 0$ en $y(0) = \sin(-\frac{1}{2}\pi) = -1$, dus door $(0, -1)$.

$\left.\begin{array}{l}y = ax^2 + b\\ \text{door } (0, -1)\end{array}\right\}$ $a \cdot 0^2 + b = -1$
$b = -1$

$\left.\begin{array}{l}y = ax^2 - 1\\ \text{door } (2, 1)\end{array}\right\}$ $a \cdot 2^2 - 1 = 1$
$4a = 2$
$a = \frac{1}{2}$

Vermoedelijk hoort bij K de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.
Substitutie van $x = 2\sin(t)$ en $y = \sin(2t - \frac{1}{2}\pi)$ in $y = \frac{1}{2}x^2 - 1$ geeft

$\sin(2t - \frac{1}{2}\pi) = \frac{1}{2}(2\sin(t))^2 - 1$

$\cos(2t - \frac{1}{2}\pi - \frac{1}{2}\pi) = \frac{1}{2} \cdot 4\sin^2(t) - 1$

$\cos(2t - \pi) = 2\sin^2(t) - 1$

$\cos(2t + \pi) = -1 + 2\sin^2(t)$

$-\cos(2t) = -(1 - 2\sin^2(t))$

$\cos(2t) = \cos(2t)$

Dit klopt voor elke t, dus bij K hoort de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.

56 Substitutie van $x = \sin(t)$ en $y = \sin(2t)$ in $y^2 = 4x^2 - 4x^4$ geeft

$\sin^2(2t) = 4\sin^2(t) - 4\sin^4(t)$

$(2\sin(t)\cos(t))^2 = 4\sin^2(t)(1 - \sin^2(t))$

$4\sin^2(t)\cos^2(t) = 4\sin^2(t)\cos^2(t)$

$\left.\begin{array}{l}\text{Dit klopt voor elke } t.\\ x = \sin(t)\\ -1 \leq \sin(t) \leq 1\end{array}\right\}$ Bij de baan van P hoort de formule $y^2 = 4x^2 - 4x^4$ met $-1 \leq x \leq 1$.

57 **a** $-1 \leq \sin(t) \leq 1$, dus $-1 \leq x \leq 1$.
$-1 \leq \sin(3t) \leq 1$, dus $-1 \leq y \leq 1$.
Dus de keerpunten van K zijn $(-1, 1)$ en $(1, -1)$.

b Substitutie van $x = \sin(t)$ en $y = \sin(3t)$ in $y = 3x - 4x^3$ geeft

$\sin(3t) = 3\sin(t) - 4\sin^3(t)$

$\sin(t + 2t) = 3\sin(t) - 4\sin^3(t)$

$\sin(t)\cos(2t) + \cos(t)\sin(2t) = 3\sin(t) - 4\sin^3(t)$

$\sin(t)(1 - 2\sin^2(t)) + \cos(t) \cdot 2\sin(t)\cos(t) = 3\sin(t) - 4\sin^3(t)$

$\sin(t) - 2\sin^3(t) + 2\sin(t)\cos^2(t) = 3\sin(t) - 4\sin^3(t)$

$\sin(t) - 2\sin^3(t) + 2\sin(t) \cdot (1 - \sin^2(t)) = 3\sin(t) - 4\sin^3(t)$

$\sin(t) - 2\sin^3(t) + 2\sin(t) - 2\sin^3(t) = 3\sin(t) - 4\sin^3(t)$

$3\sin(t) - 4\sin^3(t) = 3\sin(t) - 4\sin^3(t)$

Dit klopt voor elke t, dus alle punten van K liggen op de grafiek van $y = 3x - 4x^3$.

58 $-2 \le 2\cos(t) \le 2$, dus $-2 \le x \le 2$ en dit geeft $c = -2$ en $d = 2$.
$-1 \le \cos(3t) \le 1$, dus $-1 \le y \le 1$.
Dus de keerpunten zijn $(-2, -1)$ en $(2, 1)$.
$x(\frac{1}{3}\pi) = 2\cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} = 1$ en $y(\frac{1}{3}\pi) = \cos(\pi) = -1$, dus door $(1, -1)$.

$$\left.\begin{array}{l} y = ax^3 + bx \\ \text{door } (2, 1) \end{array}\right\} \begin{array}{l} a \cdot 2^3 + b \cdot 2 = 1 \\ 8a + 2b = 1 \end{array}$$

$$\left.\begin{array}{l} y = ax^3 + bx \\ \text{door } (1, -1) \end{array}\right\} \begin{array}{l} a \cdot 1^3 + b \cdot 1 = -1 \\ a + b = -1 \end{array}$$

$$\begin{cases} 8a + 2b = 1 \\ a + b = -1 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 8a + 2b = 1 \\ 2a + 2b = -2 \end{cases} -$$

$$6a = 3$$

$$\left.\begin{array}{l} a = \frac{1}{2} \\ a + b = -1 \end{array}\right\} \begin{array}{l} \frac{1}{2} + b = -1 \\ b = -1\frac{1}{2} \end{array}$$

Vermoedelijk hoort bij de baan van P de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \le x \le 2$.
Substitutie van $x = 2\cos(t)$ en $y = \cos(3t)$ in $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ geeft

$\cos(3t) = \frac{1}{2}(2\cos(t))^3 - 1\frac{1}{2} \cdot 2\cos(t)$

$\cos(2t + t) = \frac{1}{2} \cdot 8\cos^3(t) - 3\cos(t)$

$\cos(2t)\cos(t) - \sin(2t)\sin(t) = 4\cos^3(t) - 3\cos(t)$
$(2\cos^2(t) - 1)\cos(t) - 2\sin(t)\cos(t)\sin(t) = 4\cos^3(t) - 3\cos(t)$
$2\cos^3(t) - \cos(t) - 2\sin^2(t)\cos(t) = 4\cos^3(t) - 3\cos(t)$
$2\cos^3(t) - \cos(t) - 2(1 - \cos^2(t))\cos(t) = 4\cos^3(t) - 3\cos(t)$
$2\cos^3(t) - \cos(t) - 2\cos(t) + 2\cos^3(t) = 4\cos^3(t) - 3\cos(t)$
$4\cos^3(t) - 3\cos(t) = 4\cos^3(t) - 3\cos(t)$
Dit klopt voor elke t, dus bij de baan van P hoort de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \le x \le 2$.

59 Noem M het midden van OP.
Er geldt $MS \perp OP$ en $MS = OM = \frac{1}{2}OP$
Dus $\vec{s} = \vec{m} + \overrightarrow{MS} = \frac{1}{2}\vec{p} + \vec{m}_L = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_L$.

Bladzijde 172

60 $\vec{t} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2}\begin{pmatrix} 2\sin(t) \\ 2\sin(2t) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2\sin(2t) \\ -2\sin(t) \end{pmatrix} = \begin{pmatrix} \sin(t) + \sin(2t) \\ \sin(2t) - \sin(t) \end{pmatrix}$

Dus $\begin{cases} x_T(t) = \sin(t) + \sin(2t) \\ y_T(t) = \sin(2t) - \sin(t) \end{cases}$

61 **a** $\vec{q} = \vec{a} + \overrightarrow{AQ} = \vec{a} + \frac{1}{2}\overrightarrow{AP} + \frac{1}{2}\overrightarrow{AP}_R$

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 2\sin(t) \\ 2\sin(2t) \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\sin(t) - 2 \\ 2\sin(2t) \end{pmatrix}$

$\overrightarrow{AP}_R = \begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix}$

$\vec{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2\sin(t) - 2 \\ 2\sin(2t) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2 + \sin(t) - 1 + \sin(2t) \\ \sin(2t) + 1 - \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t) + 1 \\ \sin(2t) - \sin(t) + 1 \end{pmatrix}$

Dus $\begin{cases} x_Q(t) = \sin(2t) + \sin(t) + 1 \\ y_Q(t) = \sin(2t) - \sin(t) + 1 \end{cases}$

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b $\vec{r} = \vec{a} + \overrightarrow{AR} = \vec{a} + \overrightarrow{AP}_R = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2 + 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix}$

$y = 1$ geeft $2 - 2\sin(t) = 1$

$-2\sin(t) = -1$

$\sin(t) = \frac{1}{2}$

$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$

t in $\langle 0, \pi \rangle$ geeft $t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$

$x(\frac{1}{6}\pi) = 2 + 2\sin(\frac{1}{3}\pi) = 2 + 2 \cdot \frac{1}{2}\sqrt{3} = 2 + \sqrt{3}$

$x(\frac{5}{6}\pi) = 2 + 2\sin(1\frac{2}{3}\pi) = 2 + 2 \cdot -\frac{1}{2}\sqrt{3} = 2 - \sqrt{3}$

Dus de snijpunten zijn $(2 + \sqrt{3}, 1)$ en $(2 - \sqrt{3}, 1)$.

62 **a** $\vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{PA}_L$

$\overrightarrow{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2\sin(t) \\ 2\sin(t - \frac{1}{4}\pi) \end{pmatrix} = \begin{pmatrix} 4 - 2\sin(t) \\ -2\sin(t - \frac{1}{4}\pi) \end{pmatrix}$

$\overrightarrow{PA}_L = \begin{pmatrix} 2\sin(t - \frac{1}{4}\pi) \\ 4 - 2\sin(t) \end{pmatrix}$

$\vec{q} = \begin{pmatrix} 2\sin(t) \\ 2\sin(t - \frac{1}{4}\pi) \end{pmatrix} + \begin{pmatrix} 2\sin(t - \frac{1}{4}\pi) \\ 4 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2\sin(t) + 2\sin(t - \frac{1}{4}\pi) \\ 2\sin(t - \frac{1}{4}\pi) + 4 - 2\sin(t) \end{pmatrix}$

Dus $\begin{cases} x_Q(t) = 2\sin(t) + 2\sin(t - \frac{1}{4}\pi) \\ y_Q(t) = 2\sin(t - \frac{1}{4}\pi) - 2\sin(t) + 4 \end{cases}$

b Voer in $y_1 = 2\sin(x - \frac{1}{4}\pi) - 2\sin(x) + 4$.

De optie minimum geeft $x = 0{,}392...$ en $y = 2{,}469...$

Het maximum van $y_P(t) = 2\sin(t - \frac{1}{4}\pi)$ is 2.

Dus het laagste punt van de baan van Q ligt hoger dan het hoogste punt van de baan van P.

Bladzijde 173

63 **a** $\vec{s} = \vec{a} + \overrightarrow{AS} = \vec{a} + \frac{1}{2}\overrightarrow{AP} + \frac{1}{2}\overrightarrow{AP}_R$

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 4\cos(t) \\ 6\sin(t) \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4\cos(t) - 4 \\ 6\sin(t) \end{pmatrix}$ geeft $\overrightarrow{AP}_R = \begin{pmatrix} 6\sin(t) \\ 4 - 4\cos(t) \end{pmatrix}$

$\vec{s} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 4\cos(t) - 4 \\ 6\sin(t) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6\sin(t) \\ 4 - 4\cos(t) \end{pmatrix} = \begin{pmatrix} 4 + 2\cos(t) - 2 + 3\sin(t) \\ 3\sin(t) + 2 - 2\cos(t) \end{pmatrix} = \begin{pmatrix} 2 + 3\sin(t) + 2\cos(t) \\ 2 + 3\sin(t) - 2\cos(t) \end{pmatrix}$

Dus $\begin{cases} x_S(t) = 2 + 3\sin(t) + 2\cos(t) \\ y_S(t) = 2 + 3\sin(t) - 2\cos(t) \end{cases}$

b Substitueren van $x = 4\cos(t)$ en $y = 6\sin(t)$ in $9x^2 + 4y^2 = 144$ geeft

$9 \cdot (4\cos(t))^2 + 4 \cdot (6\sin(t))^2 = 144$

$9 \cdot 16\cos^2(t) + 4 \cdot 36\sin^2(t) = 144$

$144\cos^2(t) + 144\sin^2(t) = 144$

$\cos^2(t) + \sin^2(t) = 1$.

Dit klopt voor elke t, dus bij de baan van P hoort de formule $9x^2 + 4y^2 = 144$.

c $x = 2 + 3\sin(t) + 2\cos(t)$ en $y = 2 + 3\sin(t) - 2\cos(t)$ substitueren in $9x^2 + 4y^2 = 144$ geeft

$9(2 + 3\sin(t) + 2\cos(t))^2 + 4(2 + 3\sin(t) - 2\cos(t))^2 = 144$.

Voer in $y_1 = 9(2 + 3\sin(x) + 2\cos(x))^2 + 4(2 + 3\sin(x) - 2\cos(x))^2$ en $y_2 = 144$.

De optie snijpunt geeft $x = 2{,}583...$

$t = 2{,}583...$ geeft $x = 2 + 3\sin(2{,}583...) + 2\cos(2{,}5683...) \approx 1{,}89$ en

$y = 2 + 3\sin(2{,}583...) - 2\cos(2{,}583...) \approx 5{,}29$.

Dus $B(1{,}89; 5{,}29)$.

64 $\vec{q} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2}\begin{pmatrix} 2\sin(2t) \\ 2\sin(t + \frac{1}{4}\pi) \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2\sin(t + \frac{1}{4}\pi) \\ -2\sin(2t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{pmatrix}$

Dus $\begin{cases} x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{cases}$

$x = \sin(2t) + \sin(t + \frac{1}{4}\pi)$ en $y = \sin(t + \frac{1}{4}\pi) - \sin(2t)$ substitueren in $y = -x$ geeft

$\sin(t + \frac{1}{4}\pi) - \sin(2t) = -\sin(2t) - \sin(t + \frac{1}{4}\pi)$

$2\sin(t + \frac{1}{4}\pi) = 0$

$\sin(t + \frac{1}{4}\pi) = 0$

$t + \frac{1}{4}\pi = k \cdot \pi$

$t = -\frac{1}{4}\pi + k \cdot \pi$

$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi$ geeft $t = \frac{3}{4}\pi$

$t = \frac{3}{4}$ geeft $x = \sin(1\frac{1}{2}\pi) + \sin(\pi) = -1$

Dus $A(-1, 1)$.

65 $x = 2\sin(2t)$ en $y = 2\sin(t + \frac{1}{4}\pi)$ substitueren in $y = -x$ geeft

$2\sin(t + \frac{1}{4}\pi) = -2\sin(2t)$

$\sin(t + \frac{1}{4}\pi) = -\sin(2t)$

$\sin(t + \frac{1}{4}\pi) = \sin(2t + \pi)$

$t + \frac{1}{4}\pi = 2t + \pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - (2t + \pi) + k \cdot 2\pi$

$-t = \frac{3}{4}\pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - 2t - \pi + k \cdot 2\pi$

$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{4}\pi + k \cdot 2\pi$

$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$

$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi$ geeft $t = \frac{7}{12}\pi \vee t = 1\frac{1}{4}\pi$

$x_P(\frac{7}{12}\pi) = 2\sin(1\frac{1}{6}\pi) = 2 \cdot -\frac{1}{2} = -1$ en $y_P(\frac{7}{12}\pi) = 2\sin(\frac{5}{6}\pi) = 2 \cdot \frac{1}{2} = 1$

Dus het punt $A(-1, 1)$ ligt op beide banen.

$x_P(t) = 2\sin(2t)$ geeft $x_P'(t) = 4\cos(2t)$

$y_P(t) = 2\sin(t + \frac{1}{4}\pi)$ geeft $y_P'(t) = 2\cos(t + \frac{1}{4}\pi)$

$\vec{v}_P(\frac{7}{12}\pi) = \begin{pmatrix} x_p'(\frac{7}{12}\pi) \\ y_p'(\frac{7}{12}\pi) \end{pmatrix} = \begin{pmatrix} 4\cos(1\frac{1}{6}\pi) \\ 2\cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cdot -\frac{1}{2}\sqrt{3} \\ 2 \cdot -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$

$x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi)$ geeft $x_Q'(t) = 2\cos(2t) + \cos(t + \frac{1}{4}\pi)$

$y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t)$ geeft $y_Q'(t) = \cos(t + \frac{1}{4}\pi) - 2\cos(2t)$

$\vec{v}_Q(\frac{3}{4}\pi) = \begin{pmatrix} x_Q'(\frac{3}{4}\pi) \\ y_Q'(\frac{3}{4}\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(1\frac{1}{2}\pi) + \cos(\pi) \\ \cos(\pi) - 2\cos(1\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cdot 0 + -1 \\ -1 - 2 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$ is niet evenwijdig met $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus de banen raken elkaar niet op de lijn $y = -x$.

Diagnostische toets

Bladzijde 176

1 **a** $\sin(3x - \frac{1}{4}\pi) = \cos(2x)$

$\cos(3x - \frac{3}{4}\pi) = \cos(2x)$

$3x - \frac{3}{4}\pi = 2x + k \cdot 2\pi \vee 3x - \frac{3}{4}\pi = -2x + k \cdot 2\pi$

$x = \frac{3}{4}\pi + k \cdot 2\pi \vee 5x = \frac{3}{4}\pi + k \cdot 2\pi$

$x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{20}\pi + k \cdot \frac{2}{5}\pi$

x in $[0, \pi]$ geeft $x = \frac{3}{4}\pi \vee x = \frac{3}{20}\pi \vee x = \frac{11}{20}\pi \vee x = \frac{19}{20}\pi$

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b $2\cos^2(2x) + \sin(2x) = 1$
$2(1 - \sin^2(2x)) + \sin(2x) = 1$
$2 - 2\sin^2(2x) + \sin(2x) = 1$
$2\sin^2(2x) - \sin(2x) - 1 = 0$
Stel $\sin(2x) = u$.
$2u^2 - u - 1 = 0$
$D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$
$u = \dfrac{1+3}{4} = 1 \vee u = \dfrac{1-3}{4} = -\frac{1}{2}$
$\sin(2x) = 1 \vee \sin(2x) = -\frac{1}{2}$
$2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$
x in $[0, 2\pi]$ geeft $x = \frac{1}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = \frac{11}{12}\pi \vee x = 1\frac{11}{12}\pi \vee x = \frac{7}{12}\pi \vee x = 1\frac{7}{12}\pi$

c $\cos(\frac{2}{5}\pi t) = -\sin(\frac{1}{6}\pi t)$
$\cos(\frac{2}{5}\pi t) = \sin(\frac{1}{6}\pi t + \pi)$
$\cos(\frac{2}{5}\pi t) = \cos(\frac{1}{6}\pi t + \frac{1}{2}\pi)$
$\frac{2}{5}\pi t = \frac{1}{6}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{2}{5}\pi t = -\frac{1}{6}\pi t - \frac{1}{2}\pi + k \cdot 2\pi$
$\frac{7}{30}\pi t = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{17}{30}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi$
$t = 2\frac{1}{7} + k \cdot 8\frac{4}{7} \vee t = -\frac{15}{17} + k \cdot 3\frac{9}{17}$
t in $[0, 10]$ geeft $t = 2\frac{1}{7} \vee t = 2\frac{11}{17} \vee t = 6\frac{3}{17} \vee t = 9\frac{12}{17}$

2 **a** $\sin(\frac{1}{2}x)\cos(1\frac{1}{2}x) + \cos(\frac{1}{2}x)\sin(1\frac{1}{2}x) = \sin(\frac{1}{2}x + 1\frac{1}{2}x) = \sin(2x)$
b $\sin(3x)\sin(5x) + \cos(3x)\cos(5x) = \cos(3x - 5x) = \cos(-2x) = \cos(2x)$
c $2\sin(3x)\cos(3x) = \sin(2 \cdot 3x) = \sin(6x)$
d $2\cos^2(5x) - 1 = \cos(2 \cdot 5x) = \cos(10x)$

3 **a** $\sin(x + \frac{1}{3}\pi) = 2\sin(2x)\cos(2x)$
$\sin(x + \frac{1}{3}\pi) = \sin(4x)$
$x + \frac{1}{3}\pi = 4x + k \cdot 2\pi \vee x + \frac{1}{3}\pi = \pi - 4x + k \cdot 2\pi$
$-3x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 5x = \frac{2}{3}\pi + k \cdot 2\pi$
$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{2}{15}\pi + k \cdot \frac{2}{5}\pi$
b $\sin^2(2x) + \frac{1}{4} = \cos(4x)$
$\sin^2(2x) + \frac{1}{4} = 1 - 2\sin^2(2x)$
$3\sin^2(2x) = \frac{3}{4}$
$\sin^2(2x) = \frac{1}{4}$
$\sin(2x) = \frac{1}{2} \vee \sin(2x) = -\frac{1}{2}$
$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$

4 $y = \sin^2(2x) + 2\cos^2(2x) + 3\cos(4x) = \sin^2(2x) + \cos^2(2x) + \cos^2(2x) + 3(2\cos^2(2x) - 1)$
$= 1 + \cos^2(2x) + 6\cos^2(2x) - 3 = -2 + 7\cos^2(2x) = -2 + 7\cos^2(x) - 3\frac{1}{2} + 3\frac{1}{2}$
$= 1\frac{1}{2} + 3\frac{1}{2}(2\cos^2(x) - 1) = 1\frac{1}{2} + 3\frac{1}{2}\cos(4x)$

5 **a** $f(1\frac{1}{2}\pi - p) + f(1\frac{1}{2}\pi + p) = \sin(3\pi - 2p) + \cos(1\frac{1}{2}\pi - p) + \sin(3\pi + 2p) + \cos(1\frac{1}{2}\pi + p)$
$= \sin(3\pi)\cos(2p) - \cos(3\pi)\sin(2p) + \cos(1\frac{1}{2}\pi)\cos(p) + \sin(1\frac{1}{2}\pi)\sin(p)$
$+ \sin(3\pi)\cos(2p) + \cos(3\pi)\sin(2p) + \cos(1\frac{1}{2}\pi)\cos(p) - \sin(1\frac{1}{2}\pi)\sin(p)$
$= 2\sin(3\pi)\cos(2p) + 2\cos(1\frac{1}{2}\pi)\cos(p) = 2 \cdot 0 \cdot \cos(2p) + 2 \cdot 0 \cdot \cos(p) = 0$
Er geldt $f(1\frac{1}{2}\pi - p) + f(1\frac{1}{2}\pi + p) = 0$.
Dus de grafiek van f is symmetrisch in het punt $(1\frac{1}{2}\pi, 0)$.

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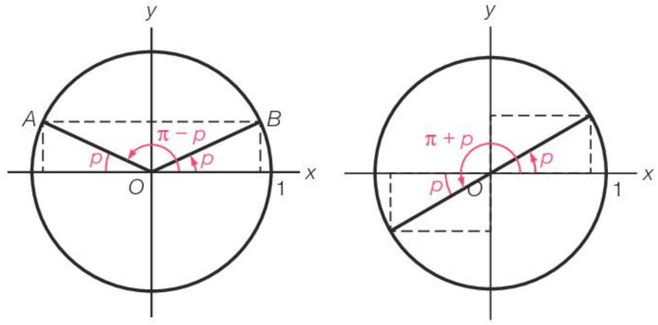
b



$f(\pi - p) = 2\sin^2(\pi - p)\cos(\pi - p) = 2\sin^2(p) \cdot -\cos(p) = -2\sin^2(p)\cos(p)$

$f(\pi + p) = 2\sin^2(\pi + p)\cos(\pi + p) = 2(-\sin(p))^2 \cdot -\cos(p) = -2\sin^2(p)\cos(p)$

Er geldt $f(\pi - p) = f(\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \pi$.

6 $f(x) = 0$ geeft $2\sin(x) = 0$

$\sin(x) = 0$

$x = k \cdot \pi$

$$I(L) = \pi\int_0^{\pi}(2\sin(x))^2\,dx = \pi\int_0^{\pi}4\sin^2(x)\,dx = \pi\int_0^{\pi}2\,(1 - \cos(2x))\,dx = \pi\int_0^{\pi}(2 - 2\cos(2x))\,dx$$

$$= \pi\Big[2x - \sin(2x)\Big]_0^{\pi} = \pi(2\pi - 0 - 0) = 2\pi^2$$

7 $f(x) = 0$ geeft $1 - 2\cos^2(x) - \cos(x) = 0$

Stel $\cos(x) = u$.

$1 - 2u^2 - u = 0$

$2u^2 + u - 1 = 0$

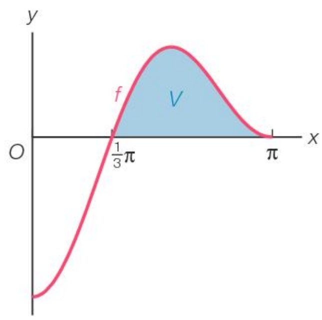
$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$

$u = \dfrac{-1 + 3}{4} = \frac{1}{2} \vee u = \dfrac{-1 - 3}{4} = -1$

$\cos(x) = \frac{1}{2} \vee \cos(x) = -1$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi$

x in $[0, \pi]$ geeft $x = \frac{1}{3}\pi \vee x = \pi$



$$O(V) = \int_{\frac{1}{3}\pi}^{\pi}(1 - 2\cos^2(x) - \cos(x))\,dx = \int_{\frac{1}{3}\pi}^{\pi}(-\cos(2x) - \cos(x))\,dx$$

$$= \Big[-\tfrac{1}{2}\sin(2x) - \sin(x)\Big]_{\frac{1}{3}\pi}^{\pi} = -\tfrac{1}{2}\sin(2\pi) - \sin(\pi) - (-\tfrac{1}{2}\sin(\tfrac{2}{3}\pi) - \sin(\tfrac{1}{3}\pi))$$

$$= 0 - 0 + \tfrac{1}{2} \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\sqrt{3} = \tfrac{3}{4}\sqrt{3}$$

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Bladzijde 177

8 **a** Het punt A doorloopt de cirkel met middelpunt O en straal 2.
Het punt B doorloopt de cirkel met middelpunt O en straal 3.
De minimale afstand is $3 - 2 = 1$ voor $\frac{1}{2}t = 2t + k \cdot 2\pi$

$$-1\tfrac{1}{2}t = k \cdot 2\pi$$
$$t = k \cdot 1\tfrac{1}{3}\pi$$

Dus de afstand is minimaal voor $t = 0$, $t = 1\frac{1}{3}\pi$, $t = 2\frac{2}{3}\pi$ en $t = 4\pi$.
De maximale afstand is $3 + 2 = 5$ voor $\frac{1}{2}t = 2t + \pi + k \cdot 2\pi$

$$-1\tfrac{1}{2}t = \pi + k \cdot 2\pi$$
$$t = -\tfrac{2}{3}\pi + k \cdot 1\tfrac{1}{3}\pi$$

Dus de afstand is maximaal voor $t = \frac{2}{3}\pi$, $t = 2\pi$ en $t = 3\frac{1}{3}\pi$.

b $AB^2 = (3\cos(2t) - 2\cos(\frac{1}{2}t))^2 + (3\sin(2t) - 2\sin(\frac{1}{2}t))^2$

$$= 9\cos^2(2t) - 12\cos(2t)\cos(\tfrac{1}{2}t) + 4\cos^2(\tfrac{1}{2}t) + 9\sin^2(2t) - 12\sin(2t)\sin(\tfrac{1}{2}t) + 4\sin^2(\tfrac{1}{2}t)$$
$$= 9 + 4 - 12(\cos(2t)\cos(\tfrac{1}{2}t) + \sin(2t)\sin(\tfrac{1}{2}t))$$
$$= 13 - 12\cos(2t - \tfrac{1}{2}t)$$
$$= 13 - 12\cos(1\tfrac{1}{2}t)$$

Dus $AB = \sqrt{13 - 12\cos(1\frac{1}{2}t)}$.

c $AB = \sqrt{7}$ geeft $\sqrt{13 - 12\cos(1\frac{1}{2}t)} = \sqrt{7}$

$$13 - 12\cos(1\tfrac{1}{2}t) = 7$$
$$12\cos(1\tfrac{1}{2}t) = 6$$
$$\cos(1\tfrac{1}{2}t) = \tfrac{1}{2}$$
$$1\tfrac{1}{2}t = \tfrac{1}{3}\pi + k \cdot 2\pi \vee 1\tfrac{1}{2}t = -\tfrac{1}{3}\pi + k \cdot 2\pi$$
$$t = \tfrac{2}{9}\pi + k \cdot 1\tfrac{1}{3}\pi \vee t = -\tfrac{2}{9}\pi + k \cdot 1\tfrac{1}{3}\pi$$

t in $[0, 4\pi]$ geeft $t = \frac{2}{9}\pi \vee t = 1\frac{5}{9}\pi \vee t = 2\frac{8}{9}\pi \vee t = 1\frac{1}{9}\pi \vee t = 2\frac{4}{9}\pi \vee t = 3\frac{7}{9}\pi$

9 **a** $u = b\sin(2\pi ft)$
b = amplitude = 5
$f = 4$ Hz
Dus $u = 5\sin(8\pi t)$.

b Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ dm = 2 m.
Er zijn 4 trillingen per seconde, dus per uur zijn er $3600 \cdot 4 = 14\,400$ trillingen.
Dus in een uur is de afgelegde afstand $14\,400 \cdot 2$ m = 28 800 m = 28,8 km.

10 **a** $y = -\frac{1}{2}\sqrt{3}$ geeft $\cos(2t - \frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

$$2t - \tfrac{1}{3}\pi = \tfrac{5}{6}\pi + k \cdot 2\pi \vee 2t - \tfrac{1}{3}\pi = -\tfrac{5}{6}\pi + k \cdot 2\pi$$
$$2t = 1\tfrac{1}{6}\pi + k \cdot 2\pi \vee 2t = -\tfrac{1}{2}\pi + k \cdot 2\pi$$
$$t = \tfrac{7}{12}\pi + k \cdot \pi \vee t = -\tfrac{1}{4}\pi + k \cdot \pi$$

t in $[0, 1\frac{1}{2}\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{3}{4}\pi$
$x(\frac{7}{12}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$
$x(\frac{3}{4}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$
Dus $AB = \frac{1}{2}\sqrt{2} - -\frac{1}{2}\sqrt{2} = \sqrt{2}$.

b $-1 \le \sin(3t) \le 1$, dus $-1 \le x \le 1$.
$-1 \le \cos(2t - \frac{1}{3}\pi) \le 1$, dus $-1 \le y \le 1$.
Dus de keerpunten zijn $(-1, 1)$ en $(1, 1)$.

c $x = 0$ geeft $\sin(3t) = 0$

$3t = k \cdot \pi$

$t = k \cdot \frac{1}{3}\pi$

x in $[0, 1\frac{1}{2}\pi]$ geeft $t = 0 \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi \vee t = \pi \vee t = 1\frac{1}{3}\pi$

$y(0) = \cos(-\frac{1}{3}\pi) = \frac{1}{2}$

$y(\frac{1}{3}\pi) = \cos(\frac{1}{3}\pi) = \frac{1}{2}$

$y(\frac{2}{3}\pi) = \cos(\pi) = -1$

$y(\pi) = \cos(1\frac{2}{3}\pi) = \frac{1}{2}$

$y(1\frac{1}{3}\pi) = \cos(2\frac{1}{3}\pi) = \frac{1}{2}$

$x(t) = \sin(3t)$ geeft $x'(t) = 3\cos(3t)$

$y(t) = \cos(2t - \frac{1}{3}\pi)$ geeft $y'(t) = -2\sin(2t - \frac{1}{3}\pi)$

$\vec{v}(\frac{1}{3}\pi) = \begin{pmatrix} x'(\frac{1}{3}\pi) \\ y'(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3\cos(\pi) \\ -2\sin(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot \frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix}$ en

$\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 3\cos(3\pi) \\ -2\sin(1\frac{2}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|} = \frac{|9 - 3|}{\sqrt{(-3)^2 + (-\sqrt{3})^2} \cdot \sqrt{(-3)^2 + (\sqrt{3})^2}} = \frac{6}{\sqrt{12} \cdot \sqrt{12}} = \frac{6}{12} = \frac{1}{2}$$

Dus $\varphi = 60°$.

d $v(\frac{2}{3}\pi) = \sqrt{(x'(\frac{2}{3}\pi))^2 + (y'(\frac{2}{3}\pi))^2} = \sqrt{(3\cos(2\pi))^2 + (-2\sin(\pi))^2} = \sqrt{(3 \cdot 1)^2 + (-2 \cdot 0)^2}$

$= \sqrt{3^2 + 0^2} = 3$

Dus de baansnelheid is 3.

e $\vec{q} = \overrightarrow{OP} + \overrightarrow{PQ} = \vec{p} + \vec{p}_L = \begin{pmatrix} \sin(3t) \\ \cos(2t - \frac{1}{3}\pi) \end{pmatrix} + \begin{pmatrix} -\cos(2t - \frac{1}{3}\pi) \\ \sin(3t) \end{pmatrix} = \begin{pmatrix} \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{pmatrix}$

Dus $\begin{cases} x_Q(t) = \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ y_Q(t) = \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{cases}$

$y = 0$ geeft $\cos(2t - \frac{1}{3}\pi) + \sin(3t) = 0$

$\cos(2t - \frac{1}{3}\pi) = -\sin(3t)$

$\cos(2t - \frac{1}{3}\pi) = -\cos(3t - \frac{1}{2}\pi)$

$\cos(2t - \frac{1}{3}\pi) = \cos(3t + \frac{1}{2}\pi)$

$2t - \frac{1}{3}\pi = 3t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -3t - \frac{1}{2}\pi + k \cdot 2\pi$

$-t = \frac{5}{6}\pi + k \cdot 2\pi \vee 5t = -\frac{1}{6}\pi + k \cdot 2\pi$

$t = -\frac{5}{6}\pi + k \cdot 2\pi \vee t = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$

t in $[0, 1\frac{1}{2}\pi]$ geeft $t = 1\frac{1}{6}\pi \vee t = \frac{11}{30}\pi \vee t = \frac{23}{30}\pi$

$x_Q(1\frac{1}{6}\pi) = -2$

$x_Q(\frac{11}{30}\pi) \approx -0{,}618$

$x_Q(\frac{23}{30}\pi) \approx 1{,}618$

Dus voor $t = \frac{23}{30}\pi$.

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K Voortgezette integraalrekening

Voorkennis Afgeleiden en primitieven

Bladzijde 181

1 **a** $f(x) = \sqrt{6x+1}$ geeft $f'(x) = \dfrac{1}{2\sqrt{6x+1}} \cdot 6 = \dfrac{3}{\sqrt{6x+1}}$

b $f(x) = \dfrac{4}{\sqrt{2x-1}} = 4(2x-1)^{-\frac{1}{2}}$ geeft $f'(x) = -2(2x-1)^{-1\frac{1}{2}} \cdot 2 = -\dfrac{4}{(2x-1)\sqrt{2x-1}}$

c $f(x) = \ln(x^2+2x)$ geeft $f'(x) = \dfrac{1}{x^2+2x} \cdot (2x+2) = \dfrac{2x+2}{x^2+2x}$

d $f(x) = 2^{4x-1}$ geeft $f'(x) = 2^{4x-1} \cdot \ln(2) \cdot 4 = 4\ln(2) \cdot 2^{4x-1}$

e $f(x) = \sin(x^2-x)$ geeft $f'(x) = \cos(x^2-x) \cdot (2x-1) = (2x-1)\cos(x^2-x)$

f $f(x) = \tan(4x - \frac{1}{3}\pi)$ geeft $f'(x) = \dfrac{1}{\cos^2(4x-\frac{1}{3}\pi)} \cdot 4 = \dfrac{4}{\cos^2(4x-\frac{1}{3}\pi)}$

2 **a** $f(x) = e^x\sin(2x)$ geeft $f'(x) = e^x\sin(2x) + e^x \cdot 2\cos(2x) = (\sin(2x) + 2\cos(2x))\,e^x$

b $f(x) = \dfrac{\ln(x)}{x^2+1}$ geeft $f'(x) = \dfrac{(x^2+1)\cdot\dfrac{1}{x} - \ln(x)\cdot 2x}{(x^2+1)^2} = \dfrac{x^2+1-2x^2\ln(x)}{x(x^2+1)^2}$

c $f(x) = \frac{1}{2}x^2\ln(x)$ geeft $f'(x) = x \cdot \ln(x) + \frac{1}{2}x^2 \cdot \dfrac{1}{x} = x\ln(x) + \frac{1}{2}x$

d $f(x) = \dfrac{2^x+1}{2^x} = 1 + 2^{-x}$ geeft $f'(x) = 2^{-x} \cdot \ln(2) \cdot -1 = -\ln(2) \cdot \dfrac{1}{2^x} = -\dfrac{\ln(2)}{2^x}$

e $f(x) = 2x\tan(x)$ geeft $f'(x) = 2 \cdot \tan(x) + 2x \cdot \dfrac{1}{\cos^2(x)} = 2\tan(x) + \dfrac{2x}{\cos^2(x)}$

f $f(x) = \dfrac{4x+2}{\sqrt{2x+1}} = \dfrac{2(2x+1)}{\sqrt{2x+1}} = 2\sqrt{2x+1}$ geeft $f'(x) = 2 \cdot \dfrac{1}{2\sqrt{2x+1}} \cdot 2 = \dfrac{2}{\sqrt{2x+1}}$

Bladzijde 182

3 **a** $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3}x^3 - \cos(x) + c$

b $f(x) = \dfrac{x^2-2}{x^3} = \dfrac{1}{x} - 2x^{-3}$ geeft $F(x) = \ln|x| + x^{-2} + c = \ln|x| + \dfrac{1}{x^2} + c$

c $f(x) = 4 \cdot 3^x$ geeft $F(x) = 4 \cdot \dfrac{3^x}{\ln(3)} + c = \dfrac{4 \cdot 3^x}{\ln(3)} + c$

d $f(x) = \ln\left(\dfrac{2}{\sqrt{x}}\right) = \ln(2) - \ln(x^{\frac{1}{2}}) = \ln(2) - \frac{1}{2}\ln(x)$ geeft

$F(x) = x\ln(2) - \frac{1}{2}(x\ln(x) - x) + c = x\ln(2) - \frac{1}{2}x\ln(x) + \frac{1}{2}x + c$

e $f(x) = 6 \cdot \log(3x)$ geeft $F(x) = 6 \cdot \frac{1}{3} \cdot \dfrac{1}{\ln(10)}(3x\ln(3x) - 3x) + c = \dfrac{6x(\ln(3x)-1)}{\ln(10)} + c$

f $f(x) = \dfrac{4^x - 2^x + 1}{2^x} = 2^x - 1 + 2^{-x}$ geeft $F(x) = \dfrac{2^x}{\ln(2)} - x - \dfrac{2^{-x}}{\ln(2)} + c = \dfrac{2^x}{\ln(2)} - x - \dfrac{1}{2^x\ln(2)} + c$

4 **a** $f(x) = e^{\frac{1}{2}x-1}$ geeft $F(x) = 2\,e^{\frac{1}{2}x-1} + c$

b $f(x) = \dfrac{3}{2x-1}$ geeft $F(x) = 3 \cdot \frac{1}{2}\ln|2x-1| + c = 1\frac{1}{2}\ln|2x-1| + c$

c $f(x) = 4\sin(\pi x)$ geeft $F(x) = 4 \cdot \dfrac{1}{\pi} \cdot -\cos(\pi x) + c = -\dfrac{4\cos(\pi x)}{\pi} + c$

d $f(x) = \dfrac{4x+1}{2\sqrt{4x+1}} = \frac{1}{2}\sqrt{4x+1} = \frac{1}{2}(4x+1)^{\frac{1}{2}}$ geeft

$F(x) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{3}(4x+1)^{1\frac{1}{2}} + c = \frac{1}{12}(4x+1)\sqrt{4x+1} + c$

e $f(x) = (\frac{1}{2})^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \dfrac{(\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \dfrac{\frac{1}{2} \cdot (\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \dfrac{(\frac{1}{2})^{2x-2}}{\ln(\frac{1}{2})} + c$

f $f(x) = 4\ln(x-1)$ geeft $F(x) = 4((x-1)\ln(x-1) - (x-1)) + c = 4(x-1)\ln(x-1) - 4(x-1) + c$

5 **a** $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin(2x)\,\mathrm{d}x = \left[-\frac{1}{2}\cos(2x)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -\frac{1}{2}\cos(\frac{2}{3}\pi) + \frac{1}{2}\cos(\frac{1}{3}\pi) = -\frac{1}{2} \cdot -\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

b $\int_1^4 \frac{6}{2x-1}\,\mathrm{d}x = \left[6 \cdot \frac{1}{2}\ln|2x-1|\right]_1^4 = 3\ln(7) - 3\ln(1) = 3\ln(7)$

c $\int_{\mathrm{e}}^{\mathrm{e}^2} 10\ln(\sqrt[4]{x})\,\mathrm{d}x = \int_{\mathrm{e}}^{\mathrm{e}^2} 10\ln(x^{\frac{1}{4}})\,\mathrm{d}x = \int_{\mathrm{e}}^{\mathrm{e}^2} 2\frac{1}{2}\ln(x)\,\mathrm{d}x = \left[2\frac{1}{2}(x\ln(x) - x)\right]_{\mathrm{e}}^{\mathrm{e}^2}$

$= 2\frac{1}{2}(\mathrm{e}^2\ln(\mathrm{e}^2) - \mathrm{e}^2 - (\mathrm{e}\ln(\mathrm{e}) - \mathrm{e})) = 2\frac{1}{2}(\mathrm{e}^2 \cdot 2 - \mathrm{e}^2 - (\mathrm{e} \cdot 1 - \mathrm{e})) = 2\frac{1}{2}\mathrm{e}^2$

d $\int_0^4 6\,\mathrm{e}^{\frac{1}{2}x-3}\mathrm{d}x = \left[6 \cdot 2\,\mathrm{e}^{\frac{1}{2}x-3}\right]_0^4 = 12\mathrm{e}^{-1} - 12\mathrm{e}^{-3} = \frac{12}{\mathrm{e}} - \frac{12}{\mathrm{e}^3}$

K.1 De substitutiemethode

Bladzijde 183

1 **a** $f(x) = \sin(x^2 + x)$ geeft $f'(x) = \cos(x^2 + x) \cdot (2x + 1) = (2x + 1)\cos(x^2 + x)$

b $G'(x) = (2x + 1)\cos(x^2 + x)$, dus $G(x) = \sin(x^2 + x) + 3$ is een primitieve van $g(x) = (2x + 1)\cos(x^2 + x)$.

Bladzijde 184

2 **a** Omdat $\frac{\mathrm{d}(x^2+1)}{\mathrm{d}x} = 2x$ en $6x = 3 \cdot 2x$ lukt het primitiveren van $f(x)$ wel op deze manier.

Omdat $\frac{\mathrm{d}(x^3+1)}{\mathrm{d}x} = 3x^2$ en $6x$ niet geschreven kan worden als het product van een constante en $3x^2$ lukt het primitiveren van $h(x)$ niet op deze manier.

b Omdat $\frac{\mathrm{d}(x^4+7)}{\mathrm{d}x} = 4x^3$ en $10x^3 = 2\frac{1}{2} \cdot 4x^3$ lukt het primitiveren van $g(x)$ wel op deze manier.

Omdat $\frac{\mathrm{d}(x^4+x)}{\mathrm{d}x} = 4x^3 + 1$ en $10x^3$ niet geschreven kan worden als het product van een constante en $4x^3 + 1$ lukt het primitiveren van $k(x)$ niet op deze manier.

c $\frac{\mathrm{d}(x^4+x)}{\mathrm{d}x} = 4x^3 + 1$

$10x^3 + a = 2\frac{1}{2}(4x^3 + \frac{2}{5}a)$ moet gelijk zijn aan $2\frac{1}{2}(4x^3 + 1)$, dus $\frac{2}{5}a = 1$ en dit geeft $a = 2\frac{1}{2}$.

3 **a** $3x^2\,\mathrm{d}x = \mathrm{d}(x^3 + 5)$

b $5x^4\,\mathrm{d}x = \mathrm{d}(x^5 - 3)$

c $\cos(2x)\,\mathrm{d}x = \mathrm{d}\,(\frac{1}{2}\sin(2x) + \pi)$

d $\frac{1}{x}\mathrm{d}x = \mathrm{d}\ln(x)$

e $(5 - 2x)\,\mathrm{d}x = \mathrm{d}(5x - x^2 + 5)$

f $2\cos(4x)\,\mathrm{d}x = \mathrm{d}\frac{1}{2}\sin(4x)$

4 **a** $F(x) = \int 2x(x^2 + 4)^3\,\mathrm{d}x = \int (x^2 + 4)^3 \cdot 2x\,\mathrm{d}x = \int (x^2 + 4)^3\,\mathrm{d}(x^2 + 4) = \int u^3\,\mathrm{d}u = \frac{1}{4}u^4 + c$

$= \frac{1}{4}(x^2 + 4)^4 + c$

b $G(x) = \int 6x\sqrt{x^2 + 1}\,\mathrm{d}x = \int 3\sqrt{x^2 + 1} \cdot 2x\,\mathrm{d}x = \int 3\sqrt{x^2 + 1}\,\mathrm{d}(x^2 + 1) = \int 3u^{\frac{1}{2}}\,\mathrm{d}u = \frac{3}{1\frac{1}{2}}u^{1\frac{1}{2}} + c$

$= 2u\sqrt{u} + c = 2(x^2 + 1)\sqrt{x^2 + 1} + c$

c $H(x) = \int 6x^2(x^3 - 1)^4\,\mathrm{d}x = \int 2(x^3 - 1)^4 \cdot 3x^2\,\mathrm{d}x = \int 2(x^3 - 1)^4\,\mathrm{d}(x^3 - 1) = \int 2u^4\,\mathrm{d}u = \frac{2}{5}u^5 + c$

$= \frac{2}{5}(x^3 - 1)^5 + c$

d $J(x) = \int 3x^2\sin(x^3 - 1)\,\mathrm{d}x = \int \sin(x^3 - 1) \cdot 3x^2\,\mathrm{d}x = \int \sin(x^3 - 1)\,\mathrm{d}(x^3 - 1) = \int \sin(u)\,\mathrm{d}u$

$= -\cos(u) + c = -\cos(x^3 - 1) + c$

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5 **a** $F(x) = \int x(3x^2-4)^3\,dx = \int \frac{1}{6}(3x^2-4)^3 \cdot 6x\,dx = \int \frac{1}{6}(3x^2-4)^3\,d(3x^2-4) = \int \frac{1}{6}u^3\,du = \frac{1}{24}u^4 + c$

$= \frac{1}{24}(3x^2-4)^4 + c$

b $F(x) = \int 2x\sqrt{x^2-3}\,dx = \int \sqrt{x^2-3}\,d(x^2-3) = \int \sqrt{u}\,du = \frac{2}{3}u^{1\frac{1}{2}} + c = \frac{2}{3}(x^2-3)\sqrt{x^2-3} + c$

c $F(x) = \int \frac{6x}{3x^2+2}\,dx = \int \frac{1}{3x^2+2} \cdot 6x\,dx = \int \frac{1}{3x^2+2}\,d(3x^2+2) = \int \frac{1}{u}\,du = \ln|u| + c = \ln|3x^2+2| + c = \ln(3x^2+2) + c$

d $F(x) = \int x\ln(x^2+1)\,dx = \int \frac{1}{2}\ln(x^2+1) \cdot 2x\,dx = \int \frac{1}{2}\ln(x^2+1)\,d(x^2+1) = \int \frac{1}{2}\ln(u)\,du$

$= \frac{1}{2}(u\ln(u) - u) + c = \frac{1}{2}((x^2+1)\ln(x^2+1) - (x^2+1)) + c$

6 **a** $F(x) = \int \frac{\ln(x)}{x}\,dx = \int \ln(x) \cdot \frac{1}{x}\,dx = \int \ln(x)\,d\ln(x) = \int u\,du = \frac{1}{2}u^2 + c = \frac{1}{2}\ln^2(x) + c$

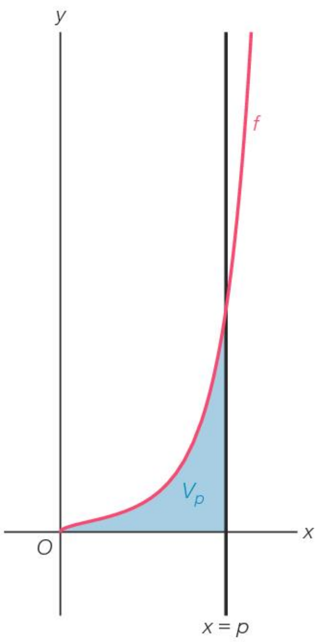
b $G(x) = \int x\,e^{-x^2}\,dx = \int -\frac{1}{2}e^{-x^2} \cdot -2x\,dx = \int -\frac{1}{2}e^{-x^2}\,d(-x^2) = \int -\frac{1}{2}e^u\,du = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-x^2} + c$

c $H(x) = \int x\sqrt{5-x^2}\,dx = \int -\frac{1}{2}\sqrt{5-x^2} \cdot -2x\,dx = \int -\frac{1}{2}\sqrt{5-x^2}\,d(5-x^2) = \int -\frac{1}{2}u^{\frac{1}{2}}\,du$

$= -\frac{1}{2} \cdot \frac{1}{1\frac{1}{2}}u^{1\frac{1}{2}} + c = -\frac{1}{3}u\sqrt{u} + c = -\frac{1}{3}(5-x^2)\sqrt{5-x^2} + c$

d $J(x) = \int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+1}} \cdot 2x\,dx = \int \frac{1}{2\sqrt{x^2+1}}\,d(x^2+1) = \int \frac{1}{2\sqrt{u}}\,du = \sqrt{u} + c = \sqrt{x^2+1} + c$

7



$\int (f(x))^2\,dx = \int \frac{1}{100} \cdot 3x \cdot (e^{x^2})^2\,dx = \int \frac{3}{400} \cdot e^{2x^2} \cdot 4x\,dx = \int \frac{3}{400} \cdot e^{2x^2}\,d\,2x^2 = \int \frac{3}{400} \cdot e^u\,du = \frac{3}{400}e^u = \frac{3}{400}e^{2x^2}$

$I(L_p) = \pi\int_0^p (f(x))^2\,dx = \pi\left[\frac{3}{400}e^{2x^2}\right]_0^p = \pi(\frac{3}{400} \cdot e^{2p^2} - \frac{3}{400} \cdot 1) = \frac{3}{400}\pi(e^{2p^2} - 1)$

$I(L_p) = 3\frac{3}{10}\pi$ geeft $\frac{3}{400}\pi(e^{2p^2} - 1) = 3\frac{3}{10}\pi$

$e^{2p^2} - 1 = 440$

$e^{2p^2} = 441$

$2p^2 = \ln(441)$

$p^2 = \frac{1}{2}\ln(441)$

$p^2 = \ln(\sqrt{441})$

$p^2 = \ln(21)$

$p = \sqrt{\ln(21)}$

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8 **a** $f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)} = \dfrac{1}{\cos(x)} \cdot \sin(x)$

b $\int \tan(x)\,dx = \int \dfrac{\sin(x)}{\cos(x)}\,dx = \int \dfrac{-1}{\cos(x)} \cdot -\sin(x)\,dx = \int \dfrac{-1}{\cos(x)}\,d\cos(x)$

Bladzijde 186

9 **a** $\int_0^{\frac{1}{6}\pi} \tan(2x)\,dx = \left[-\tfrac{1}{2}\ln|\cos(2x)|\right]_0^{\frac{1}{6}\pi} = -\tfrac{1}{2}\ln|\cos(\tfrac{1}{3}\pi)| + \tfrac{1}{2}\ln|\cos(0)| = -\tfrac{1}{2}\ln(\tfrac{1}{2}) + \tfrac{1}{2}\ln(1)$

$= -\tfrac{1}{2}\ln(2^{-1}) + \tfrac{1}{2} \cdot 0 = \tfrac{1}{2}\ln(2)$

b $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^3(x)\,dx = \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^2(x) \cdot \sin(x)\,dx = \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} -(1 - \cos^2(x)) \cdot -\sin(x)\,dx = \int_{x=\frac{1}{4}\pi}^{x=\frac{1}{3}\pi} -(1 - \cos^2(x))\,d\cos(x)$

$= \int_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} (-1 + u^2)\,du = \left[-u + \tfrac{1}{3}u^3\right]_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} = -\tfrac{1}{2} + \tfrac{1}{3} \cdot (\tfrac{1}{2})^3 - (-\tfrac{1}{2}\sqrt{2} + \tfrac{1}{3} \cdot (\tfrac{1}{2}\sqrt{2})^3)$

$= -\tfrac{1}{2} + \tfrac{1}{24} + \tfrac{1}{2}\sqrt{2} - \tfrac{1}{12}\sqrt{2} = -\tfrac{11}{24} + \tfrac{5}{12}\sqrt{2}$

c $\int_0^{\frac{1}{6}\pi} \dfrac{\sin(x)}{\cos^3(x)}\,dx = \int_0^{\frac{1}{6}\pi} -\dfrac{1}{\cos^3(x)} \cdot -\sin(x)\,dx = \int_{x=0}^{x=\frac{1}{6}\pi} -\dfrac{1}{\cos^3(x)}\,d\cos(x) = \int_1^{\frac{1}{2}\sqrt{3}} -\dfrac{1}{u^3}\,du = \int_1^{\frac{1}{2}\sqrt{3}} -u^{-3}\,du$

$= \left[\tfrac{1}{2}u^{-2}\right]_1^{\frac{1}{2}\sqrt{3}} = \left[\dfrac{1}{2u^2}\right]_1^{\frac{1}{2}\sqrt{3}} = \dfrac{1}{2 \cdot \frac{3}{4}} - \dfrac{1}{2 \cdot 1} = \tfrac{2}{3} - \tfrac{1}{2} = \tfrac{1}{6}$

Bladzijde 187

10 **a** $\int \dfrac{\cos(x)}{\sin^2(x)}\,dx = \int \dfrac{1}{\sin^2(x)} \cdot \cos(x)\,dx = \int \dfrac{1}{\sin^2(x)}\,d\sin(x) = \int \dfrac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + c = -\dfrac{1}{u} + c$

$= -\dfrac{1}{\sin(x)} + c$

$f(x) = \cos(x) - \dfrac{\cos(x)}{\sin^2(x)}$ geeft $F(x) = \sin(x) + \dfrac{1}{\sin(x)} + c$

b $G(x) = \int \sin^5(x)\,dx = \int -\sin^4(x) \cdot -\sin(x)\,dx = \int -(\sin^2(x))^2 \cdot -\sin(x)\,dx = \int -(1 - \cos^2(x))^2\,d\cos(x)$

$= \int -(1 - u^2)^2\,du = \int -(1 - 2u^2 + u^4)\,du = \int (-1 + 2u^2 - u^4)\,du = -u + \tfrac{2}{3}u^3 - \tfrac{1}{5}u^5 + c$

$= -\tfrac{1}{5}u^5 + \tfrac{2}{3}u^3 - u + c = -\tfrac{1}{5}\cos^5(x) + \tfrac{2}{3}\cos^3(x) - \cos(x) + c$

11 **a** $\int_1^2 \dfrac{\ln^2(x)}{x}\,dx = \int_1^2 \ln^2(x) \cdot \dfrac{1}{x}\,dx = \int_{x=1}^{x=2} \ln^2(x)\,d\ln(x) = \int_0^{\ln(2)} u^2\,du = \left[\tfrac{1}{3}u^3\right]_0^{\ln(2)} = \tfrac{1}{3}\ln^3(2)$

b $\int_e^{e^2} \dfrac{1}{x\ln(x)}\,dx = \int_e^{e^2} \dfrac{1}{\ln(x)} \cdot \dfrac{1}{x}\,dx = \int_{x=e}^{x=e^2} \dfrac{1}{\ln(x)}\,d\ln(x) = \int_1^2 \dfrac{1}{u}\,du = \left[\ln|u|\right]_1^2 = \ln(2) - \ln(1) = \ln(2)$

c $\int_0^1 \dfrac{x^2}{e^{2x^3}}\,dx = \int_0^1 -\tfrac{1}{6} \cdot e^{-2x^3} \cdot -6x^2\,dx = \int_{x=0}^{x=1} -\tfrac{1}{6} \cdot e^{-2x^3}\,d{-2x^3} = \int_0^{-2} -\tfrac{1}{6}e^u\,du = \left[-\tfrac{1}{6}e^u\right]_0^{-2} = -\tfrac{1}{6}e^{-2} + \tfrac{1}{6}e^0$

$= -\dfrac{1}{6e^2} + \tfrac{1}{6} = \dfrac{e^2 - 1}{6e^2}$

12 **a** $x = 0$ geeft $u = \sqrt{0 + 4} = 2$
$x = 5$ geeft $u = \sqrt{5 + 4} = 3$

b $\sqrt{x + 4} = u$
kwadrateren geeft
$x + 4 = u^2$
$x = u^2 - 4$
Omdat $x = u^2 - 4$ en $\sqrt{x + 4} = u$ krijg je $\int x\sqrt{x + 4}\,dx = \int (u^2 - 4) \cdot u\,d(u^2 - 4)$.

c $\int (u^2 - 4) \cdot u\,d(u^2 - 4) = \int (u^2 - 4) \cdot u \cdot 2u\,du = \int (2u^4 - 8u^2)\,du$

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d $\int_0^5 x\sqrt{x+4}\,dx = \int_2^3 (2u^4 - 8u^2)\,du = \left[\frac{2}{5}u^5 - \frac{8}{3}u^3\right]_2^3 = \frac{2}{5}\cdot 3^5 - \frac{8}{3}\cdot 3^3 - \frac{2}{5}\cdot 2^5 + \frac{8}{3}\cdot 2^3$

$= 97\frac{1}{5} - 72 - 12\frac{4}{5} + 21\frac{1}{3} = 33\frac{11}{15}$

13 **a** $\sqrt{x+1} = u$

kwadrateren geeft

$x + 1 = u^2$

$x = u^2 - 1$

$\int_0^3 x\sqrt{x+1}\,dx = \int_{u=1}^{u=2} (u^2-1)\cdot u\,d(u^2-1) = \int_1^2 (u^2-1)\cdot u\cdot 2u\,du = \int_1^2 (2u^4 - 2u^2)\,du = \left[\frac{2}{5}u^5 - \frac{2}{3}u^3\right]_1^2$

$= \frac{2}{5}\cdot 2^5 - \frac{2}{3}\cdot 2^3 - (\frac{2}{5}\cdot 1^5 - \frac{2}{3}\cdot 1^3) = 12\frac{4}{5} - 5\frac{1}{3} - \frac{2}{5} + \frac{2}{3} = 7\frac{11}{15}$

b $\sqrt{x+9} = u$

kwadrateren geeft

$x + 9 = u^2$

$x = u^2 - 9$

$\int_0^7 \frac{5x}{\sqrt{x+9}}\,dx = \int_{u=3}^{u=4} \frac{5(u^2-9)}{u}\,d(u^2-9) = \int_3^4 \frac{5(u^2-9)}{u}\cdot 2u\,du = \int_3^4 (10u^2 - 90)\,du = \left[\frac{10}{3}u^3 - 90u\right]_3^4$

$= \frac{10}{3}\cdot 4^3 - 90\cdot 4 - (\frac{10}{3}\cdot 3^3 - 90\cdot 3) = 213\frac{1}{3} - 360 - 90 + 270 = 33\frac{1}{3}$

14 **a** $f(x) = 0$ geeft $\frac{2+\ln(x)}{x} = 0$

$2 + \ln(x) = 0$

$\ln(x) = -2$

$x = e^{-2}$

$O(V) = \int_{e^{-2}}^{e} \frac{2+\ln(x)}{x}\,dx = \int_{e^{-2}}^{e} (2+\ln(x))\cdot\frac{1}{x}\,dx = \int_{x=e^{-2}}^{x=e} (2+\ln(x))\,d\ln(x) = \int_{-2}^{1} (2+u)\,du$

$= \left[2u + \frac{1}{2}u^2\right]_{-2}^{1} = 2 + \frac{1}{2} - (-4+2) = 4\frac{1}{2}$

b $\int_1^p \frac{2+\ln(x)}{x}\,dx = \int_1^p (2+\ln(x))\cdot\frac{1}{x}\,dx = \int_{x=1}^{x=p} (2+\ln(x))\,d\ln(x) = \int_0^{\ln(p)} (2+u)\,du = \left[2u + \frac{1}{2}u^2\right]_0^{\ln(p)}$

$= 2\ln(p) + \frac{1}{2}\ln^2(p)$

$\int_1^p f(x)\,dx = 6$ geeft $2\ln(p) + \frac{1}{2}\ln^2(p) = 6$

$\ln^2(p) + 4\ln(p) - 12 = 0$

$(\ln(p) - 2)(\ln(p) + 6) = 0$

$\ln(p) = 2 \vee \ln(p) = -6$

$p = e^2 \vee p = e^{-6}$

vold. niet

Dus $p = e^2$.

Bladzijde 188

15 **a** $f(x) = g(x)$ geeft $\frac{4\ln^2(x)}{x} = \frac{1}{x}$

$4\ln^2(x) = 1$

$\ln^2(x) = \frac{1}{4}$

$\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$

$x = e^{\frac{1}{2}} \vee x = e^{-\frac{1}{2}}$

$x = \sqrt{e} \vee x = \frac{1}{\sqrt{e}}$

$f(x) \leq g(x)$ geeft $\frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}$

b $O(V) = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (g(x) - f(x))\,dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} \left(\frac{1}{x} - \frac{4\ln^2(x)}{x}\right) dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (1 - 4\ln^2(x)) \cdot \frac{1}{x}\,dx$

$= \int_{x=\frac{1}{\sqrt{e}}}^{x=\sqrt{e}} (1 - 4\ln^2(x))\,d\ln(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 4u^2)\,du = \left[u - \frac{4}{3}u^3\right]_{-\frac{1}{2}}^{\frac{1}{2}}$

$= \frac{1}{2} - \frac{4}{3} \cdot \frac{1}{8} - (-\frac{1}{2} - \frac{4}{3} \cdot -\frac{1}{8}) = \frac{1}{2} - \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$

16 **a** $f(x) = \frac{3x^2}{x^3 + 4}$ geeft $f'(x) = \frac{(x^3 + 4) \cdot 6x - 3x^2 \cdot 3x^2}{(x^3 + 4)^2} = \frac{6x^4 + 24x - 9x^4}{(x^3 + 4)^2} = \frac{-3x^4 + 24x}{(x^3 + 4)^2}$

$f'(x) = 0$ geeft $-3x^4 + 24x = 0$

$-3x(x^3 - 8) = 0$

$x = 0 \vee x^3 = 8$

$x = 0 \vee x = 2$

$x^3 + 4 = 0$

$x^3 = -4$

$x = \sqrt[3]{-4}$

min. is $f(0) = 0$

max. is $f(2) = 1$

$f(x) = p$ heeft drie oplossingen voor $0 < p < 1$.

b $\int_0^p \frac{3x^2}{x^3 + 4}\,dx = \int_0^p \frac{1}{x^3 + 4} \cdot 3x^2\,dx = \int_{x=0}^{x=p} \frac{1}{x^3 + 4}\,d(x^3 + 4) = \int_4^{p^3+4} \frac{1}{u}\,du \;\; = [\ln|u|]_4^{p^3+4} = \ln(p^3 + 4) - \ln(4)$

$\int_0^p f(x)\,dx = 2$ geeft $\ln(p^3 + 4) - \ln(4) = 2$

$\ln\left(\frac{p^3 + 4}{4}\right) = 2$

$\frac{p^3 + 4}{4} = e^2$

$p^3 + 4 = 4e^2$

$p^3 = 4e^2 - 4$

$p = \sqrt[3]{4e^2 - 4}$

K.2 Partieel integreren

Bladzijde 190

17 **a** $k(x) = (2x + 3)\sin(x)$ differentiëren met de productregel geeft $k'(x) = 2\sin(x) + (2x + 3)\cos(x)$. Dus is k geen primitieve van h.

b Integreren van het linker- en rechterlid van de vergelijking $(2x + 3)\cos(x) = k'(x) - 2\sin(x)$ geeft

$\int (2x + 3)\cos(x)\,dx = \int (k'(x) - 2\sin(x))\,dx$

$\int (2x + 3)\cos(x)\,dx = (2x + 3)\sin(x) - \int 2\sin(x)\,dx.$

c $H(x) = \int (2x + 3)\cos(x)\,dx = (2x + 3)\sin(x) - \int 2\sin(x)\,dx = (2x + 3)\sin(x) + 2\cos(x) + c$

Bladzijde 192

18 **a** $\int \frac{\ln(x)}{\sqrt{x}}\,dx = \int \ln(x) \cdot x^{-\frac{1}{2}}\,dx = \int \ln(x)\,d\,2\sqrt{x} = \ln(x) \cdot 2\sqrt{x} - \int 2\sqrt{x}\,d\ln(x)$

$= 2\sqrt{x} \cdot \ln(x) - \int 2\sqrt{x} \cdot \frac{1}{x}\,dx = 2\sqrt{x} \cdot \ln(x) - \int 2x^{-\frac{1}{2}}\,dx$

$= 2\sqrt{x} \cdot \ln(x) - 4x^{\frac{1}{2}} + c = 2\sqrt{x}(\ln(x) - 2) + c$

b $\int x\sin(x)\,dx = \int \sin(x)\,d\tfrac{1}{2}x^2 = \tfrac{1}{2}x^2\sin(x) - \int \tfrac{1}{2}x^2\cos(x)\,dx = \ldots$

Het wordt op deze manier alleen maar ingewikkelder, dus het lukt zo niet.

c $F(x) = \int \ln(x)\,dx = x\ln(x) - \int x\,d\ln(x) = x\ln(x) - \int x \cdot \frac{1}{x}\,dx = x\ln(x) - \int 1\,dx = x\ln(x) - x + c$

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19 **a** $F(x) = \int x\,\mathrm{e}^{2x}\,\mathrm{d}x = \int x\,\mathrm{d}\tfrac{1}{2}\mathrm{e}^{2x} = x \cdot \tfrac{1}{2}\mathrm{e}^{2x} - \int \tfrac{1}{2}\mathrm{e}^{2x}\,\mathrm{d}x = \tfrac{1}{2}x\,\mathrm{e}^{2x} - \tfrac{1}{4}\mathrm{e}^{2x} + c$

b $F(x) = \int 2x\cos(x)\,\mathrm{d}x = \int 2x\,\mathrm{d}\sin(x) = 2x\sin(x) - \int \sin(x)\,\mathrm{d}2x = 2x\sin(x) - \int 2\sin(x)\,\mathrm{d}x$

$= 2x\sin(x) + 2\cos(x) + c$

c $F(x) = \int x\ln(x)\,\mathrm{d}x = \int \ln(x)\,\mathrm{d}\tfrac{1}{2}x^2 = \tfrac{1}{2}x^2\ln(x) - \int \tfrac{1}{2}x^2\,\mathrm{d}\ln(x) = \tfrac{1}{2}x^2\ln(x) - \int \tfrac{1}{2}x^2 \cdot \dfrac{1}{x}\mathrm{d}x$

$= \tfrac{1}{2}x^2\ln(x) - \int \tfrac{1}{2}x\,\mathrm{d}x = \tfrac{1}{2}x^2\ln(x) - \tfrac{1}{4}x^2 + c$

d $\int x^3\ln(x)\,\mathrm{d}x = \int \ln(x)\,\mathrm{d}\tfrac{1}{4}x^4 = \tfrac{1}{4}x^4\ln(x) - \int \tfrac{1}{4}x^4\,\mathrm{d}\ln(x) = \tfrac{1}{4}x^4\ln(x) - \int \tfrac{1}{4}x^4 \cdot \dfrac{1}{x}\mathrm{d}x$

$= \tfrac{1}{4}x^4\ln(x) - \int \tfrac{1}{4}x^3\,\mathrm{d}x = \tfrac{1}{4}x^4\ln(x) - \tfrac{1}{16}x^4 + c$

Dus $f(x) = x^3\ln(x) + 3$ geeft $F(x) = \tfrac{1}{4}x^4\ln(x) - \tfrac{1}{16}x^4 + 3x + c$

20 **a** $\int 2x\,\mathrm{e}^{x+1}\,\mathrm{d}x = \int 2x\,\mathrm{d}\,\mathrm{e}^{x+1} = 2x\,\mathrm{e}^{x+1} - \int \mathrm{e}^{x+1}\,\mathrm{d}2x = 2x\,\mathrm{e}^{x+1} - \int \mathrm{e}^{x+1} \cdot 2\,\mathrm{d}x = 2x\,\mathrm{e}^{x+1} - 2\,\mathrm{e}^{x+1} + c$

$\int_0^1 2x\,\mathrm{e}^{x+1}\,\mathrm{d}x = \left[2x\,\mathrm{e}^{x+1} - 2\,\mathrm{e}^{x+1}\right]_0^1 = 2\mathrm{e}^2 - 2\mathrm{e}^2 - (0 - 2\mathrm{e}) = 2\mathrm{e}$

b $\int (3x+1)\sin(x)\,\mathrm{d}x = \int -(3x+1)\,\mathrm{d}\cos(x) = -(3x+1) \cdot \cos(x) - \int -\cos(x)\,\mathrm{d}(3x+1)$

$= -(3x+1)\cos(x) + \int 3\cos(x)\,\mathrm{d}x = -(3x+1)\cos(x) + 3\sin(x) + c$

$\int_0^{\pi} (3x+1)\sin(x)\,\mathrm{d}x = \left[-(3x+1)\cos(x) + 3\sin(x)\right]_0^{\pi}$

$= -(3\pi+1)\cos(\pi) + 3\sin(\pi) - (-\cos(0) + 3\sin(0)) = 3\pi + 1 + 1 = 3\pi + 2$

21 **a** $f(x) = x^2\ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \dfrac{1}{x} = 2x\ln(x) + x$

$f'(x) = 0$ geeft $2x\ln(x) + x = 0$

$x(2\ln(x) + 1) = 0$

$x = 0 \vee 2\ln(x) + 1 = 0$

$x = 0 \vee \ln(x) = -\tfrac{1}{2}$

$x = 0 \quad \vee \quad x = \mathrm{e}^{-\frac{1}{2}}$

vold. niet

$f(\mathrm{e}^{-\frac{1}{2}}) = (\mathrm{e}^{-\frac{1}{2}})^2 \cdot -\tfrac{1}{2} = -\tfrac{1}{2}\mathrm{e}^{-1}$

Dus $A(\mathrm{e}^{-\frac{1}{2}}, -\tfrac{1}{2}\mathrm{e}^{-1}) = A\left(\dfrac{1}{\sqrt{\mathrm{e}}}, -\dfrac{1}{2\mathrm{e}}\right)$.

b $\int x^2\ln(x)\,\mathrm{d}x = \int \ln(x)\,\mathrm{d}\tfrac{1}{3}x^3 = \tfrac{1}{3}x^3\ln(x) - \int \tfrac{1}{3}x^3\,\mathrm{d}\ln(x) = \tfrac{1}{3}x^3\ln(x) - \int \tfrac{1}{3}x^3 \cdot \dfrac{1}{x}\mathrm{d}x$

$= \tfrac{1}{3}x^3\ln(x) - \int \tfrac{1}{3}x^2\,\mathrm{d}x = \tfrac{1}{3}x^3\ln(x) - \tfrac{1}{9}x^3 + c$

$O(V) = \int_1^{\mathrm{e}} x^2\ln(x)\,\mathrm{d}x = \left[\tfrac{1}{3}x^3\ln(x) - \tfrac{1}{9}x^3\right]_1^{\mathrm{e}} = \tfrac{1}{3}\mathrm{e}^3\ln(\mathrm{e}) - \tfrac{1}{9}\mathrm{e}^3 - (\tfrac{1}{3}\ln(1) - \tfrac{1}{9}) = \tfrac{2}{9}\mathrm{e}^3 + \tfrac{1}{9}$

Bladzijde 193

22 **a** $f(x) = \dfrac{\ln(x)}{x^2}$ geeft $f'(x) = \dfrac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{x^4} = \dfrac{x - 2x\ln(x)}{x^4} = \dfrac{1 - 2\ln(x)}{x^3}$

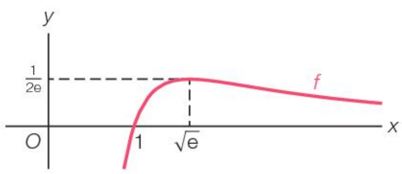
$f'(x) = 0$ geeft $1 - 2\ln(x) = 0$

$\ln(x) = \tfrac{1}{2}$

$x = \mathrm{e}^{\frac{1}{2}}$

$x = \sqrt{\mathrm{e}}$

Zie de figuur hiernaast.



max. is $f(\sqrt{\mathrm{e}}) = \dfrac{\frac{1}{2}}{(\sqrt{\mathrm{e}})^2} = \dfrac{\frac{1}{2}}{\mathrm{e}} = \dfrac{1}{2\mathrm{e}}$

Dus $B_f = \left\langle \leftarrow, \dfrac{1}{2\mathrm{e}} \right]$.

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b $\int \frac{\ln(x)}{x^2}\,dx = \int \ln(x)\cdot x^{-2}\,dx = \int \ln(x)\,d(-x^{-1}) = -x^{-1}\ln(x) - \int -x^{-1}\,d\ln(x) = -\frac{1}{x}\ln(x) + \int \frac{1}{x}\cdot\frac{1}{x}\,dx$

$= -\frac{\ln(x)}{x} + \int x^{-2}\,dx = -\frac{\ln(x)}{x} - x^{-1} + c = -\frac{\ln(x)+1}{x} + c$

$O(V) = \int_1^e \frac{\ln(x)}{x^2}\,dx = \left[-\frac{\ln(x)+1}{x}\right]_1^e = -\frac{1+1}{e} + \frac{0+1}{1} = -\frac{2}{e} + 1$

23 **a** $\int x^2 e^x\,dx = \int x^2\,d\,e^x = x^2 e^x - \int e^x\,dx^2 = x^2 e^x - \int e^x \cdot 2x\,dx = x^2 e^x - \int 2x\,e^x\,dx$

b $\int 2x\,e^x\,dx = 2x\,e^x - \int e^x\,d\,2x = 2x\,e^x - \int e^x \cdot 2\,dx = 2x\,e^x - 2\,e^x + c$

c $\int x^2 e^x\,dx = x^2 e^x - \int 2x\,e^x\,dx = x^2 e^x - (2x\,e^x - 2e^x + c) = x^2 e^x - 2x\,e^x + 2e^x + c = (x^2 - 2x + 2)\,e^x + c$

Dus de primitieven van f zijn $F(x) = (x^2 - 2x + 2)\,e^x + c$

Bladzijde 194

24 **a** $F(x) = (ax^2 + bx + c)\,e^x$ geeft
$F'(x) = (2ax + b)\,e^x + (ax^2 + bx + c)\,e^x = (ax^2 + (2a + b)x + b + c)\,e^x$
Dit moet gelijk zijn aan $f(x) = (ax^2 + px + q)\,e^x$.
Dit geeft $p = 2a + b$ en $q = b + c$.

b $f(x) = (4x^2 - 5x + 6)\,e^x$, dus $a = 4$, $p = -5$ en $q = 6$.
$a = 4$ en $2a + b = -5$ geeft $8 + b = -5$, dus $b = -13$.
$b = -13$ en $b + c = 6$ geeft $-13 + c = 6$, dus $c = 19$.
Dus de primitieven van $f(x) = (4x^2 - 5x + 6)\,e^x$ zijn $F(x) = (4x^2 - 13x + 19)\,e^x + c$.

25 **a** $\int e^x \sin(x)\,dx = \int \sin(x)\,d\,e^x = e^x \sin(x) - \int e^x\,d\sin(x) = e^x \sin(x) - \int e^x \cos(x)\,dx$

$= e^x \sin(x) - \int \cos(x)\,d\,e^x = e^x \sin(x) - (e^x \cos(x) - \int e^x\,d\cos(x))$

$= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)\,dx$

Uit $\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)\,dx$ volgt $2\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x)$

dus $\int e^x \sin(x)\,dx = \frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$.

De primitieven zijn $G(x) = \frac{1}{2}e^x(\sin(x) - \cos(x)) + c$.

b $F(x) = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + c$ geeft
$F'(x) = -2x \cdot \cos(x) - x^2 \cdot -\sin(x) + 2 \cdot \sin(x) + 2x \cdot \cos(x) - 2\sin(x) = x^2 \sin(x)$

26 **a** $F(x) = \int \frac{1}{4}x^2 \cos(x)\,dx = \int \frac{1}{4}x^2\,d\sin(x) = \frac{1}{4}x^2 \sin(x) - \int \sin(x)\,d\frac{1}{4}x^2 = \frac{1}{4}x^2 \sin(x) - \int \frac{1}{2}x\sin(x)\,dx$

$= \frac{1}{4}x^2 \sin(x) + \int \frac{1}{2}x\,d\cos(x) = \frac{1}{4}x^2 \sin(x) + \frac{1}{2}x\cos(x) - \int \cos(x)\,d\frac{1}{2}x$

$= \frac{1}{4}x^2 \sin(x) + \frac{1}{2}x\cos(x) - \int \frac{1}{2}\cos(x)\,dx = \frac{1}{4}x^2 \sin(x) + \frac{1}{2}x\cos(x) - \frac{1}{2}\sin(x) + c$

b $\int e^{-x} \cos(x)\,dx = \int -\cos(x)\,d\,e^{-x} = -\cos(x)\,e^{-x} + \int e^{-x}\,d\cos(x) = -e^{-x}\cos(x) - \int e^{-x}\sin(x)\,dx$

$= -e^{-x}\cos(x) + \int \sin(x)\,d\,e^{-x} = -e^{-x}\cos(x) + e^{-x}\sin(x) - \int e^{-x}\,d\sin(x)$

$= -e^{-x}\cos(x) + e^{-x}\sin(x) - \int e^{-x}\cos(x)\,dx$

Uit $\int e^{-x}\cos(x)\,dx = -e^{-x}\cos(x) + e^{-x}\sin(x) - \int e^{-x}\cos(x)\,dx$ volgt $2\int e^{-x}\cos(x)\,dx = -e^{-x}\cos(x) + e^{-x}\sin(x)$

dus $\int e^{-x}\cos(x)\,dx = \frac{1}{2}e^{-x}(\sin(x) - \cos(x))$.

De primitieven zijn $G(x) = \frac{1}{2}e^{-x}(\sin(x) - \cos(x)) + c$.

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c $\int e^{2x}\sin(x)\,dx = \int e^{2x}\,d(-\cos(x)) = -e^{2x}\cos(x) + \int \cos(x)\,d\,e^{2x} = -e^{2x}\cos(x) + \int \cos(x)\cdot 2\,e^{2x}\,dx$

$= -e^{2x}\cos(x) + \int 2\,e^{2x}\,d\sin(x) = -e^{2x}\cos(x) + 2\,e^{2x}\sin(x) - \int \sin(x)\,d\,2\,e^{2x}$

$= -e^{2x}\cos(x) + 2\,e^{2x}\sin(x) - \int 4\,e^{2x}\sin(x)\,dx$

$= -e^{2x}\cos(x) + 2\,e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx$

Uit $\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2\,e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx$ volgt $5\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2\,e^{2x}\sin(x)$

dus $\int e^{2x}\sin(x)\,dx = -\frac{1}{5}e^{2x}\cos(x) + \frac{2}{5}e^{2x}\sin(x)$.

De primitieven zijn $H(x) = \frac{2}{5}e^{2x}\sin(x) - \frac{1}{5}e^{2x}\cos(x) + c = \frac{1}{5}e^{2x}(2\sin(x) - \cos(x)) + c$.

d $K(x) = \int \ln^2(x)\,dx = x\ln^2(x) - \int x\,d\ln^2(x) = x\ln^2(x) - \int x\cdot 2\ln(x)\cdot\frac{1}{x}dx = x\ln^2(x) - \int 2\ln(x)\,dx$

$= x\ln^2(x) - 2(x\ln(x) - x) + c = x\ln^2(x) - 2x\ln(x) + 2x + c$

Bladzijde 195

27 **a** $\int (x^2 - x)\,e^x\,dx = \int (x^2 - x)\,d\,e^x = (x^2 - x)\,e^x - \int e^x\,d(x^2 - x) = (x^2 - x)\,e^x - \int (2x - 1)\,e^x\,dx$

$= (x^2 - x)\,e^x - \int (2x - 1)\,d\,e^x = (x^2 - x)\,e^x - (2x - 1)\,e^x + \int e^x\,d(2x - 1)$

$= (x^2 - x)\,e^x - (2x - 1)\,e^x + \int 2\,e^x\,dx = (x^2 - x)\,e^x - (2x - 1)\,e^x + 2\,e^x + c$

$= (x^2 - 3x + 3)\,e^x + c$

$\int_1^3 (x^2 - x)\,e^x\,dx = \left[(x^2 - 3x + 3)\,e^x\right]_1^3 = (9 - 9 + 3)e^3 - (1 - 3 + 3)e^1 = 3e^3 - e$

b $\int x\ln^2(x)\,dx = \int \ln^2(x)\,d\tfrac{1}{2}x^2 = \tfrac{1}{2}x^2\ln^2(x) - \int \tfrac{1}{2}x^2\,d\ln^2(x) = \tfrac{1}{2}x^2\ln^2(x) - \int \tfrac{1}{2}x^2\cdot 2\ln(x)\cdot\frac{1}{x}dx$

$= \tfrac{1}{2}x^2\ln^2(x) - \int x\ln(x)\,dx = \tfrac{1}{2}x^2\ln^2(x) - \int \ln(x)\,d\tfrac{1}{2}x^2$

$= \tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \int \tfrac{1}{2}x^2\,d\ln(x) = \tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \int \tfrac{1}{2}x^2\cdot\frac{1}{x}dx$

$= \tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \int \tfrac{1}{2}x\,dx = \tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \tfrac{1}{4}x^2 + c$

$\int_1^e x\ln^2(x)\,dx = \left[\tfrac{1}{2}x^2\ln^2(x) - \tfrac{1}{2}x^2\ln(x) + \tfrac{1}{4}x^2\right]_1^e$

$= \tfrac{1}{2}e^2\ln^2(e) - \tfrac{1}{2}e^2\ln(e) + \tfrac{1}{4}e^2 - (\tfrac{1}{2}\ln^2(1) - \tfrac{1}{2}\ln(1) + \tfrac{1}{4}) = \tfrac{1}{4}e^2 - \tfrac{1}{4}$

28 **a** $f(x) = (2x^2 + x - 1)\,e^x$ geeft $f'(x) = (4x + 1)\cdot e^x + (2x^2 + x - 1)\cdot e^x = (2x^2 + 5x)\,e^x$

$f'(x) = 0$ geeft $(2x^2 + 5x)\,e^x = 0$

$2x^2 + 5x = 0$

$x(2x + 5) = 0$

$x = 0 \lor x = -2\tfrac{1}{2}$

max. is $f(-2\tfrac{1}{2}) = (2\cdot 6\tfrac{1}{4} - 2\tfrac{1}{2} - 1)e^{-2\frac{1}{2}} = \dfrac{9}{e^2\sqrt{e}}$

min. is $f(0) = -1$

b $f(x) = 0$ geeft $(2x^2 + x - 1)\,e^x = 0$

$2x^2 + x - 1 = 0$

$D = 1^2 - 4\cdot 2\cdot -1 = 9$

$x = \dfrac{-1 + 3}{4} = \tfrac{1}{2} \lor x = \dfrac{-1 - 3}{4} = -1$

$\int (2x^2 + x - 1)\,e^x\,dx = \int (2x^2 + x - 1)\,d\,e^x = (2x^2 + x - 1)\,e^x - \int e^x\,d(2x^2 + x - 1)$

$= (2x^2 + x - 1)\,e^x - \int (4x + 1)\,e^x\,dx = (2x^2 + x - 1)\,e^x - \int (4x + 1)\,d\,e^x$

$= (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + \int e^x\,d(4x + 1)$

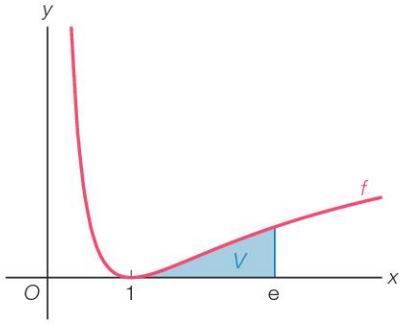
$= (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + \int 4\,e^x\,dx$

$= (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + 4\,e^x + c = (2x^2 - 3x + 2)\,e^x + c$

$\int_{-1}^{\frac{1}{2}} (2x^2 + x - 1)\,e^x\,dx = \left[(2x^2 - 3x + 2)\,e^x\right]_{-1}^{\frac{1}{2}} = (\tfrac{1}{2} - 1\tfrac{1}{2} + 2)e^{\frac{1}{2}} - (2 + 3 + 2)e^{-1} = \sqrt{e} - \dfrac{7}{e}$

Dus $O(V) = \dfrac{7}{e} - \sqrt{e}$.

29 **a** $f(x) = 0$ geeft $\ln^2(x) = 0$
$\ln(x) = 0$
$x = 1$



$$\int \frac{\ln^2(x)}{\sqrt{x}}\,dx = \int 2\ln^2(x) \cdot \frac{1}{2\sqrt{x}}\,dx = \int 2\ln^2(x)\,d\sqrt{x} = 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x}\,d\ln^2(x)$$
$$= 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x} \cdot 2\ln(x) \cdot \frac{1}{x}\,dx = 2\sqrt{x} \cdot \ln^2(x) - \int 4\ln(x) \cdot x^{-\frac{1}{2}}\,dx$$
$$= 2\sqrt{x} \cdot \ln^2(x) - \int 8\ln(x)\,dx^{\frac{1}{2}} = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8\sqrt{x}\,d\ln(x)$$
$$= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{\frac{1}{2}} \cdot \frac{1}{x}\,dx = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{-\frac{1}{2}}\,dx$$
$$= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16x^{\frac{1}{2}} + c = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x} + c$$

$$O(V) = \int_1^e \frac{\ln^2(x)}{\sqrt{x}}\,dx = \left[2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x}\right]_1^e$$
$$= 2\sqrt{e} \cdot \ln^2(e) - 8\sqrt{e} \cdot \ln(e) + 16\sqrt{e} - (2\ln^2(1) - 8\ln(1) + 16)$$
$$= 2\sqrt{e} - 8\sqrt{e} + 16\sqrt{e} - 16 = 10\sqrt{e} - 16$$

b $$I(L) = \pi\int_1^e \frac{\ln^4(x)}{x}\,dx = \pi\int_1^e \ln^4(x) \cdot \frac{1}{x}\,dx = \pi\int_{x=1}^{x=e} \ln^4(x)\,d\ln(x) = \pi\int_0^1 u^4\,du = \pi\left[\tfrac{1}{5}u^5\right]_0^1 = \tfrac{1}{5}\pi$$

K.3 Cyclometrische functies

Bladzijde 197

30 $$\int_0^1 \frac{1}{x^2+1}\,dx = \int_{t=0}^{t=\frac{1}{4}\pi} \frac{1}{\tan^2(t)+1}\,d\tan(t) = \int_0^{\frac{1}{4}\pi} \frac{1}{\tan^2(t)+1} \cdot (\tan^2(t)+1)\,dt = \int_0^{\frac{1}{4}\pi} 1\,dt = \left[t\right]_0^{\frac{1}{4}\pi} = \tfrac{1}{4}\pi$$

Bladzijde 199

31 **a** Het bereik van $f(x) = \arctan(x)$ is $\langle -\frac{1}{2}\pi, \frac{1}{2}\pi \rangle$.
$1\frac{1}{6}\pi$ zit niet in het bereik, dus $\arctan(\frac{1}{3}\sqrt{3}) \neq 1\frac{1}{6}\pi$.

b $\sqrt{3} > \frac{1}{2}\pi$, dus de vergelijking $\arctan(x) = \sqrt{3}$ heeft geen oplossing.

c Een primitieve van $f(ax+b)$ is $\frac{1}{a}F(ax+b)$.

32

x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
$\arctan(x)$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

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33 **a** $\arctan(x) = \frac{1}{3}\pi$
$x = \tan(\frac{1}{3}\pi)$
$x = \sqrt{3}$
b $\arctan(x - 2) = -\frac{1}{4}\pi$
$x - 2 = \tan(-\frac{1}{4}\pi)$
$x - 2 = -1$
$x = 1$
c $\arctan(x^2 - 1) = \frac{1}{4}\pi$
$x^2 - 1 = \tan(\frac{1}{4}\pi)$
$x^2 - 1 = 1$
$x^2 = 2$
$x = \sqrt{2} \vee x = -\sqrt{2}$
d $\arctan(x) = \frac{2}{3}\pi$
geen oplossing, want $\frac{2}{3}\pi > \frac{1}{2}\pi$
e $\arctan(x) = \sqrt{2}$
$x = \tan(\sqrt{2})$
$x \approx 6{,}334$
f $\arctan(x^2 - 1) = 1$
$x^2 - 1 = \tan(1)$
$x^2 - 1 = 1{,}557...$
$x^2 = 2{,}557...$
$x = \sqrt{2{,}557...} \vee x = -\sqrt{2{,}557...}$
$x \approx 1{,}599 \vee x = -1{,}599$

Bladzijde 200

34 **a** $f(x) = 2\arctan(\frac{1}{2}x)$ geeft $f'(x) = 2 \cdot \frac{1}{(\frac{1}{2}x)^2 + 1} \cdot \frac{1}{2} = \frac{1}{\frac{1}{4}x^2 + 1} = \frac{4}{x^2 + 4}$

b $g(x) = \arctan(x - 2)$ geeft $g'(x) = \frac{1}{(x-2)^2 + 1} = \frac{1}{x^2 - 4x + 4 + 1} = \frac{1}{x^2 - 4x + 5}$

c $h(x) = \arctan(x^2)$ geeft $h'(x) = \frac{1}{(x^2)^2 + 1} \cdot 2x = \frac{2x}{x^4 + 1}$

35 **a** $\int_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2+1}\,dx = \left[\arctan(x)\right]_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(-\frac{1}{3}\sqrt{3}) = \frac{1}{3}\pi - -\frac{1}{6}\pi = \frac{1}{2}\pi$

b $\int_{-2}^{-1} \frac{1}{(x+1)^2+1}\,dx = \left[\arctan(x+1)\right]_{-2}^{-1} = \arctan(0) - \arctan(-1) = 0 - -\frac{1}{4}\pi = \frac{1}{4}\pi$

c $\int_{0}^{\frac{1}{3}\sqrt{3}} \frac{3}{9x^2+1}\,dx = \int_{0}^{\frac{1}{3}\sqrt{3}} \frac{3}{(3x)^2+1}\,dx = \left[3 \cdot \frac{1}{3}\arctan(3x)\right]_{0}^{\frac{1}{3}\sqrt{3}} = \left[\arctan(3x)\right]_{0}^{\frac{1}{3}\sqrt{3}}$
$= \arctan(\sqrt{3}) - \arctan(0) = \frac{1}{3}\pi - 0 = \frac{1}{3}\pi$

36 **a** $\arctan(2x - 1) = \frac{1}{6}\pi$
$2x - 1 = \tan(\frac{1}{6}\pi)$
$2x - 1 = \frac{1}{3}\sqrt{3}$
$2x = 1 + \frac{1}{3}\sqrt{3}$
$x = \frac{1}{2} + \frac{1}{6}\sqrt{3}$
b $\arctan(1\frac{1}{2}\pi + \sqrt{x}) = \sqrt{2}$
$1\frac{1}{2}\pi + \sqrt{x} = \tan(\sqrt{2})$
$\sqrt{x} = \tan(\sqrt{2}) - 1\frac{1}{2}\pi$
$x = (\tan(\sqrt{2}) - 1\frac{1}{2}\pi)^2 \approx 2{,}63$
c $f(x) = \arctan(2x^2 + 1)$ geeft $f'(x) = \frac{1}{(2x^2+1)^2 + 1} \cdot 4x = \frac{4x}{4x^4 + 4x^2 + 1 + 1} = \frac{2x}{2x^4 + 2x^2 + 1}$

d $\int_{\frac{1}{5}}^{\frac{1}{5}\sqrt{3}} \frac{10}{25x^2+1}\,dx = \int_{\frac{1}{5}}^{\frac{1}{5}\sqrt{3}} \frac{10}{(5x)^2+1}\,dx = \left[10 \cdot \frac{1}{5}\arctan(5x)\right]_{\frac{1}{5}}^{\frac{1}{5}\sqrt{3}} = 2\arctan(\sqrt{3}) - 2\arctan(1) = 2 \cdot \frac{1}{3}\pi - 2 \cdot \frac{1}{4}\pi = \frac{1}{6}\pi$

37 **a** $f(x) = \frac{10}{x^2 + 4} = \frac{2\frac{1}{2}}{\frac{1}{4}x^2 + 1} = \frac{2\frac{1}{2}}{(\frac{1}{2}x)^2 + 1}$
Dus $a = 2\frac{1}{2}$ en $b = \frac{1}{2}$.
b $F(x) = 2\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \cdot \arctan(\frac{1}{2}x) + c = 5\arctan(\frac{1}{2}x) + c$

Bladzijde 201

38 **a** $F(x) = \int \frac{5}{32x^2+2}\,dx = \int \frac{2\frac{1}{2}}{16x^2+1}\,dx = \int \frac{2\frac{1}{2}}{(4x)^2+1}\,dx = 2\frac{1}{2} \cdot \frac{1}{4}\arctan(4x) + c = \frac{5}{8}\arctan(4x) + c$

b $G(x) = \int \frac{4}{x^2+4}\,dx = \int \frac{1}{\frac{1}{4}x^2+1}\,dx = \int \frac{1}{(\frac{1}{2}x)^2+1}\,dx = 2\arctan(\frac{1}{2}x) + c$

c $H(x) = \int \frac{5}{x^2+6x+10}\,dx = \int \frac{5}{(x+3)^2-9+10}\,dx = \int \frac{5}{(x+3)^2+1}\,dx = 5\arctan(x+3) + c$

d $J(x) = \int \frac{3}{x^2+4x+5}\,dx = \int \frac{3}{(x+2)^2-4+5}\,dx = \int \frac{3}{(x+2)^2+1}\,dx = 3\arctan(x+2) + c$

39 **a** $\int_0^1 \frac{3}{x^2+3}\,dx = \int_0^1 \frac{1}{\frac{1}{3}x^2+1}\,dx = \int_0^1 \frac{1}{\left(\frac{x}{\sqrt{3}}\right)^2+1}\,dx = \left[\sqrt{3} \cdot \arctan\left(\frac{x}{\sqrt{3}}\right)\right]_0^1$

$= \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan\left(\frac{0}{\sqrt{3}}\right) = \sqrt{3} \cdot \arctan(\frac{1}{3}\sqrt{3}) - \sqrt{3} \cdot \arctan(0)$

$= \sqrt{3} \cdot \frac{1}{6}\pi - 0 = \frac{1}{6}\pi\sqrt{3}$

b $\int_0^1 \frac{2x}{x^4+1}\,dx = \int_{x=0}^{x=1} \frac{1}{(x^2)^2+1}\,dx^2 = \left[\arctan(x^2)\right]_0^1 = \arctan(1) - \arctan(0) = \frac{1}{4}\pi$

c $\int_3^6 \frac{5}{x^2-6x+18}\,dx = \int_3^6 \frac{5}{(x-3)^2-9+18}\,dx = \int_3^6 \frac{5}{(x-3)^2+9}\,dx = \int_3^6 \frac{\frac{5}{9}}{\frac{1}{9}(x-3)^2+1}\,dx = \int_3^6 \frac{\frac{5}{9}}{(\frac{1}{3}(x-3))^2+1}\,dx$

$= \left[\frac{5}{9} \cdot 3\arctan(\frac{1}{3}(x-3))\right]_3^6 = \frac{5}{3}\arctan(1) - \frac{5}{3}\arctan(0) = \frac{5}{3} \cdot \frac{1}{4}\pi = \frac{5}{12}\pi$

d $\int_0^{\sqrt{3}} \frac{\arctan(x)}{x^2+1}\,dx = \int_0^{\sqrt{3}} \arctan(x) \cdot \frac{1}{x^2+1}\,dx = \int_{x=0}^{x=\sqrt{3}} \arctan(x)\,d\arctan(x) = \int_0^{\frac{1}{3}\pi} u\,du = \left[\frac{1}{2}u^2\right]_0^{\frac{1}{3}\pi}$

$= \frac{1}{2} \cdot \frac{1}{9}\pi^2 - 0 = \frac{1}{18}\pi^2$

Bladzijde 202

40 **a** $\int_0^{1\frac{1}{2}} \frac{2}{4x^2+9}\,dx = \int_0^{1\frac{1}{2}} \frac{\frac{2}{9}}{\frac{4}{9}x^2+1}\,dx = \int_0^{1\frac{1}{2}} \frac{\frac{2}{9}}{(\frac{2}{3}x)^2+1}\,dx = \left[\frac{2}{9} \cdot \frac{3}{2}\arctan(\frac{2}{3}x)\right]_0^{1\frac{1}{2}} = \frac{1}{3}\arctan(1) - \frac{1}{3}\arctan(0)$

$= \frac{1}{3} \cdot \frac{1}{4}\pi = \frac{1}{12}\pi$

b $\int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x}+1}\,dx = \int_{x=0}^{x=\frac{1}{2}\ln(3)} \frac{1}{(e^x)^2+1}\,de^x = \int_1^{\sqrt{3}} \frac{1}{u^2+1}\,du = \left[\arctan(u)\right]_1^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(1) = \frac{1}{3}\pi - \frac{1}{4}\pi = \frac{1}{12}\pi$

c $\int_0^1 \frac{1}{4x^2-4x+2}\,dx = \int_0^1 \frac{1}{(2x-1)^2-1+2}\,dx = \int_0^1 \frac{1}{(2x-1)^2+1}\,dx = \left[\frac{1}{2}\arctan(2x-1)\right]_0^1$

$= \frac{1}{2}\arctan(1) - \frac{1}{2}\arctan(-1) = \frac{1}{2} \cdot \frac{1}{4}\pi - \frac{1}{2} \cdot -\frac{1}{4}\pi = \frac{1}{4}\pi$

d $\int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{\cos^2(x)+1}\,dx = \int_{x=0}^{x=\frac{1}{2}\pi} -\frac{1}{\cos^2(x)+1}\,d\cos(x) = \int_1^0 -\frac{1}{u^2+1}\,du = \left[-\arctan(u)\right]_1^0$

$= -\arctan(0) + \arctan(1) = \frac{1}{4}\pi$

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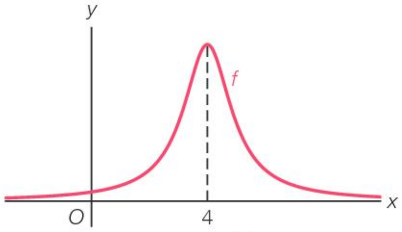
41 **a** $f(x)=\dfrac{10}{x^2-8x+17}$ geeft $f'(x)=\dfrac{(x^2-8x+17)\cdot 0-10(2x-8)}{(x^2-8x+17)^2}=\dfrac{-20x+80}{(x^2-8x+17)^2}$

$f'(x)=0$ geeft $-20x+80=0$

$-20x=-80$

$x=4$



max. is $f(4)=\dfrac{10}{16-32+17}=10$

b $O(V)=\displaystyle\int_4^5 \frac{10}{x^2-8x+17}\,dx=\int_4^5 \frac{10}{(x-4)^2-16+17}\,dx=\int_4^5\frac{10}{(x-4)^2+1}\,dx=\big[10\arctan(x-4)\big]_4^5$

$=10\arctan(1)-10\arctan(0)=10\cdot\frac{1}{4}\pi=2\frac{1}{2}\pi$

c $O(W_p)=10$ geeft $\displaystyle\int_4^p f(x)\,dx=10$

$\big[10\arctan(x-4)\big]_4^p=10$

$10\arctan(p-4)-10\arctan(0)=10$

$\arctan(p-4)=1$

$p-4=\tan(1)$

$p=4+\tan(1)$

42 $\displaystyle\int\frac{4}{x\ln^2(x)+x}\,dx=\int\frac{1}{x}\cdot\frac{4}{\ln^2(x)+1}\,dx=\int\frac{4}{\ln^2(x)+1}\,d\ln(x)=\int\frac{4}{u^2+1}\,du=4\arctan(u)+c=4\arctan(\ln(x))+c$

$O(V_p)=\displaystyle\int_1^p f(x)\,dx=\big[4\arctan(\ln(x))\big]_1^p=4\arctan(\ln(p))-4\arctan(0)=4\arctan(\ln(p))$

$O(V_p)=\pi$ geeft $4\arctan(\ln(p))=\pi$

$\arctan(\ln(p))=\frac{1}{4}\pi$

$\ln(p)=\tan(\frac{1}{4}\pi)$

$\ln(p)=1$

$p=\mathrm{e}$

43 $\displaystyle\int_0^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}\,dx=\int_{t=0}^{t=\frac{1}{6}\pi}\frac{1}{\sqrt{1-\sin^2(t)}}\,d\sin(t)=\int_0^{\frac{1}{6}\pi}\frac{1}{\sqrt{\cos^2(t)}}\cdot\cos(t)\,dt=\int_0^{\frac{1}{6}\pi}\frac{1}{\cos(t)}\cdot\cos(t)\,dt=\int_0^{\frac{1}{6}\pi}1\,dt=\big[t\big]_0^{\frac{1}{6}\pi}=\frac{1}{6}\pi$

Bladzijde 204

44

x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$

45 **a** $\arcsin(x)=\frac{1}{2}\pi$

$x=\sin(\frac{1}{2}\pi)$

$x=1$

b $\arcsin(x)=-\frac{1}{6}\pi$

$x=\sin(-\frac{1}{6}\pi)$

$x=-\frac{1}{2}$

c $\arcsin(x) = 2$

geen oplossing, want $2 > \frac{1}{2}\pi$

d $3\arcsin(x - \sqrt{3}) = \pi$

$\arcsin(x - \sqrt{3}) = \frac{1}{3}\pi$

$x - \sqrt{3} = \sin(\frac{1}{3}\pi)$

$x - \sqrt{3} = \frac{1}{2}\sqrt{3}$

$x = 1\frac{1}{2}\sqrt{3} \approx 2{,}598$

46 **a** $\int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-9x^2}}\,dx = \int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-(3x)^2}}\,dx = \left[\frac{1}{3}\arcsin(3x)\right]_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} = \frac{1}{3}\arcsin(\frac{1}{2}\sqrt{3}) - \frac{1}{3}\arcsin(\frac{1}{2})$

$= \frac{1}{3} \cdot \frac{1}{3}\pi - \frac{1}{3} \cdot \frac{1}{6}\pi = \frac{1}{9}\pi - \frac{1}{18}\pi = \frac{1}{18}\pi$

b $\int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{9-x^2}}\,dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}x^2}}\,dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-(\frac{1}{3}x)^2}}\,dx = \left[\arcsin(\frac{1}{3}x)\right]_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}}$

$= \arcsin(\frac{1}{3} \cdot 1\frac{1}{2}\sqrt{2}) - \arcsin(\frac{1}{3} \cdot -1\frac{1}{2}\sqrt{2}) = \arcsin(\frac{1}{2}\sqrt{2}) - \arcsin(-\frac{1}{2}\sqrt{2})$

$= \frac{1}{4}\pi - -\frac{1}{4}\pi = \frac{1}{2}\pi$

c $\int_{0}^{\frac{1}{2}\sqrt{2}} \frac{x}{\sqrt{1-x^4}}\,dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{2}} \frac{1}{2} \cdot \frac{1}{\sqrt{1-(x^2)^2}}\,dx^2 = \int_{0}^{\frac{1}{2}} \frac{1}{2} \cdot \frac{1}{\sqrt{1-u^2}}\,du = \left[\frac{1}{2}\arcsin(u)\right]_0^{\frac{1}{2}}$

$= \frac{1}{2}\arcsin(\frac{1}{2}) - \frac{1}{2}\arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

d $\int_{0}^{1} \frac{x}{\sqrt{4-x^4}}\,dx = \int_{0}^{1} \frac{\frac{1}{2}x}{\sqrt{1-\frac{1}{4}x^4}}\,dx = \int_{x=0}^{x=1} \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{2}x^2)^2}}\,d\frac{1}{2}x^2 = \int_{0}^{\frac{1}{2}} \frac{\frac{1}{2}}{\sqrt{1-u^2}}\,du = \left[\frac{1}{2}\arcsin(u)\right]_0^{\frac{1}{2}}$

$= \frac{1}{2}\arcsin(\frac{1}{2}) - \frac{1}{2}\arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

47 **a** $F(x) = \int \arctan(x)\,dx = x\arctan(x) - \int x\,d\arctan(x) = x\arctan(x) - \int x \cdot \frac{1}{x^2+1}\,dx$

$= x\arctan(x) - \int \frac{1}{2} \cdot \frac{1}{x^2+1}\,d(x^2+1) = x\arctan(x) - \frac{1}{2}\ln(x^2+1) + c$

b $G(x) = \int \arcsin(x)\,dx = x\arcsin(x) - \int x\,d\arcsin(x) = x\arcsin(x) - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$

$= x\arcsin(x) - \int -\frac{1}{2\sqrt{1-x^2}}\,d(1-x^2) = x\arcsin(x) + \sqrt{1-x^2} + c$

Bladzijde 205

48 **a** $F(x) = \int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \arcsin(x) \cdot \frac{1}{\sqrt{1-x^2}}\,dx = \int \arcsin(x)\,d\arcsin(x) = \frac{1}{2}\arcsin^2(x) + c$

b $G(x) = \int \frac{1}{x\sqrt{1-\ln^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} \cdot \frac{1}{x}\,dx = \int \frac{1}{\sqrt{1-\ln^2(x)}}\,d\ln(x) = \arcsin(\ln(x)) + c$

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49 **a** $25 - x^4 > 0$

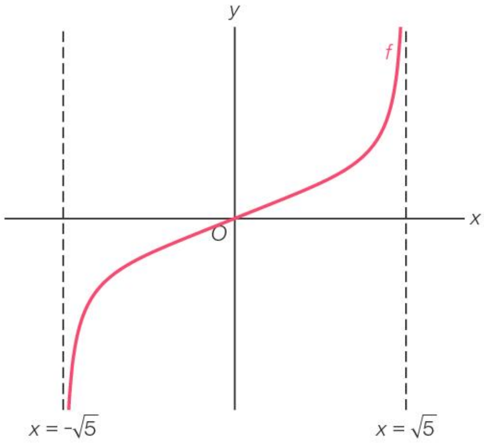
$x^4 < 25$

$-5 < x^2 < 5$

$x^2 < 5$

$-\sqrt{5} < x < \sqrt{5}$

Dus $D_f = \langle -\sqrt{5}, \sqrt{5} \rangle$.



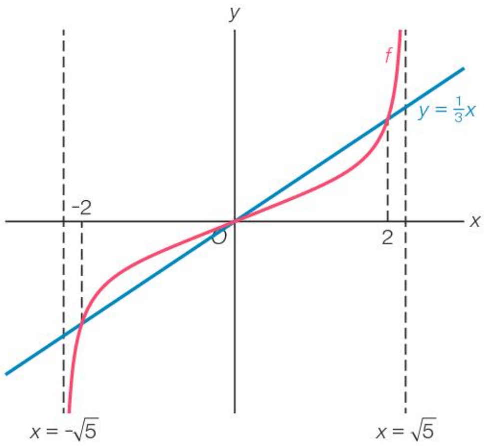
b $f(x) = \frac{1}{3}x$ geeft $\dfrac{x}{\sqrt{25 - x^4}} = \dfrac{x}{3}$

$x = 0 \vee \sqrt{25 - x^4} = 3$

$x = 0 \vee 25 - x^4 = 9$

$x = 0 \vee x^4 = 16$

$x = 0 \vee x = 2 \vee x = -2$



$f(x) \leq \frac{1}{3}x$ geeft $-\sqrt{5} < x \leq -2 \vee 0 \leq x \leq 2$

c $$O(V) = \int_0^{\frac{1}{2}\sqrt{10}} \frac{x}{\sqrt{25 - x^4}}\,dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1 - \frac{1}{25}x^4}}\,dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1 - (\frac{1}{5}x^2)^2}}\,dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{10}} \frac{\frac{1}{2}}{\sqrt{1 - (\frac{1}{5}x^2)^2}}\,d\tfrac{1}{5}x^2$$

$$= \int_0^{\frac{1}{2}} \frac{1}{2} \cdot \frac{1}{\sqrt{1 - u^2}}\,du = \left[\tfrac{1}{2}\arcsin(u)\right]_0^{\frac{1}{2}} = \tfrac{1}{2}\arcsin(\tfrac{1}{2}) - \tfrac{1}{2}\arcsin(0) = \tfrac{1}{2} \cdot \tfrac{1}{6}\pi = \tfrac{1}{12}\pi$$

K.4 Breuksplitsen

Bladzijde 207

50 **a** $F(x) = \int \dfrac{2x}{x^2 + 1}\,dx = \int \dfrac{1}{x^2 + 1}\,d(x^2 + 1) = \ln(x^2 + 1)$

$G(x) = \int \dfrac{1}{x^2 + 1}\,dx = \arctan(x)$

b Uitdelen geeft $h(x) = \dfrac{2x + 1}{x^2 + 1} = \dfrac{2x}{x^2 + 1} + \dfrac{1}{x^2 + 1} = f(x) + g(x)$.

Dus $H(x) = \ln(x^2 + 1) + \arctan(x) + c$.

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51 **a** Uitdelen geeft $f(x) = \frac{2x+5}{x+1} = \frac{2x}{x+1} + \frac{5}{x+1}$.

f is zo niet te primitiveren, want er is geen primitieve te geven van $\frac{2x}{x+1}$.

b $f(x) = 2 + \frac{3}{x+1} = \frac{2x+2}{x+1} + \frac{3}{x+1} = \frac{2x+2+3}{x+1} = \frac{2x+5}{x+1}$

Dus de splitsing is correct.

$F(x) = \int\left(2 + \frac{3}{x+1}\right)\mathrm{d}x = 2x + 3\ln|x+1| + c$

Bladzijde 209

52 **a** $x+1\ /\ 2x+1\ \backslash\ 2$

$\underline{2x+2}\ -$

-1

Dus $\frac{2x+1}{x+1} = 2 - \frac{1}{x+1}$.

$F(x) = \int\frac{2x+1}{x+1}\mathrm{d}x = \int\left(2 - \frac{1}{x+1}\right)\mathrm{d}x = 2x - \ln|x+1| + c$

b $x+1\ /\ x\quad \backslash\ 1$

$\underline{x+1}\ -$

-1

Dus $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.

$F(x) = \int\frac{x}{x+1}\mathrm{d}x = \int\left(1 - \frac{1}{x+1}\right)\mathrm{d}x = x - \ln|x+1| + c$

c $2x+1\ /\ x+1\ \backslash\ \frac{1}{2}$

$\underline{x+\frac{1}{2}}\ -$

$\frac{1}{2}$

Dus $\frac{x+1}{2x+1} = \frac{1}{2} + \frac{\frac{1}{2}}{2x+1}$.

$F(x) = \int\frac{x+1}{2x+1}\mathrm{d}x = \int\left(\frac{1}{2} + \frac{\frac{1}{2}}{2x+1}\right)\mathrm{d}x = \frac{1}{2}x + \frac{1}{4}\ln|2x+1| + c$

d $f(x) = \frac{2-x}{x+1} = \frac{-x+2}{x+1}$

$x+1\ /\ -x+2\ \backslash\ -1$

$\underline{-x-1}\ -$

3

Dus $\frac{2-x}{x+1} = -1 + \frac{3}{x+1}$.

$F(x) = \int\frac{2-x}{x+1}\mathrm{d}x = \int\left(-1 + \frac{3}{x+1}\right)\mathrm{d}x = -x + 3\ln|x+1| + c$

53 **a** $x-1\ /\ x^2-2x+3\ \backslash\ x-1$

$\underline{x^2-x}\ -$

$-x+3$

$\underline{-x+1}\ -$

2

Dus $\frac{x^2-2x+3}{x-1} = x - 1 + \frac{2}{x-1}$.

$\int_2^3\frac{x^2-2x+3}{x-1}\mathrm{d}x = \int_2^3\left(x - 1 + \frac{2}{x-1}\right)\mathrm{d}x = \left[\frac{1}{2}x^2 - x + 2\ln|x-1|\right]_2^3 = 4\frac{1}{2} - 3 + 2\ln(2) - (2 - 2 + 2\ln(1))$

$= 1\frac{1}{2} + 2\ln(2)$

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b $x+1\ /\ -2x^2-x\quad \backslash\ -2x+1$

$$\begin{array}{l} \underline{-2x^2-2x}\ - \\ \quad x \\ \quad \underline{x+1}\ - \\ \quad\ -1 \end{array}$$

Dus $\dfrac{-2x^2-x}{x+1}=-2x+1-\dfrac{1}{x+1}$.

$$\int_{-4}^{-2}\frac{-2x^2-x}{x+1}\,\mathrm{d}x=\int_{-4}^{-2}\left(-2x+1-\frac{1}{x+1}\right)\mathrm{d}x=\left[-x^2+x-\ln|x+1|\right]_{-4}^{-2}$$
$$=-4-2-\ln(1)-(-16-4-\ln(3))=14+\ln(3)$$

54 **a** $f(x)=\dfrac{3-4x}{2x+1}=\dfrac{-4x+3}{2x+1}$

$2x+1\ /\ -4x+3\ \backslash\ -2$

$$\begin{array}{l} \underline{-4x-2}\ - \\ \quad 5 \end{array}$$

Dus $\dfrac{3-4x}{2x+1}=-2+\dfrac{5}{2x+1}$.

$F(x)=\displaystyle\int\frac{3-4x}{2x+1}\,\mathrm{d}x=\int\left(-2+\frac{5}{2x+1}\right)\mathrm{d}x=-2x+2\tfrac{1}{2}\ln|2x+1|+c$

$g(x)=\dfrac{6x-1}{1-2x}=\dfrac{6x-1}{-2x+1}$

$-2x+1\ /\ 6x-1\ \backslash\ -3$

$$\begin{array}{l} \underline{6x-3}\ - \\ \quad 2 \end{array}$$

Dus $\dfrac{6x-1}{1-2x}=-3+\dfrac{2}{1-2x}$.

$G(x)=\displaystyle\int\frac{6x-1}{1-2x}\,\mathrm{d}x=\int\left(-3+\frac{2}{1-2x}\right)\mathrm{d}x=-3x-\ln|1-2x|+c$

b $2x-1\ /\ 2x^2-5x\quad \backslash\ x-2$

$$\begin{array}{l} \underline{2x^2-x}\ - \\ \quad -4x \\ \quad \underline{-4x+2}\ - \\ \quad\quad -2 \end{array}$$

Dus $\dfrac{2x^2-5x}{2x-1}=x-2-\dfrac{2}{2x-1}$.

$$\int_3^6\frac{2x^2-5x}{2x-1}\,\mathrm{d}x=\int_3^6\left(x-2-\frac{2}{2x-1}\right)\mathrm{d}x=\left[\tfrac{1}{2}x^2-2x-\ln|2x-1|\right]_3^6=18-12-\ln(11)-(4\tfrac{1}{2}-6-\ln(5))$$
$$=6-\ln(11)+1\tfrac{1}{2}+\ln(5)=7\tfrac{1}{2}+\ln(\tfrac{5}{11})$$

55 **a** $f(x)=\dfrac{x^3+x}{x+1}$ geeft

$$f'(x)=\frac{(x+1)\cdot(3x^2+1)-(x^3+x)\cdot 1}{(x+1)^2}=\frac{3x^3+x+3x^2+1-x^3-x}{(x+1)^2}=\frac{2x^3+3x^2+1}{(x+1)^2}$$

$f'(0)=1$, dus $k{:}\ y=x$.

$f(x)=x$ geeft $\dfrac{x^3+x}{x+1}=x$

$$x\cdot\frac{x^2+1}{x+1}=x$$
$$x=0\lor\frac{x^2+1}{x+1}=1$$
$$x=0\lor x^2+1=x+1$$
$$x=0\lor x^2=x$$
$$x=0\lor x=0\lor x=1$$

$f(1)=1$, dus $A(1,1)$.

b $x+1\,/\,x^3 \quad\quad +x \quad\quad \backslash\, x^2-x+2$

$\quad\underline{x^3+x^2}\,-$

$\quad\quad -x^2+x$

$\quad\quad \underline{-x^2-x}\,-$

$\quad\quad\quad 2x$

$\quad\quad\quad \underline{2x+2}\,-$

$\quad\quad\quad\quad -2$

Dus $\dfrac{x^3+x}{x+1}=x^2-x+2-\dfrac{2}{x+1}$.

$$O(V)=\int_0^2 \frac{x^3+x}{x+1}\,\mathrm{d}x=\int_0^2\left(x^2-x+2-\frac{2}{x+1}\right)\mathrm{d}x=\left[\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+2x-2\ln|x+1|\right]_0^2$$

$$=\tfrac{8}{3}-2+4-2\ln(3)-(0-0+0-2\ln(1))=4\tfrac{2}{3}-2\ln(3)$$

56 **a** $f(x)=\dfrac{3x+1}{x+1}=\dfrac{3(x+1)-3+1}{x+1}=3-\dfrac{2}{x+1}$

Dus $F(x)=3x-2\ln|x+1|+c$.

b $g(x)=\dfrac{4x}{2x-1}=\dfrac{2(2x-1)+2}{2x-1}=2+\dfrac{2}{2x-1}$

Dus $G(x)=2x+\ln|2x-1|+c$.

Bladzijde 210

57 **a** $f(x)$ is voor elke x gedefinieerd, want de noemer x^2+4x+5 heeft geen nulpunten.
$x^2+4x+4=(x+2)^2$ heeft één nulpunt, namelijk -2, dus de grafiek van g heeft één verticale asymptoot.
$x^2+4x+3=(x+1)(x+3)$ heeft twee nulpunten, namelijk -1 en -3, dus de grafiek van h heeft twee verticale asymptoten.

b $x^2+4x+5=(x+2)^2-4+5=(x+2)^2+1$

c $\displaystyle\int\frac{2x+4}{x^2+4x+5}\,\mathrm{d}x=\int\frac{1}{x^2+4x+5}\,\mathrm{d}(x^2+4x+5)=\ln(x^2+4x+5)+c$

$\displaystyle\int\frac{c}{(x+2)^2+1}\,\mathrm{d}x=c\arctan(x+2)+d$

d $\displaystyle\int\frac{a}{x+2}\,\mathrm{d}x=a\ln|x+2|+c$

$\displaystyle\int\frac{b}{(x+2)^2}\,\mathrm{d}x=\int b(x+2)^{-2}\,\mathrm{d}x=-b(x+2)^{-1}+c=\frac{-b}{x+2}+c$

e $\displaystyle\int\frac{a}{x+1}\,\mathrm{d}x=a\ln|x+1|+c$

$\displaystyle\int\frac{b}{x+3}\,\mathrm{d}x=b\ln|x+3|+c$

Bladzijde 213

58 $p(x)=x^2-3x+5$, $q(x)=1$, $l(x)=x+1$, $b=-4$ en $c=4$.

59 **a** $x^2+1\,/\,x^2+x \quad \backslash\, 1$

$\quad\underline{x^2 \quad +1}\,-$

$\quad\quad x-1$

$f(x)=\dfrac{x^2+x}{x^2+1}=1+\dfrac{x-1}{x^2+1}=1+\dfrac{\frac{1}{2}\cdot 2x-1}{x^2+1}=1+\dfrac{\frac{1}{2}\cdot 2x}{x^2+1}-\dfrac{1}{x^2+1}$

$\displaystyle\int\frac{\frac{1}{2}\cdot 2x}{x^2+1}\,\mathrm{d}x=\int\frac{\frac{1}{2}}{x^2+1}\,\mathrm{d}(x^2+1)=\tfrac{1}{2}\ln(x^2+1)+c$

Dus $F(x)=x+\tfrac{1}{2}\ln(x^2+1)-\arctan(x)+c$.

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b $x^2-6x+9\,/\,x^2-6x+8\,\backslash\,1$

$$\begin{array}{l} \underline{x^2-6x+9}\,- \\ \quad\quad\quad -1 \end{array}$$

$f(x)=\dfrac{x^2-6x+8}{x^2-6x+9}=1-\dfrac{1}{x^2-6x+9}=1-\dfrac{1}{(x-3)^2}=1-(x-3)^{-2}$ geeft

$F(x)=x+(x-3)^{-1}+c=x+\dfrac{1}{x-3}+c$

c $x^2+2x-3\,/\,x^3\quad\quad\quad\quad\backslash\,x-2$

$$\begin{array}{l} \underline{x^3+2x^2-3x}\,- \\ \quad -2x^2+3x \\ \quad \underline{-2x^2-4x+6}\,- \\ \quad\quad\quad 7x-6 \end{array}$$

$f(x)=\dfrac{x^3}{x^2+2x-3}=x-2+\dfrac{7x-6}{x^2+2x-3}=x-2+\dfrac{7x-6}{(x-1)(x+3)}=x-2+\dfrac{a}{x-1}+\dfrac{b}{x+3}$

$=x-2+\dfrac{a(x+3)+b(x-1)}{(x-1)(x+3)}=x-2+\dfrac{ax+3a+bx-b}{(x-1)(x+3)}=x-2+\dfrac{(a+b)x+3a-b}{(x-1)(x+3)}$

$$\begin{cases} a+b=7 \\ 3a-b=-6 \end{cases}\,+$$

$4a=1$

$\left.\begin{array}{l} a=\frac{1}{4} \\ a+b=7 \end{array}\right\}\begin{array}{l} \frac{1}{4}+b=7 \\ b=6\frac{3}{4} \end{array}$

Dus $f(x)=x-2+\dfrac{\frac{1}{4}}{x-1}+\dfrac{6\frac{3}{4}}{x+3}$.

Dit geeft $F(x)=\frac{1}{2}x^2-2x+\frac{1}{4}\ln|x-1|+6\frac{3}{4}\ln|x+3|+c$.

60 **a** $f(x)=\dfrac{x^3+x}{x^2+1}=\dfrac{x(x^2+1)}{x^2+1}=x$ geeft $F(x)=\frac{1}{2}x^2+c$

b $x^2-6x+9\,/\,x^3\quad\quad +x\quad\quad\backslash\,x+6$

$$\begin{array}{l} \underline{x^3-6x^2+9x}\,- \\ \quad 6x^2-8x \\ \quad \underline{6x^2-36x+54}\,- \\ \quad\quad 28x-54 \end{array}$$

$f(x)=\dfrac{x^3+x}{x^2-6x+9}=x+6+\dfrac{28x-54}{x^2-6x+9}=x+6+\dfrac{28(x-3)+84-54}{(x-3)^2}=x+6+\dfrac{28}{x-3}+\dfrac{30}{(x-3)^2}$

$=x+6+\dfrac{28}{x-3}+30(x-3)^{-2}$ geeft

$F(x)=\frac{1}{2}x^2+6x+28\ln|x-3|-30(x-3)^{-1}+c=\frac{1}{2}x^2+6x+28\ln|x-3|-\dfrac{30}{x-3}+c$

c $2x^2+x\,/\,2x^3+\quad\quad 1\,\backslash\,x-\frac{1}{2}$

$$\begin{array}{l} \underline{2x^3+x^2}\,- \\ \quad -x^2 \\ \quad \underline{-x^2-\frac{1}{2}x}\,- \\ \quad\quad \frac{1}{2}x+1 \end{array}$$

$f(x)=\dfrac{2x^3+1}{2x^2+x}=x-\frac{1}{2}+\dfrac{\frac{1}{2}x+1}{2x^2+x}=x-\frac{1}{2}+\dfrac{\frac{1}{2}x+1}{x(2x+1)}=x-\frac{1}{2}+\dfrac{a}{x}+\dfrac{b}{2x+1}$

$=x-\frac{1}{2}+\dfrac{a(2x+1)+bx}{x(2x+1)}=x-\frac{1}{2}+\dfrac{2ax+a+bx}{x(2x+1)}=x-\frac{1}{2}+\dfrac{(2a+b)x+a}{x(2x+1)}$

$\left.\begin{array}{l} 2a+b=\frac{1}{2} \\ a=1 \end{array}\right\}\begin{array}{l} 2+b=\frac{1}{2} \\ b=-1\frac{1}{2} \end{array}$

Dus $f(x)=x-\frac{1}{2}+\dfrac{1}{x}-\dfrac{1\frac{1}{2}}{2x+1}$.

Dit geeft $F(x)=\frac{1}{2}x^2-\frac{1}{2}x+\ln|x|-\frac{3}{4}\ln|2x+1|+c$.

61 **a** $x^2 + 4 \,/\, x^3 \qquad \backslash\, x$

$\underline{x^3 + 4x} \; -$

$-4x$

Dus $\dfrac{x^3}{x^2+4} = x - \dfrac{4x}{x^2+4}$.

$$\int \frac{4x}{x^2+4}\,dx = \int \frac{2}{x^2+4}\,d(x^2+4) = 2\ln(x^2+4) + c$$

Dus $\displaystyle\int_0^2 \frac{x^3}{x^2+4}\,dx = \int_0^2 \left(x - \frac{4x}{x^2+4}\right)dx = \left[\tfrac{1}{2}x^2 - 2\ln(x^2+4)\right]_0^2 = 2 - 2\ln(8) - (0 - 2\ln(4))$

$= 2 - 2\ln(8) + 2\ln(4) = 2 - 2(\ln(8) - \ln(4)) = 2 - 2\ln(2)$

b $x^2 + 4x + 4 \,/\, x^3 \qquad \backslash\, x - 4$

$\underline{x^3 + 4x^2 + 4x} \; -$

$-4x^2 - 4x$

$\underline{-4x^2 - 16x - 16} \; -$

$12x + 16$

Dus $\dfrac{x^3}{x^2+4x+4} = x - 4 + \dfrac{12x+16}{x^2+4x+4} = x - 4 + \dfrac{12x+16}{(x+2)^2} = x - 4 + \dfrac{12(x+2) - 8}{(x+2)^2}$

$= x - 4 + \dfrac{12}{x+2} - \dfrac{8}{(x+2)^2} = x - 4 + \dfrac{12}{x+2} - 8(x+2)^{-2}$.

$\displaystyle\int_0^8 \frac{x^3}{x^2+4x+4}\,dx = \int_0^8 \left(x - 4 + \frac{12}{x+2} - 8(x+2)^{-2}\right)dx = \left[\tfrac{1}{2}x^2 - 4x + 12\ln|x+2| + 8(x+2)^{-1}\right]_0^8$

$= \left[\tfrac{1}{2}x^2 - 4x + 12\ln|x+2| + \dfrac{8}{x+2}\right]_0^8$

$= 32 - 32 + 12\ln(10) + \tfrac{4}{5} - (0 - 0 + 12\ln(2) + 4) = 12\ln(10) + \tfrac{4}{5} - 12\ln(2) - 4$

$= 12\ln(5) - 3\tfrac{1}{5}$

c $\dfrac{4x-8}{x^2-4x-5} = \dfrac{4x-8}{(x+1)(x-5)} = \dfrac{a}{x+1} + \dfrac{b}{x-5} = \dfrac{a(x-5) + b(x+1)}{(x+1)(x-5)} = \dfrac{ax - 5a + bx + b}{(x+1)(x-5)}$

$= \dfrac{(a+b)x - 5a + b}{(x+1)(x-5)}$

$\begin{cases} a + b = 4 \\ -5a + b = -8 \end{cases} \; -$

$6a = 12$

$\left.\begin{array}{l} a = 2 \\ a + b = 4 \end{array}\right\} \begin{array}{l} 2 + b = 4 \\ b = 2 \end{array}$

$\displaystyle\int_0^2 \frac{4x-8}{x^2-4x-5}\,dx = \int_0^2 \left(\frac{2}{x+1} + \frac{2}{x-5}\right)dx = \left[2\ln|x+1| + 2\ln|x-5|\right]_0^2$

$= 2\ln(3) + 2\ln(3) - (2\ln(1) + 2\ln(5)) = 4\ln(3) - 2\ln(5)$

62 **a** $x^2 + 1 \,/\, x^4 \qquad + 1 \,\backslash\, x^2 - 1$

$\underline{x^4 + x^2} \; -$

$-x^2$

$\underline{-x^2 - 1} \; -$

2

$\displaystyle\int_0^1 \frac{x^4+1}{x^2+1}\,dx = \int_0^1 \left(x^2 - 1 + \frac{2}{x^2+1}\right)dx = \left[\tfrac{1}{3}x^3 - x + 2\arctan(x)\right]_0^1$

$= \tfrac{1}{3} - 1 + 2\arctan(1) - (0 - 0 + 2\arctan(0)) = -\tfrac{2}{3} + 2 \cdot \tfrac{1}{4}\pi = -\tfrac{2}{3} + \tfrac{1}{2}\pi$

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b $x^2 + 2x + 1 \,/\, x^4 \qquad\qquad\qquad + 1 \,\backslash\, x^2 - 2x + 3$

$$\begin{array}{l} \underline{x^4 + 2x^3 + x^2} - \\ \quad -2x^3 - x^2 \\ \quad \underline{-2x^3 - 4x^2 - 2x} - \\ \qquad\quad 3x^2 + 2x \\ \qquad\quad \underline{3x^2 + 6x + 3} - \\ \qquad\qquad -4x - 2 \end{array}$$

Dus $\dfrac{x^4+1}{x^2+2x+1} = x^2 - 2x + 3 - \dfrac{4x+2}{x^2+2x+1} = x^2 - 2x + 3 - \dfrac{4x+2}{(x+1)^2} = x^2 - 2x + 3 - \dfrac{4(x+1)-2}{(x+1)^2}$

$= x^2 - 2x + 3 - \dfrac{4}{x+1} + \dfrac{2}{(x+1)^2} = x^2 - 2x + 3 - \dfrac{4}{x+1} + 2(x+1)^{-2}$

$\displaystyle\int_0^1 \frac{x^4+1}{x^2+2x+1}\,\mathrm{d}x = \int_0^1 \left(x^2 - 2x + 3 - \frac{4}{x+1} + 2(x+1)^{-2}\right)\mathrm{d}x$

$= \left[\frac{1}{3}x^3 - x^2 + 3x - 4\ln|x+1| - 2(x+1)^{-1}\right]_0^1 = \left[\frac{1}{3}x^3 - x^2 + 3x - 4\ln|x+1| - \dfrac{2}{x+1}\right]_0^1$

$= \frac{1}{3} - 1 + 3 - 4\ln(2) - 1 - (0 - 0 + 0 - 4\ln(1) - 2) = 1\frac{1}{3} - 4\ln(2) + 2 = 3\frac{1}{3} - 4\ln(2)$

c $x^2 + 4x + 3 \,/\, x^4 \qquad\qquad\qquad \backslash\, x^2 - 4x + 13$

$$\begin{array}{l} \underline{x^4 + 4x^3 + 3x^2} - \\ \quad -4x^3 - 3x^2 \\ \quad \underline{-4x^3 - 16x^2 - 12x} - \\ \qquad\quad 13x^2 + 12x \\ \qquad\quad \underline{13x^2 + 52x + 39} - \\ \qquad\qquad -40x - 39 \end{array}$$

$\dfrac{-40x-39}{x^2+4x+3} = \dfrac{-40x-39}{(x+1)(x+3)} = \dfrac{a}{x+1} + \dfrac{b}{x+3} = \dfrac{a(x+3)+b(x+1)}{(x+1)(x+3)} = \dfrac{ax+3a+bx+b}{(x+1)(x+3)}$

$= \dfrac{(a+b)x+3a+b}{(x+1)(x+3)}$

$\begin{cases} a + b = -40 \\ 3a + b = -39 \end{cases} -$

$-2a = -1$

$\left.\begin{array}{l} a = \frac{1}{2} \\ a + b = -40 \end{array}\right\} \begin{array}{l} \frac{1}{2} + b = -40 \\ b = -40\frac{1}{2} \end{array}$

$\displaystyle\int_0^2 \frac{x^4}{x^2+4x+3}\,\mathrm{d}x = \int_0^2 \left(x^2 - 4x + 13 + \frac{\frac{1}{2}}{x+1} - \frac{40\frac{1}{2}}{x+3}\right)\mathrm{d}x$

$= \left[\frac{1}{3}x^3 - 2x^2 + 13x + \frac{1}{2}\ln|x+1| - 40\frac{1}{2}\ln|x+3|\right]_0^2$

$= \frac{8}{3} - 8 + 26 + \frac{1}{2}\ln(3) - 40\frac{1}{2}\ln(5) - (0 - 0 + 0 + \frac{1}{2}\ln(1) - 40\frac{1}{2}\ln(3))$

$= 20\frac{2}{3} + \frac{1}{2}\ln(3) - 40\frac{1}{2}\ln(5) + 40\frac{1}{2}\ln(3) = 20\frac{2}{3} + 41\ln(3) - 40\frac{1}{2}\ln(5)$

63 I $\dfrac{2x-5}{x^2-5x+6} = \dfrac{2x-5}{(x-2)(x-3)} = \dfrac{a}{x-2} + \dfrac{b}{x-3} = \dfrac{a(x-3)+b(x-2)}{(x-2)(x-3)} = \dfrac{ax-3a+bx-2b}{(x-2)(x-3)}$

$= \dfrac{(a+b)x-3a-2b}{(x-2)(x-3)}$

$\begin{cases} a + b = 2 \\ -3a - 2b = -5 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 2a + 2b = 4 \\ -3a - 2b = -5 \end{cases} +$

$-a = -1$

$\left.\begin{array}{l} a = 1 \\ a + b = 2 \end{array}\right\} \begin{array}{l} 1 + b = 2 \\ b = 1 \end{array}$

$f(x) = \dfrac{1}{x-2} + \dfrac{1}{x-3}$ geeft $F(x) = \ln|x-2| + \ln|x-3| + c$

II $F(x) = \displaystyle\int \frac{2x-5}{x^2-5x+6}\,\mathrm{d}x = \int \frac{1}{x^2-5x+6}\,\mathrm{d}(x^2-5x+6) = \ln|x^2-5x+6| + c$

$\ln|x-2| + \ln|x-3| = \ln|(x-2)(x-3)| = \ln|x^2-5x+6|$

Dus de primitieven komen op hetzelfde neer.

64 **a** $f(x) = \dfrac{10x+5}{4x^2-4x+1}$ geeft

$$f'(x) = \frac{(4x^2-4x+1)\cdot 10 - (10x+5)\cdot(8x-4)}{(4x^2-4x+1)^2} = \frac{40x^2-40x+10-(80x^2-40x+40x-20)}{(2x-1)^4}$$

$$= \frac{40x^2-40x+10-80x^2+20}{(2x-1)^4} = \frac{-40x^2-40x+30}{(2x-1)^4}$$

$f'(x) = 0$ geeft $-40x^2 - 40x + 30 = 0$

$4x^2 + 4x - 3 = 0$

$D = 4^2 - 4 \cdot 4 \cdot -3 = 64$

$x = \dfrac{-4+8}{8} = \frac{1}{2} \vee x = \dfrac{-4-8}{8} = -1\frac{1}{2}$

vold. niet

min. is $f(-1\frac{1}{2}) = \dfrac{-15+5}{9+6+1} = \dfrac{-10}{16} = -\frac{5}{8}$

Dus $B_f = \left[-\frac{5}{8}, \rightarrow\right\rangle$.

b $O(V) = \displaystyle\int_1^3 \frac{10x+5}{4x^2-4x+1}\,dx = \int_1^3 \frac{5(2x-1)+10}{(2x-1)^2}\,dx = \int_1^3 \left(\frac{5}{2x-1} + \frac{10}{(2x-1)^2}\right)dx$

$$= \int_1^3 \left(\frac{5}{2x-1} + 10(2x-1)^{-2}\right)dx = \left[\tfrac{5}{2}\ln|2x-1| - 5(2x-1)^{-1}\right]_1^3 = \left[2\tfrac{1}{2}\ln|2x-1| - \frac{5}{2x-1}\right]_1^3$$

$$= 2\tfrac{1}{2}\ln(5) - \frac{5}{5} - \left(2\tfrac{1}{2}\ln(1) - \frac{5}{1}\right) = 2\tfrac{1}{2}\ln(5) - 1 + 5 = 4 + 2\tfrac{1}{2}\ln(5)$$

Bladzijde 214

65 **a** $f(x) = \dfrac{x^2+4}{x^2+5x+4}$ geeft

$$f'(x) = \frac{(x^2+5x+4)\cdot 2x - (x^2+4)\cdot(2x+5)}{(x^2+5x+4)^2} = \frac{2x^3+10x^2+8x-(2x^3+5x^2+8x+20)}{(x^2+5x+4)^2}$$

$$= \frac{2x^3+10x^2+8x-2x^3-5x^2-8x-20}{(x^2+5x+4)^2} = \frac{5x^2-20}{(x^2+5x+4)^2}$$

$f'(x) = 0$ geeft $5x^2 - 20 = 0$

$5x^2 = 20$

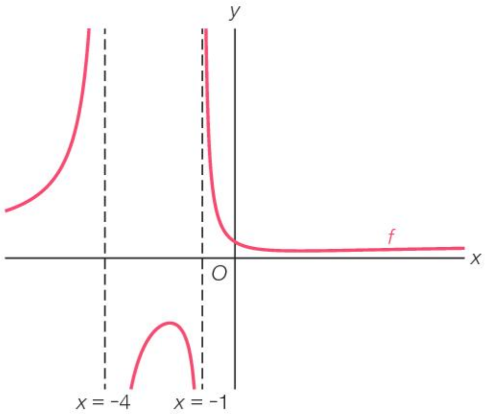
$x^2 = 4$

$x = 2 \vee x = -2$

$x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

$x = -1 \vee x = -4$



max. is $f(-2) = -4$

min. is $f(2) = \frac{4}{9}$

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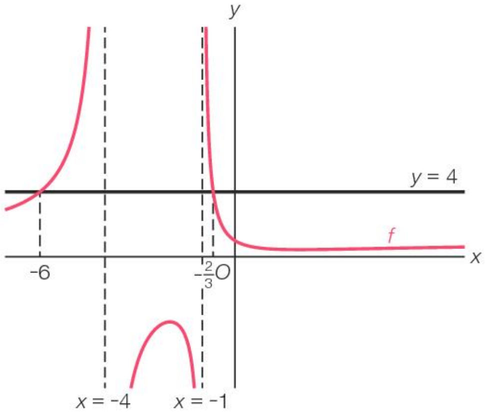
b $f(x) = 4$ geeft $\frac{x^2+4}{x^2+5x+4} = 4$

$4x^2 + 20x + 16 = x^2 + 4$

$3x^2 + 20x + 12 = 0$

$D = 20^2 - 4 \cdot 3 \cdot 12 = 256$

$x = \frac{-20+16}{6} = -\frac{2}{3} \vee x = \frac{-20-16}{6} = -6$



$f(x) \leq 4$ geeft $x \leq -6 \vee -4 < x < -1 \vee x \geq -\frac{2}{3}$

c $x^2 + 5x + 4 \,/\, x^2 \qquad + 4 \,\backslash\, 1$

$\underline{x^2 + 5x + 4} \; -$

$-5x$

$f(x) = 1 - \frac{5x}{x^2+5x+4} = 1 - \frac{5x}{(x+1)(x+4)} = 1 - \left(\frac{a}{x+1} + \frac{b}{x+4}\right) = 1 - \frac{a(x+4)+b(x+1)}{(x+1)(x+4)}$

$= 1 - \frac{ax + 4a + bx + b}{(x+1)(x+4)} = 1 - \frac{(a+b)x + 4a + b}{(x+1)(x+4)}$

$\begin{cases} a + b = 5 \\ 4a + b = 0 \end{cases} \; -$

$-3a = 5$

$\left.\begin{array}{l} a = -1\frac{2}{3} \\ a + b = 5 \end{array}\right\} -1\frac{2}{3} + b = 5$

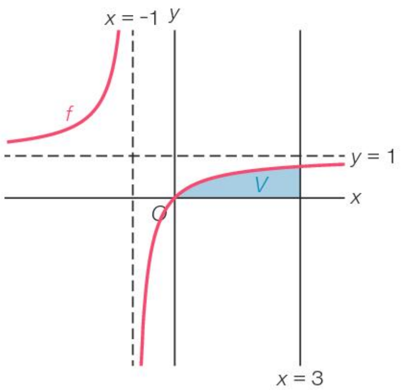
$b = 6\frac{2}{3}$

$O(V) = \int_0^6 \left(1 - \frac{5x}{x^2+5x+4}\right) dx = \int_0^6 \left(1 + \frac{1\frac{2}{3}}{x+1} - \frac{6\frac{2}{3}}{x+4}\right) dx = \left[x + 1\frac{2}{3}\ln|x+1| - 6\frac{2}{3}\ln|x+4|\right]_0^6$

$= 6 + 1\frac{2}{3}\ln(7) - 6\frac{2}{3}\ln(10) - (0 + 1\frac{2}{3}\ln(1) - 6\frac{2}{3}\ln(4)) = 6 + 1\frac{2}{3}\ln(7) - 6\frac{2}{3}\ln(10) + 6\frac{2}{3}\ln(4)$

$= 6 + 1\frac{2}{3}\ln(7) + 6\frac{2}{3}\ln(\frac{2}{5})$

66 **a**



$f(x) = \frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$

$O(V) = \int_0^3 \left(1 - \frac{1}{x+1}\right) dx = \left[x - \ln|x+1|\right]_0^3 = 3 - \ln(4) - (0 - \ln(1)) = 3 - \ln(4)$

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b $I(L) = \pi\int_0^3 \left(\frac{x}{x+1}\right)^2 dx = \pi\int_0^3 \left(1 - \frac{1}{x+1}\right)^2 dx = \pi\int_0^3 \left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right) dx$

$= \pi\int_0^3 \left(1 - \frac{2}{x+1} + (x+1)^{-2}\right) dx = \pi\left[x - 2\ln|x+1| - (x+1)^{-1}\right]_0^3 = \pi\left[x - 2\ln|x+1| - \frac{1}{x+1}\right]_0^3$

$= \pi(3 - 2\ln(4) - \frac{1}{4} - (0 - 2\ln(1) - 1)) = \pi(3 - 2\ln(4) - \frac{1}{4} + 1) = 3\frac{3}{4}\pi - 2\pi\ln(4)$

67 **a** $f(x) = \ln(x^2+1)$ geeft $f'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$f'(x) = \frac{3}{5}$ geeft $\frac{2x}{x^2+1} = \frac{3}{5}$

$3x^2 + 3 = 10x$

$3x^2 - 10x + 3 = 0$

$D = (-10)^2 - 4 \cdot 3 \cdot 3 = 64$

$x = \frac{10+8}{6} = 3 \vee x = \frac{10-8}{6} = \frac{1}{3}$

$f(3) = \ln(10)$ en $f(\frac{1}{3}) = \ln(1\frac{1}{9})$

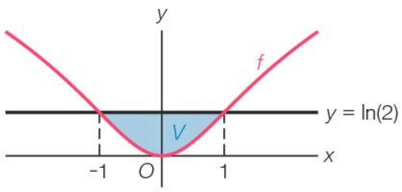
Dus de raakpunten zijn $(3, \ln(10))$ en $(\frac{1}{3}, \ln(1\frac{1}{9}))$.

b $f(x) = \ln(2)$ geeft $\ln(x^2+1) = \ln(2)$

$x^2 + 1 = 2$

$x^2 = 1$

$x = 1 \vee x = -1$



$\int f(x)\,dx = \int \ln(x^2+1)\,dx = x\ln(x^2+1) - \int x\,d\ln(x^2+1) = x\ln(x^2+1) - \int x \cdot \frac{1}{x^2+1} \cdot 2x\,dx$

$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1} dx = x\ln(x^2+1) - \int \frac{2x^2+2-2}{x^2+1} dx$

$= x\ln(x^2+1) - \int \left(2 - \frac{2}{x^2+1}\right) dx = x\ln(x^2+1) - (2x - 2\arctan(x)) + c$

$= x\ln(x^2+1) - 2x + 2\arctan(x) + c$

$O(V) = \int_{-1}^{1} (\ln(2) - f(x))\,dx = \left[x\ln(2) - (x\ln(x^2+1) - 2x + 2\arctan(x))\right]_{-1}^{1}$

$= \left[x\ln(2) - x\ln(x^2+1) + 2x - 2\arctan(x)\right]_{-1}^{1}$

$= \ln(2) - \ln(2) + 2 - 2\arctan(1) - (-\ln(2) + \ln(2) - 2 - 2\arctan(-1))$

$= 2 - 2 \cdot \frac{1}{4}\pi + 2 + 2 \cdot -\frac{1}{4}\pi = 4 - \pi$

K.5 Integralen bij parameterkrommen

Bladzijde 216

68 **a** $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a) = -(F(a) - F(b)) = -\left[F(x)\right]_b^a = -\int_b^a f(x)\,dx$

b $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \left[F(x)\right]_a^b + \left[F(x)\right]_b^c = F(b) - F(a) + F(c) - F(b) = F(c) - F(a)$

$= \left[F(x)\right]_a^c = \int_a^c f(x)\,dx$

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69 **a** Bij A hoort $t = 0$, bij B hoort $t = 2$ en bij C hoort $t = 4$, dus

$$O(V) = \int_{x_A}^{x_B} y\,\mathrm{d}x - \int_{x_C}^{x_B} y\,\mathrm{d}x = \int_{t=0}^{t=2} y\,\mathrm{d}x - \int_{t=4}^{t=2} y\,\mathrm{d}x.$$

Herleiden geeft $O(V) = \int_{t=0}^{t=2} y\,\mathrm{d}x - \int_{t=4}^{t=2} y\,\mathrm{d}x = \int_{t=0}^{t=2} y\,\mathrm{d}x + \int_{t=2}^{t=4} y\,\mathrm{d}x = \int_{t=0}^{t=4} y\,\mathrm{d}x.$

b $O(V) = \int_{t=0}^{t=4} (-\frac{1}{2}t^2 + 2)\,\mathrm{d}(-\frac{1}{2}t^2 + 2t) = \int_0^4 (-\frac{1}{2}t^2 + 2)(-t + 2)\,\mathrm{d}t = \int_0^4 (\frac{1}{2}t^3 - t^2 - 2t + 4)\,\mathrm{d}t$

$= [\frac{1}{8}t^4 - \frac{1}{3}t^3 - t^2 + 4t]_0^4 = 32 - \frac{64}{3} - 16 + 16 - 0 = 10\frac{2}{3}$

c $O(V) = \int_{t=4}^{t=0} x\,\mathrm{d}y = \int_{t=4}^{t=0} (-\frac{1}{2}t^2 + 2t)\,\mathrm{d}(-\frac{1}{2}t^2 + 2) = \int_{t=4}^{t=0} (-\frac{1}{2}t^2 + 2t) \cdot -t\,\mathrm{d}t = \int_{t=4}^{t=0} (\frac{1}{2}t^3 - 2t^2)\,\mathrm{d}t$

$= [\frac{1}{8}t^4 - \frac{2}{3}t^3]_4^0 = \frac{1}{8} \cdot 0^4 - \frac{2}{3} \cdot 0^3 - (\frac{1}{8} \cdot 4^4 - \frac{2}{3} \cdot 4^3) = 0 - 32 + 42\frac{2}{3} = 10\frac{2}{3}$

Dus hetzelfde antwoord als bij b.

Bladzijde 217

70 **a** $O(V) = -\int_{t=2}^{t=-2} x\,\mathrm{d}y + \int_{t=2}^{t=4} x\,\mathrm{d}y = \int_{t=-2}^{t=2} x\,\mathrm{d}y + \int_{t=2}^{t=4} x\,\mathrm{d}y = \int_{t=-2}^{t=4} x\,\mathrm{d}y$

b $I(L) = \pi\int_{t=2}^{t=-2} x^2\,\mathrm{d}y = \pi\int_{t=2}^{t=-2} (\frac{1}{2}t^2 - 2)^2\,\mathrm{d}(\frac{1}{2}t^2 - 2t) = \pi\int_2^{-2} (\frac{1}{2}t^2 - 2)^2(t - 2)\,\mathrm{d}t$

$= \pi\int_2^{-2} (\frac{1}{4}t^4 - 2t^2 + 4)(t - 2)\,\mathrm{d}t = \pi\int_2^{-2} (\frac{1}{4}t^5 - \frac{1}{2}t^4 - 2t^3 + 4t^2 + 4t - 8)\,\mathrm{d}t$

$= \pi[\frac{1}{24}t^6 - \frac{1}{10}t^5 - \frac{1}{2}t^4 + 1\frac{1}{3}t^3 + 2t^2 - 8t]_2^{-2}$

$= \pi(\frac{64}{24} + \frac{32}{10} - 8 - 10\frac{2}{3} + 8 + 16 - (\frac{64}{24} - \frac{32}{10} - 8 + 10\frac{2}{3} + 8 - 16))$

$= \pi(2 \cdot \frac{32}{10} - 2 \cdot 10\frac{2}{3} + 2 \cdot 16) = 17\frac{1}{15}\pi$

Bladzijde 218

71 $x = 0$ geeft $t^2 - 2t = 0$

$t(t - 2) = 0$

$t = 0 \vee t = 2$

$t = 0$ geeft $(0, 0)$

$t = 2$ geeft $(0, 6)$

$y = 0$ geeft $t^3 - t = 0$

$t(t^2 - 1) = 0$

$t = 0 \vee t^2 = 1$

$t = 0 \vee t = 1 \vee t = -1$

$t = 1$ geeft $(-1, 0)$

$t = -1$ geeft $(3, 0)$

$O(V) = -\int_{t=1}^{t=0} y\,\mathrm{d}x + \int_{t=1}^{t=2} y\,\mathrm{d}x = \int_{t=0}^{t=1} y\,\mathrm{d}x + \int_{t=1}^{t=2} y\,\mathrm{d}x = \int_{t=0}^{t=2} y\,\mathrm{d}x = \int_{t=0}^{t=2} (t^3 - t)\,\mathrm{d}(t^2 - 2t) = \int_0^2 (t^3 - t)(2t - 2)\,\mathrm{d}t$

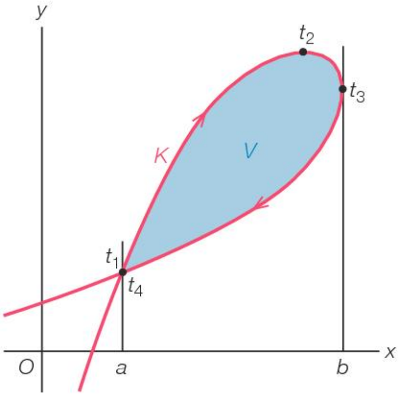
$= \int_0^2 (2t^4 - 2t^3 - 2t^2 + 2t)\,\mathrm{d}t = [\frac{2}{5}t^5 - \frac{1}{2}t^4 - \frac{2}{3}t^3 + t^2]_0^2 = \frac{2}{5} \cdot 32 - \frac{1}{2} \cdot 16 - \frac{2}{3} \cdot 8 + 4 - 0 = 3\frac{7}{15}$

72 $O(V) = -\int_{t=3}^{t=1} y\,\mathrm{d}x + \int_{t=0}^{t=1} y\,\mathrm{d}x = \int_{t=1}^{t=3} y\,\mathrm{d}x + \int_{t=0}^{t=1} y\,\mathrm{d}x = \int_{t=0}^{t=3} y\,\mathrm{d}x = \int_{t=0}^{t=3} (\frac{2}{3}t^2 - 2\frac{2}{3}t + 2)\,\mathrm{d}(-t^2 + 2t)$

$= \int_{t=0}^{t=3} (\frac{2}{3}t^2 - 2\frac{2}{3}t + 2)(-2t + 2)\,\mathrm{d}t = \int_0^3 (-\frac{4}{3}t^3 + \frac{20}{3}t^2 - \frac{28}{3}t + 4)\,\mathrm{d}t = [-\frac{1}{3}t^4 + \frac{20}{9}t^3 - \frac{14}{3}t^2 + 4t]_0^3$

$= -27 + 60 - 42 + 12 - 0 = 3$

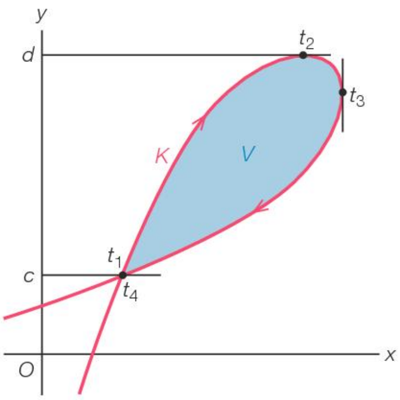
73 **a** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_3$.



Noem V_1 het vlakdeel ingesloten door de x-as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en V_2 het vlakdeel ingesloten door de x-as, de lijnen $x = a$ en $x = b$ en het onderste deel van K.

Dan geldt $O(V) = O(V_1) - O(V_2) = \int_{t=t_1}^{t=t_3} y\,\mathrm{d}x - \int_{t=t_4}^{t=t_3} y\,\mathrm{d}x = \int_{t=t_1}^{t=t_3} y\,\mathrm{d}x + \int_{t=t_3}^{t=t_4} y\,\mathrm{d}x = \int_{t=t_1}^{t=t_4} y\,\mathrm{d}x.$

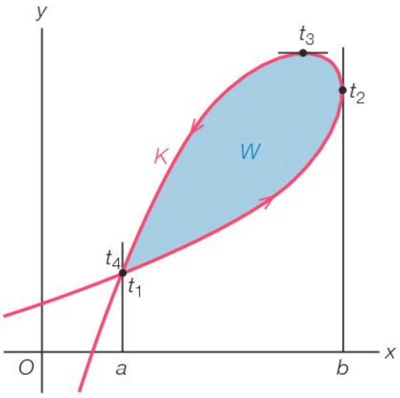
b Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_2$.



Noem V_3 het vlakdeel ingesloten door de y-as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en V_4 het vlakdeel ingesloten door de y-as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K.

Dan geldt $O(V) = O(V_3) - O(V_4) = \int_{t=t_4}^{t=t_2} x\,\mathrm{d}y - \int_{t=t_1}^{t=t_2} x\,\mathrm{d}y = \int_{t=t_4}^{t=t_2} x\,\mathrm{d}y + \int_{t=t_2}^{t=t_1} x\,\mathrm{d}y = \int_{t=t_4}^{t=t_1} x\,\mathrm{d}y.$

c Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_2$.



Noem W_1 het vlakdeel ingesloten door de x-as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en W_2 het vlakdeel ingesloten door de x-as, de lijnen $x = a$ en $x = b$ en het onderste deel van K.

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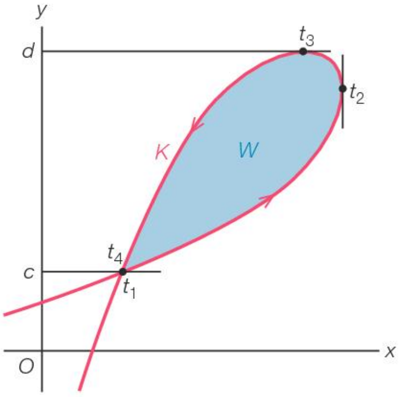
Dan geldt $O(W) = O(W_1) - O(W_2) = \int_{t=t_4}^{t=t_2} y\,dx - \int_{t=t_1}^{t=t_2} y\,dx = \int_{t=t_4}^{t=t_2} y\,dx + \int_{t=t_2}^{t=t_1} y\,dx = \int_{t=t_4}^{t=t_1} y\,dx$.

Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_3$.



Noem W_3 het vlakdeel ingesloten door de y-as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en W_4 het vlakdeel ingesloten door de y-as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K.

Dan geldt $O(W) = O(W_3) - O(W_4) = \int_{t=t_1}^{t=t_3} x\,dy - \int_{t=t_4}^{t=t_3} x\,dy = \int_{t=t_1}^{t=t_3} x\,dy + \int_{t=t_3}^{t=t_4} x\,dy = \int_{t=t_1}^{t=t_4} x\,dy$.

Bladzijde 220

74 **a** $y = 0$ geeft $4t - t^3 = 0$

$t(4 - t^2) = 0$

$t = 0 \vee t^2 = 4$

$t = 0 \vee t = 2 \vee t = -2$

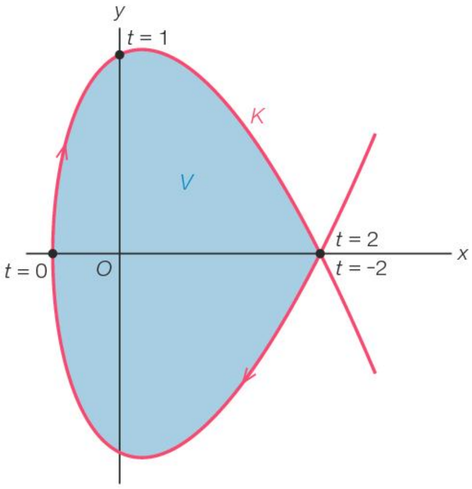
$t = -2$ geeft $(3, 0)$

$t = 0$ geeft $(-1, 0)$

$t = 2$ geeft $(3, 0)$.

$t = 1$ geeft $(0, 3)$

Dus V wordt in negatieve richting omlopen.



$$O(V) = \int_{t=-2}^{t=2} y\,dx = \int_{t=-2}^{t=2} (4t - t^3)\,d(t^2 - 1) = \int_{-2}^{2} (4t - t^3) \cdot 2t\,dt = \int_{-2}^{2} (8t^2 - 2t^4)\,dt = \left[2\tfrac{2}{3}t^3 - \tfrac{2}{5}t^5\right]_{-2}^{2}$$

$$= 2\tfrac{2}{3} \cdot 2^3 - \tfrac{2}{5} \cdot 2^5 - (2\tfrac{2}{3} \cdot (-2)^3 - \tfrac{2}{5} \cdot (-2)^5) = 2\tfrac{2}{3} \cdot 8 - \tfrac{2}{5} \cdot 32 - 2\tfrac{2}{3} \cdot -8 + \tfrac{2}{5} \cdot -32 = 17\tfrac{1}{15}$$

b $$I(L) = \pi\int_{t=0}^{t=2} y^2\,dx = \pi\int_{t=0}^{t=2} (4t - t^3)^2\,d(t^2 - 1) = \pi\int_{0}^{2} (16t^2 - 8t^4 + t^6) \cdot 2t\,dt$$

$$= \pi\int_{0}^{2} (32t^3 - 16t^5 + 2t^7)\,dt = \pi\left[8t^4 - 2\tfrac{2}{3}t^6 + \tfrac{1}{4}t^8\right]_{0}^{2}$$

$$= \pi(8 \cdot 2^4 - 2\tfrac{2}{3} \cdot 2^6 + \tfrac{1}{4} \cdot 2^8) - \pi(0 - 0 + 0) = 21\tfrac{1}{3}\pi$$

Bladzijde 221

75 $x = 0$ geeft $-t^2 + 6t = 0$

$-t(t-6) = 0$

$t = 0 \vee t = 6$

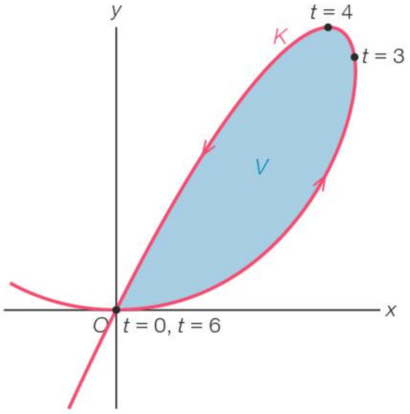
$t = 0$ geeft $(0, 0)$

$t = 6$ geeft $(0, 0)$

$t = 3$ geeft $(9, 9)$

$t = 4$ geeft $(8, 10\frac{2}{3})$

Dus V wordt in positieve richting omlopen.



$$O(V) = \int_{t=6}^{t=0} y\,dx = \int_{t=6}^{t=0} (-\tfrac{1}{3}t^3 + 2t^2)\,d(-t^2 + 6t) = \int_6^0 (-\tfrac{1}{3}t^3 + 2t^2)(-2t + 6)\,dt = \int_6^0 (\tfrac{2}{3}t^4 - 6t^3 + 12t^2)\,dt$$

$$= \left[\tfrac{2}{15}t^5 - 1\tfrac{1}{2}t^4 + 4t^3\right]_6^0 = 0 - (\tfrac{2}{15} \cdot 6^5 - 1\tfrac{1}{2} \cdot 6^4 + 4 \cdot 6^3) = 43\tfrac{1}{5}$$

76 **a** $y = 0$ geeft $\sin(2t) = 0$

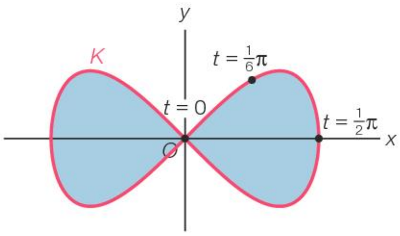
$2t = k \cdot \pi$

$t = k \cdot \frac{1}{2}\pi$

$t = 0$ geeft $(0, 0)$

$t = \frac{1}{2}\pi$ geeft $(2, 0)$

$t = \frac{1}{6}\pi$ geeft $(1, \frac{1}{2}\sqrt{3})$



Noem V het vlakdeel boven de x-as en rechts van de y-as.

$$O(V) = \int_{t=0}^{t=\frac{1}{2}\pi} y\,dx = \int_{t=0}^{t=\frac{1}{2}\pi} \sin(2t)\,d\,2\sin(t) = \int_0^{\frac{1}{2}\pi} \sin(2t) \cdot 2\cos(t)\,dt = \int_0^{\frac{1}{2}\pi} 2\sin(t)\cos(t) \cdot 2\cos(t)\,dt$$

$$= \int_0^{\frac{1}{2}\pi} 4\sin(t)\cos^2(t)\,dt = \int_{t=0}^{t=\frac{1}{2}\pi} -4\cos^2(t)\,d\cos(t) = \int_1^0 -4u^2\,du = \left[-1\tfrac{1}{3}u^3\right]_1^0 = 0 - -1\tfrac{1}{3} \cdot 1^3 = 1\tfrac{1}{3}$$

Dus de totale oppervlakte van de vlakdelen is $4 \cdot 1\frac{1}{3} = 5\frac{1}{3}$.

b $$I(L) = 2 \cdot \pi \int_{t=0}^{t=\frac{1}{2}\pi} y^2\,dx = 2\pi \int_{t=0}^{t=\frac{1}{2}\pi} \sin^2(2t)\,d\,2\sin(t) = 2\pi \int_0^{\frac{1}{2}\pi} (2\sin(t)\cos(t))^2 \cdot 2\cos(t)\,dt$$

$$= 2\pi \int_0^{\frac{1}{2}\pi} 4\sin^2(t)\cos^2(t) \cdot 2\cos(t)\,dt = 16\pi \int_{t=0}^{t=\frac{1}{2}\pi} \sin^2(t)(1 - \sin^2(t))\,d\sin(t) = 16\pi \int_0^1 u^2(1 - u^2)\,du$$

$$= 16\pi \int_0^1 (u^2 - u^4)\,du = 16\pi\left[\tfrac{1}{3}u^3 - \tfrac{1}{5}u^5\right]_0^1 = 16\pi(\tfrac{1}{3} - \tfrac{1}{5}) - 16\pi(0 - 0) = 2\tfrac{2}{15}\pi$$

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77 **a** $x = 0$ geeft $2\sin(t) = 0$

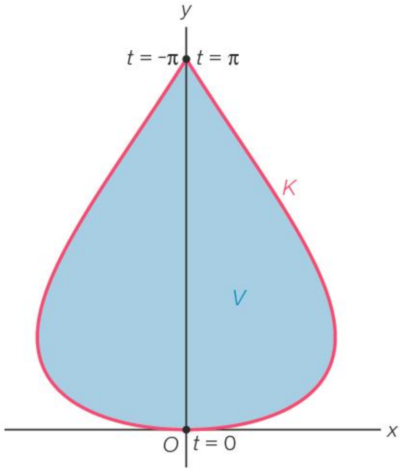
$t = k \cdot \pi$

$t = -\pi$ geeft $(0, \frac{1}{2}\pi^2)$

$t = 0$ geeft $(0, 0)$

$t = \pi$ geeft $(0, \frac{1}{2}\pi^2)$

$t = \frac{1}{2}\pi$ geeft $(2, \frac{1}{8}\pi^2)$



Dus V wordt in positieve richting omlopen.

K is symmetrisch ten opzichte van de y-as, want

$x(-t) = 2\sin(-t) = -2\sin(t) = -x(t)$ en $y(-t) = \frac{1}{2}(-t)^2 = \frac{1}{2}t^2 = y(t)$.

$$O(V) = 2 \cdot \int_{t=0}^{t=\pi} x\,\mathrm{d}y = 2 \cdot \int_{t=0}^{t=\pi} 2\sin(t)\,\mathrm{d}\tfrac{1}{2}t^2 = 2 \cdot \int_0^{\pi} 2\sin(t) \cdot t\,\mathrm{d}t = \int_0^{\pi} 4t\sin(t)\,\mathrm{d}t$$

$$\int 4t\sin(t)\,\mathrm{d}t = \int 4t\,\mathrm{d}{-\cos(t)} = 4t \cdot -\cos(t) + \int \cos(t)\,\mathrm{d}(4t) = -4t\cos(t) + \int 4\cos(t)\,\mathrm{d}t$$

$$= -4t\cos(t) + 4\sin(t) + c$$

Dus $O(V) = \left[-4t\cos(t) + 4\sin(t)\right]_0^{\pi} = -4\pi \cdot -1 + 0 - (0 + 0) = 4\pi$.

b $$I(L) = \pi\int_{t=0}^{t=\pi} x^2\,\mathrm{d}y = \pi\int_{t=0}^{t=\pi} 4\sin^2(t)\,\mathrm{d}\tfrac{1}{2}t^2 = \pi\int_0^{\pi} 4\sin^2(t) \cdot t\,\mathrm{d}t = \pi\int_0^{\pi} 4t\sin^2(t)\,\mathrm{d}t$$

$$\int 4t\sin^2(t)\,\mathrm{d}t = \int 4t(\tfrac{1}{2} - \tfrac{1}{2}\cos(2t))\,\mathrm{d}t = \int (2t - 2t\cos(2t))\,\mathrm{d}t = \int 2t\,\mathrm{d}t - \int 2t\cos(2t)\,\mathrm{d}t =$$

$$t^2 - \int 2t\,\mathrm{d}\tfrac{1}{2}\sin(2t) = t^2 - 2t \cdot \tfrac{1}{2}\sin(2t) + \int \tfrac{1}{2}\sin(2t)\,\mathrm{d}2t = t^2 - t\sin(2t) + \int \sin(2t)\,\mathrm{d}t =$$

$$t^2 - t\sin(2t) - \tfrac{1}{2}\cos(2t) + c$$

Dus $I(L) = \pi\int_0^{\pi} 4t\sin^2(t)\,\mathrm{d}t = \pi[t^2 - t\sin(2t) - \frac{1}{2}\cos(2t)]_0^{\pi} = \pi(\pi^2 - 0 - \frac{1}{2} - (0 - 0 - \frac{1}{2})) = \pi^3$.

Diagnostische toets

Bladzijde 224

1 **a** $F(x) = \int 3x\sqrt{x^2+2}\,\mathrm{d}x = \int 1\frac{1}{2}(x^2+2)^{\frac{1}{2}}\,\mathrm{d}(x^2+2) = \int 1\frac{1}{2}u^{\frac{1}{2}}\,\mathrm{d}u = \frac{3}{2} \cdot \frac{2}{3}u^{1\frac{1}{2}} + c = (x^2+2)\sqrt{x^2+2} + c$

b $F(x) = \int 3x^2\cos(x^3+2)\,\mathrm{d}x = \int \cos(x^3+2)\,\mathrm{d}(x^3+2) = \int \cos(u)\,\mathrm{d}u = \sin(u) + c = \sin(x^3+2) + c$

c $F(x) = \int (4x+6)\ln(x^2+3x)\,\mathrm{d}x = \int \ln(x^2+3x) \cdot 2\,\mathrm{d}(x^2+3x) = \int 2\ln(u)\,\mathrm{d}u = 2(u\ln(u) - u) + c$

$= 2u\ln(u) - 2u + c = 2(x^2+3x)\ln(x^2+3x) - 2(x^2+3x) + c$

2 **a** $\int_1^{e} \frac{\sqrt{\ln(x)}}{x}\,dx = \int_{x=1}^{x=e} \sqrt{\ln(x)}\,d\ln(x) = \int_0^1 \sqrt{u}\,du = \int_0^1 u^{\frac{1}{2}}\,du = \left[\frac{2}{3}u^{1\frac{1}{2}}\right]_0^1 = \frac{2}{3}\cdot 1 - \frac{2}{3}\cdot 0 = \frac{2}{3}$

b $\int_0^{\frac{1}{2}\pi} \cos(x)\sin^3(x)\,dx = \int_{x=0}^{x=\frac{1}{2}\pi} \sin^3(x)\,d\sin(x) = \int_0^1 u^3\,du = \left[\frac{1}{4}u^4\right]_0^1 = \frac{1}{4}\cdot 1 - \frac{1}{4}\cdot 0 = \frac{1}{4}$

c $\int_1^3 \frac{6x}{x^2+1}\,dx = \int_{x=1}^{x=3} \frac{3}{x^2+1}\,d(x^2+1) = \int_2^{10} \frac{3}{u}\,du = \left[3\ln|u|\right]_2^{10} = 3\ln(10) - 3\ln(2) = 3\ln(5)$

3 $f(x) = 0$ geeft $-2x\sin(x^2) = 0$
$x = 0 \vee \sin(x^2) = 0$
$x = 0 \vee x^2 = k\cdot\pi$
$x = 0 \vee x = \sqrt{k\cdot\pi} \vee x = -\sqrt{k\cdot\pi}$
x in $[0, \sqrt{2\pi}]$ geeft $x = 0 \vee x = \sqrt{\pi} \vee x = \sqrt{2\pi}$

$\int -2x\sin(x^2)\,dx = \int -\sin(x^2)\cdot 2x\,dx = \int -\sin(x^2)\,d(x^2) = \int -\sin(u)\,du = \cos(u) + c = \cos(x^2) + c$

$O(V) = \int_0^{\sqrt{\pi}} 2x\sin(x^2)\,dx = \left[-\cos(x^2)\right]_0^{\sqrt{\pi}} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$

$O(W) = \int_{\sqrt{\pi}}^{\sqrt{2\pi}} -2x\sin(x^2)\,dx = \left[\cos(x^2)\right]_{\sqrt{\pi}}^{\sqrt{2\pi}} = \cos(2\pi) - \cos(\pi) = 1 + 1 = 2$

Dus de oppervlakten van de twee vlakdelen zijn gelijk.

4 **a** $F(x) = \int 4x\sin(2x)\,dx = \int -2x\cdot -2\sin(2x)\,dx = \int -2x\,d\cos(2x) = -2x\cos(2x) + \int 2\cos(2x)\,dx$
$= -2x\cos(2x) + \sin(2x) + c$

b $G(x) = \int x^2\ln(x)\,dx = \int \ln(x)\,d\frac{1}{3}x^3 = \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^3\,d\ln(x) = \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^3\cdot\frac{1}{x}\,dx$
$= \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^2\,dx = \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + c$

5 **a** $f(x) = (2x - x^2)e^x$ geeft $f'(x) = (2-2x)\cdot e^x + (2x - x^2)\cdot e^x = (2 - x^2)e^x$
$f'(x) = 0$ geeft $(2 - x^2)e^x = 0$
$2 - x^2 = 0$
$x^2 = 2$
$x = \sqrt{2} \vee x = -\sqrt{2}$

min. is $f(-\sqrt{2}) = (-2\sqrt{2} - 2)e^{-\sqrt{2}}$

max. is $f(\sqrt{2}) = (2\sqrt{2} - 2)e^{\sqrt{2}}$

b $f(x) = 0$ geeft $2x - x^2 = 0$
$x(2 - x) = 0$
$x = 0 \vee x = 2$

$\int (2x - x^2)e^x\,dx = \int (2x - x^2)\,de^x = (2x - x^2)e^x - \int e^x\,d(2x - x^2)$
$= (2x - x^2)e^x - \int (2 - 2x)e^x\,dx$
$= (2x - x^2)e^x - \int (2 - 2x)\,de^x$
$= (2x - x^2)e^x - (2 - 2x)e^x + \int e^x\,d(2 - 2x)$
$= (2x - x^2)e^x - (2 - 2x)e^x - \int 2e^x\,dx$
$= (2x - x^2)e^x - (2 - 2x)e^x - 2e^x = (-x^2 + 4x - 4)e^x$

$O(V) = \int_0^2 f(x)\,dx = \int_0^2 (2x - x^2)e^x\,dx = \left[(-x^2 + 4x - 4)e^x\right]_0^2 = 0 + 4 = 4$

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6 **a** $F(x) = \int 4x^2 \sin(2x)\,dx = \int 4x^2\, d(-\tfrac{1}{2}\cos(2x)) = -2x^2\cos(2x) - \int -\tfrac{1}{2}\cos(2x)\, d\,4x^2$

$= -2x^2\cos(2x) + \int 4x\cos(2x)\,dx = -2x^2\cos(2x) + \int 4x\, d(\tfrac{1}{2}\sin(2x))$

$= -2x^2\cos(2x) + 2x\sin(2x) - \int \tfrac{1}{2}\sin(2x)\, d\,4x$

$= -2x^2\cos(2x) + 2x\sin(2x) - \int 2\sin(2x)\,dx$

$= -2x^2\cos(2x) + 2x\sin(2x) + \cos(2x) + c$

b $\int e^x \sin(2x)\,dx = \int \sin(2x)\,d\,e^x = e^x\sin(2x) - \int e^x\,d\sin(2x) = e^x\sin(2x) - \int 2\cos(2x)\,e^x\,dx$

$= e^x\sin(2x) - \int 2\cos(2x)\,d\,e^x = e^x\sin(2x) - 2e^x\cos(2x) + \int 2e^x\,d\cos(2x)$

$= e^x\sin(2x) - 2e^x\cos(2x) - \int 4e^x\sin(2x)\,dx$

Dus $\int e^x\sin(2x)\,dx = e^x\sin(2x) - 2e^x\cos(2x) - 4\int e^x\sin(2x)\,dx$

$5\int e^x\sin(2x)\,dx = e^x\sin(2x) - 2e^x\cos(2x)$

$\int e^x\sin(2x)\,dx = \tfrac{1}{5}e^x\sin(2x) - \tfrac{2}{5}e^x\cos(2x)$

Dus de primitieven zijn $G(x) = \tfrac{1}{5}e^x(\sin(2x) - 2\cos(2x)) + c$.

7 **a** $\int_0^{2\sqrt{3}} \frac{4}{x^2+4}\,dx = \int_0^{2\sqrt{3}} \frac{1}{\tfrac{1}{4}x^2+1}\,dx = \int_{x=0}^{x=2\sqrt{3}} 2 \cdot \frac{1}{(\tfrac{1}{2}x)^2+1}\,d\tfrac{1}{2}x = \left[2\arctan(\tfrac{1}{2}x)\right]_0^{2\sqrt{3}}$

$= 2\arctan(\sqrt{3}) - 2\arctan(0) = 2 \cdot \tfrac{1}{3}\pi - 0 = \tfrac{2}{3}\pi$

b $\int_1^2 \frac{1}{x^2-2x+2}\,dx = \int_1^2 \frac{1}{(x-1)^2-1+2}\,dx = \int_1^2 \frac{1}{(x-1)^2+1}\,dx = \left[\arctan(x-1)\right]_1^2 = \arctan(1) - \arctan(0) = \tfrac{1}{4}\pi$

c $\int_{-1}^1 \frac{3x^2}{x^6+1}\,dx = \int_{x=-1}^{x=1} \frac{1}{(x^3)^2+1}\,dx^3 = \left[\arctan(x^3)\right]_{-1}^1 = \arctan(1) - \arctan(-1) = \tfrac{1}{4}\pi - (-\tfrac{1}{4}\pi) = \tfrac{1}{2}\pi$

d $\int_0^1 \frac{\arctan^2(x)}{x^2+1}\,dx = \int_0^1 \arctan^2(x) \cdot \frac{1}{x^2+1}\,dx = \int_{x=0}^{x=1} \arctan^2(x)\,d\arctan(x) = \int_0^{\frac{1}{4}\pi} u^2\,du = \left[\tfrac{1}{3}u^3\right]_0^{\frac{1}{4}\pi} = \tfrac{1}{3} \cdot (\tfrac{1}{4}\pi)^3 - 0 = \tfrac{1}{192}\pi^3$

Bladzijde 225

8 **a** $f(x) = \tfrac{1}{2}$ geeft $\frac{1}{x^2-4x+5} = \tfrac{1}{2}$

$x^2 - 4x + 5 = 2$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1 \vee x = 3$

$\int_1^3 f(x)\,dx = \int_1^3 \frac{1}{x^2-4x+5}\,dx = \int_1^3 \frac{1}{(x-2)^2-4+5}\,dx = \int_1^3 \frac{1}{(x-2)^2+1}\,dx = \left[\arctan(x-2)\right]_1^3$

$= \arctan(1) - \arctan(-1) = \tfrac{1}{4}\pi + \tfrac{1}{4}\pi = \tfrac{1}{2}\pi$

$O(V) = \int_1^3 f(x)\,dx - 2 \cdot \tfrac{1}{2} = \tfrac{1}{2}\pi - 1$

b $\int_2^p f(x)\,dx = \left[\arctan(x-2)\right]_2^p = \arctan(p-2) - \arctan(0) = \arctan(p-2)$

$O(W) = \tfrac{1}{3}\pi$ geeft $\arctan(p-2) = \tfrac{1}{3}\pi$

$p - 2 = \sqrt{3}$

$p = 2 + \sqrt{3}$

9 **a** $F(x) = \int \frac{1}{\sqrt{1-4x^2}}\,dx = \int \tfrac{1}{2} \cdot \frac{1}{\sqrt{1-(2x)^2}}\,d\,2x = \tfrac{1}{2}\arcsin(2x) + c$

b $G(x) = \int \frac{5x^4}{\sqrt{1-x^{10}}}\,dx = \int \frac{1}{\sqrt{1-(x^5)^2}}\,d(x^5) = \arcsin(x^5) + c$

10 **a** $\int_{-1}^{1} \frac{3x+4}{x+4}\,dx = \int_{-1}^{1} \frac{3(x+4)-8}{x+4}\,dx = \int_{-1}^{1}\left(3 - \frac{8}{x+4}\right)dx = \left[3x - 8\ln|x+4|\right]_{-1}^{1}$

$= 3 - 8\ln(5) - (-3 - 8\ln(3)) = 6 - 8\ln(5) + 8\ln(3) = 6 + 8\ln(\frac{3}{5})$

b $x+1 \;/\; x^3 \qquad -x+1 \;\backslash\; x^2 - x$

$\underline{x^3 + x^2}\;-$

$-x^2 - x + 1$

$\underline{-x^2 - x}\;-$

1

$\int_{1}^{3} \frac{x^3 - x + 1}{x+1}\,dx = \int_{1}^{3}\left(x^2 - x + \frac{1}{x+1}\right)dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + \ln|x+1|\right]_{1}^{3}$

$= 9 - 4\frac{1}{2} + \ln(4) - (\frac{1}{3} - \frac{1}{2} + \ln(2)) = 4\frac{2}{3} + \ln(4) - \ln(2) = 4\frac{2}{3} + \ln(2)$

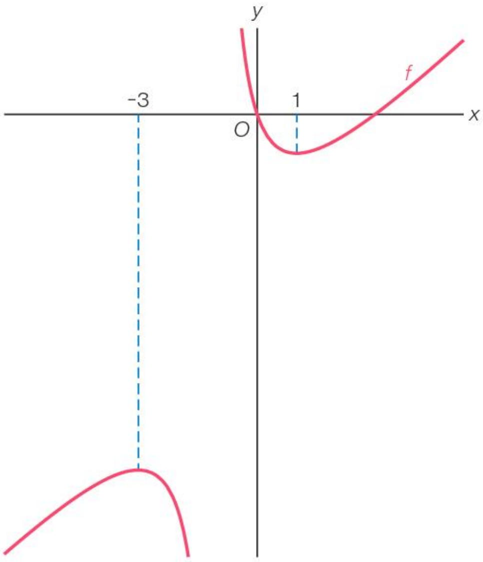
11 **a** $f(x) = \frac{x^2 - 3x}{x+1}$ geeft

$f'(x) = \frac{(x+1)\cdot(2x-3) - (x^2 - 3x)\cdot 1}{(x+1)^2} = \frac{2x^2 - 3x + 2x - 3 - x^2 + 3x}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2}$

$f'(x) = 0$ geeft $x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1 \vee x = -3$



max. is $f(-3) = -9$

min. is $f(1) = -1$

b $f(x) = 0$ geeft $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \vee x = 3$

$x+1 \;/\; x^2 - 3x \;\backslash\; x - 4$

$\underline{x^2 + \;x}\;-$

$-4x$

$\underline{-4x - 4}\;-$

4

$O(V) = -\int_{0}^{3} f(x)\,dx = -\int_{0}^{3}\left(x - 4 + \frac{4}{x+1}\right)dx = -\left[\frac{1}{2}x^2 - 4x + 4\ln|x+1|\right]_{0}^{3}$

$= -(4\frac{1}{2} - 12 + 4\ln(4)) + (0 - 0 + 4\ln(1)) = 7\frac{1}{2} - 4\ln(4)$

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12 **a** $f(x)=\frac{2x+3}{x^2-6x+10}=\frac{2x-6+9}{x^2-6x+10}=\frac{2x-6}{x^2-6x+10}+\frac{9}{x^2-6x+10}$

$\int\frac{2x-6}{x^2-6x+10}\,dx=\int\frac{1}{x^2-6x+10}\cdot(2x-6)\,dx=\int\frac{1}{x^2-6x+10}\,d(x^2-6x+10)=\ln\left|x^2-6x+10\right|+c$

$\int\frac{9}{x^2-6x+10}\,dx=\int 9\cdot\frac{1}{(x-3)^2-9+10}\,dx=\int 9\cdot\frac{1}{(x-3)^2+1}\,dx=9\arctan(x-3)+c$

Dus $F(x)=\ln\left|x^2-6x+10\right|+9\arctan(x-3)+c$

b $x^2-4x+4\ /\ x^2+4x\quad\backslash\ 1$

$\quad\underline{x^2-4x+4}\ -$

$\quad 8x-4$

$G(x)=\int\frac{x^2+4x}{x^2-4x+4}\,dx=\int\left(1+\frac{8x-4}{x^2-4x+4}\right)dx=\int\left(1+\frac{8(x-2)+12}{(x-2)^2}\right)dx$

$=\int\left(1+\frac{8}{x-2}+\frac{12}{(x-2)^2}\right)dx=\int\left(1+\frac{8}{x-2}+12(x-2)^{-2}\right)dx$

$=x+8\ln\left|x-2\right|-12(x-2)^{-1}+c=x+8\ln\left|x-2\right|-\frac{12}{x-2}+c$

c $h(x)=\frac{6}{x^2-1}=\frac{6}{(x-1)(x+1)}=\frac{a}{x-1}+\frac{b}{x+1}=\frac{a(x+1)+b(x-1)}{(x-1)(x+1)}$

$=\frac{ax+a+bx-b}{(x-1)(x+1)}=\frac{(a+b)x+a-b}{(x-1)(x+1)}$

$\begin{cases}a+b=0\\ a-b=6\end{cases}\ +$

$2a=6$

$\left.\begin{matrix}a=3\\ a+b=0\end{matrix}\right\}\ b=-3$

Dus $h(x)=\frac{3}{x-1}-\frac{3}{x+1}$ en dit geeft $H(x)=3\ln\left|x-1\right|-3\ln\left|x+1\right|+c$.

13 **a** $x(t) = 0$ geeft $4t - t^2 = 0$

$t(4 - t) = 0$

$t = 0 \vee t = 4$

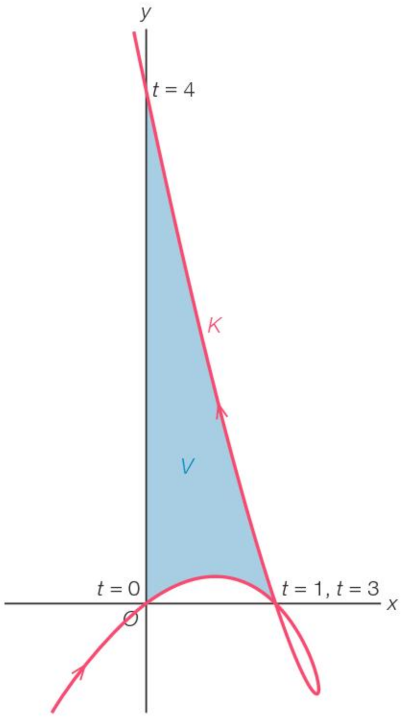
$y(t) = 0$ geeft $t^3 - 4t^2 + 3t = 0$

$t(t^2 - 4t + 3) = 0$

$t(t - 1)(t - 3) = 0$

$t = 0 \vee t = 1 \vee t = 3$

$t = 0$ geeft $(0, 0)$



$$O(V) = \int_{t=4}^{t=3} y\,\mathrm{d}x - \int_{t=0}^{t=1} y\,\mathrm{d}x$$

$$\int y\,\mathrm{d}x = \int (t^3 - 4t^2 + 3t)\,\mathrm{d}(4t - t^2) = \int (t^3 - 4t^2 + 3t)(4 - 2t)\,\mathrm{d}t = \int (-2t^4 + 12t^3 - 22t^2 + 12t)\,\mathrm{d}t$$

$$= -\tfrac{2}{5}t^5 + 3t^4 - \tfrac{22}{3}t^3 + 6t^2 + c$$

$$O(V) = \left[-\tfrac{2}{5}t^5 + 3t^4 - \tfrac{22}{3}t^3 + 6t^2\right]_4^3 - \left[-\tfrac{2}{5}t^5 + 3t^4 - \tfrac{22}{3}t^3 + 6t^2\right]_0^1 = 1\tfrac{4}{5} - -14\tfrac{14}{15} - (1\tfrac{4}{15} - 0) = 15\tfrac{7}{15}$$

b $O(W) = \int_{t=1}^{t=3} y\,\mathrm{d}x = \left[-\tfrac{2}{5}t^5 + 3t^4 - \tfrac{22}{3}t^3 + 6t^2\right]_1^3 = 1\tfrac{4}{5} - 1\tfrac{4}{15} = \tfrac{8}{15}$

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Gemengde opgaven

9 Exponentiële en logaritmische functies

Bladzijde 226

1 **a** $x + 1 > 0$ geeft $x > -1$, dus $D_f = \langle -1, \rightarrow \rangle$.

$f(x) = g(x)$ geeft $2 - {}^{3}\log(x + 1) = {}^{\frac{1}{3}}\log(5 - x)$

$${}^{\frac{1}{3}}\log(\tfrac{1}{9}) + {}^{\frac{1}{3}}\log(x + 1) = {}^{\frac{1}{3}}\log(5 - x)$$
$${}^{\frac{1}{3}}\log(\tfrac{1}{9}(x + 1)) = {}^{\frac{1}{3}}\log(5 - x)$$
$$\tfrac{1}{9}(x + 1) = 5 - x$$
$$x + 1 = 45 - 9x$$
$$10x = 44$$
$$x = 4\tfrac{2}{5}$$

$f(x) \geq g(x)$ geeft $-1 < x \leq 4\tfrac{2}{5}$

b $f(x) = 3$ geeft $2 - {}^{3}\log(x + 1) = 3$

$$-{}^{3}\log(x + 1) = 1$$
$${}^{3}\log(x + 1) = -1$$
$$x + 1 = 3^{-1}$$
$$x + 1 = \tfrac{1}{3}$$
$$x = -\tfrac{2}{3}$$

$g(x) = 3$ geeft ${}^{\frac{1}{3}}\log(5 - x) = 3$

$$5 - x = (\tfrac{1}{3})^3$$
$$5 - x = \tfrac{1}{27}$$
$$x = 4\tfrac{26}{27}$$

$AB = 4\tfrac{26}{27} - -\tfrac{2}{3} = 4\tfrac{26}{27} + \tfrac{18}{27} = 5\tfrac{17}{27}$

$f(2) = 2 - {}^{3}\log(3) = 2 - 1 = 1$ en $g(2) = {}^{\frac{1}{3}}\log(3) = {}^{\frac{1}{3}}\log((\tfrac{1}{3})^{-1}) = -1$

$CD = 1 - -1 = 2$

Dus AB is niet meer dan drie keer zo lang als CD.

c $f(x) = 2 - {}^{3}\log(x + 1)$ geeft $f'(x) = -\dfrac{1}{(x + 1)\ln(3)}$

$rc_{raaklijn} = -1$, dus $f'(x) = -1$

$$-\frac{1}{(x + 1)\ln(3)} = -1$$
$$\frac{1}{(x + 1)\ln(3)} = 1$$
$$x + 1 = \frac{1}{\ln(3)}$$
$$x = \frac{1}{\ln(3)} - 1$$
$$x = \frac{1 - \ln(3)}{\ln(3)}$$

Dus $x_P = \dfrac{1 - \ln(3)}{\ln(3)}$.

2 **a**

$$4^{\log(x)} = 2^{3 + \log(x)}$$
$$(2^2)^{\log(x)} = 2^{3 + \log(x)}$$
$$2^{2\log(x)} = 2^{3 + \log(x)}$$
$$2\log(x) = 3 + \log(x)$$
$$\log(x) = 3$$
$$x = 1000$$

b

$${}^{8}\log(2x + 1) = {}^{4}\log(25)$$
$$\frac{{}^{2}\log(2x + 1)}{{}^{2}\log(8)} = \frac{{}^{2}\log(25)}{{}^{2}\log(4)}$$
$$\frac{{}^{2}\log(2x + 1)}{3} = \frac{{}^{2}\log(25)}{2}$$
$${}^{2}\log(2x + 1) = 1\tfrac{1}{2} \cdot {}^{2}\log(25)$$
$${}^{2}\log(2x + 1) = {}^{2}\log(25^{1\frac{1}{2}})$$
$$2x + 1 = 125$$
$$2x = 124$$
$$x = 62$$

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c $^5\log^2(x+2) = 6 \cdot {}^5\log(x+2) + 7$
Stel $^5\log(x+2) = u$.
$u^2 = 6u + 7$
$u^2 - 6u - 7 = 0$
$(u+1)(u-7) = 0$
$u = -1 \vee u = 7$
$^5\log(x+2) = -1 \vee {}^5\log(x+2) = 7$
$x + 2 = 5^{-1} \vee x + 2 = 5^7$
$x = -2 + \frac{1}{5} \vee x = 5^7 - 2$
$x = -1\frac{4}{5} \vee x = 78\,123$

d $2 \cdot {}^3\log(2x-3) + {}^{\frac{1}{3}}\log(2x+1) = 2$
$^3\log((2x-3)^2) - {}^3\log(2x+1) = {}^3\log(9)$
$^3\log((2x-3)^2) = {}^3\log(9) + {}^3\log(2x+1)$
$^3\log((2x-3)^2) = {}^3\log(9(2x+1))$
$(2x-3)^2 = 9(2x+1)$
$4x^2 - 12x + 9 = 18x + 9$
$4x^2 - 30x = 0$
$4x(x - 7\frac{1}{2}) = 0$
$x = 0 \quad \vee \quad x = 7\frac{1}{2}$
vold. niet

3 **a** $\ln^2(x) + 1 = 2\frac{1}{2}\ln(x)$
Stel $\ln(x) = u$.
$u^2 + 1 = 2\frac{1}{2}u$
$u^2 - 2\frac{1}{2}u + 1 = 0$
$(u - \frac{1}{2})(u-2) = 0$
$u = \frac{1}{2} \vee u = 2$
$\ln(x) = \frac{1}{2} \vee \ln(x) = 2$
$x = \sqrt{e} \vee x = e^2$

b $\dfrac{e^x}{e^x - 2} = 2$
$e^x = 2(e^x - 2)$
$e^x = 2\,e^x - 4$
$-e^x = -4$
$e^x = 4$
$x = \ln(4)$

c $\ln(3x+2) = \frac{1}{2}$
$3x + 2 = e^{\frac{1}{2}}$
$3x + 2 = \sqrt{e}$
$3x = -2 + \sqrt{e}$
$x = -\frac{2}{3} + \frac{1}{3}\sqrt{e}$

d $\ln(4x) - \ln(x+4) = 1$
$\ln\left(\dfrac{4x}{x+4}\right) = 1$
$\dfrac{4x}{x+1} = e$
$4x = e(x+1)$
$4x = ex + 4e$
$4x - ex = 4e$
$(4-e)x = 4e$
$x = \dfrac{4e}{4-e}$

e $\ln^2(x-2) = 4$
$\ln(x-2) = 2 \vee \ln(x-2) = -2$
$x - 2 = e^2 \vee x - 2 = e^{-2}$
$x = 2 + e^2 \vee x = 2 + \dfrac{1}{e^2}$

f $3\,e^{2x} + 2 = 5\,e^x$
$3\,(e^x)^2 + 2 = 5\,e^x$
Stel $e^x = u$.
$3u^2 + 2 = 5u$
$3u^2 - 5u + 2 = 0$
$D = (-5^2) - 4 \cdot 3 \cdot 2 = 1$
$u = \dfrac{5+1}{6} = 1 \vee u = \dfrac{5-1}{6} = \frac{2}{3}$
$e^x = 1 \vee e^x = \frac{2}{3}$
$x = 0 \vee x = \ln(\frac{2}{3})$

4 **a** $g_{\text{jaar}} = 1{,}096$
$1{,}096^T = 2$
$T = {}^{1{,}096}\log(2) = 7{,}561...$
De verdubbelingstijd is 7 jaar en 0,561... · 12 ≈ 7 maanden.

b $g_{\text{dag}} = 0{,}83$
$0{,}83^T = \frac{1}{2}$
$T = {}^{0{,}83}\log(\frac{1}{2}) = 3{,}720...$
De halveringstijd is 3,720... · 24 ≈ 89 uren.

c $g_{\text{maand}} = 2$
$g_{\text{dag}} = 2^{\frac{1}{30}} = 1{,}023...$
De toename per dag is 2,3%.

d $g_{8{,}3\text{ dagen}} = \frac{1}{2}$
$g_{\text{dag}} = (\frac{1}{2})^{\frac{1}{8{,}3}} = 0{,}919...$
$0{,}919...^t = 0{,}01$
$t = {}^{0{,}919...}\log(0{,}01) = 55{,}1...$
Dus na 55 dagen is er nog 1% over.

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5 **a** $f(x) = g(x)$ geeft $3 \cdot 2^x = 6 \cdot (\frac{1}{4})^x$

$3 \cdot 2^x = 6 \cdot (2^{-2})^x$

$3 \cdot 2^x = 6 \cdot (2^x)^{-2}$

Stel $2^x = u$.

$3u = 6u^{-2}$

$3u^3 = 6$

$u^3 = 2$

$u = 2^{\frac{1}{3}}$

$2^x = 2^{\frac{1}{3}}$

$x = \frac{1}{3}$

$f(x) \leq g(x)$ geeft $x \leq \frac{1}{3}$

b $h(x) = f(x) + g(x) = 3 \cdot 2^x + 6 \cdot (\frac{1}{4})^x$ geeft $h'(x) = 3 \cdot 2^x \cdot \ln(2) + 6 \cdot (\frac{1}{4})^x \cdot \ln(\frac{1}{4})$

$h'(x) = 0$ geeft $3 \cdot 2^x \cdot \ln(2) + 6 \cdot (\frac{1}{4})^x \cdot \ln(\frac{1}{4}) = 0$

$3 \cdot 2^x \cdot \ln(2) + 6 \cdot (2^{-2})^x \cdot \ln(2^{-2}) = 0$

$3 \cdot 2^x \cdot \ln(2) + 6 \cdot 2^{-2x} \cdot -2\ln(2) = 0$

$3 \cdot 2^x \cdot \ln(2) - 12 \cdot 2^{-2x} \cdot \ln(2) = 0$

$3 \cdot 2^x \cdot \ln(2) = 12 \cdot 2^{-2x} \cdot \ln(2)$

$2^x = 4 \cdot 2^{-2x}$

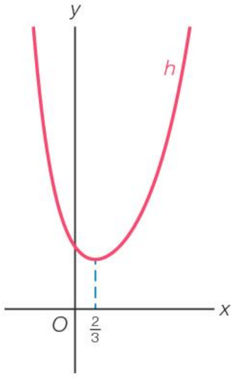
$2^x = 2^2 \cdot 2^{-2x}$

$2^x = 2^{2-2x}$

$x = 2 - 2x$

$3x = 2$

$x = \frac{2}{3}$



Dus $h(x) = f(x) + g(x)$ heeft een minimum voor $x = \frac{2}{3}$.

Bladzijde 227

6 **a** $f(x) = x^2 e^{x-1}$ geeft $f'(x) = 2x \cdot e^{x-1} + x^2 \cdot e^{x-1} = (x^2 + 2x)e^{x-1}$

b $g(x) = \ln^2(x) + \ln(x^2) = (\ln(x))^2 + \ln(x^2)$ geeft

$g'(x) = 2\ln(x) \cdot \frac{1}{x} + \frac{1}{x^2} \cdot 2x = \frac{2\ln(x)}{x} + \frac{2}{x} = \frac{2\ln(x) + 2}{x}$

c $h(x) = {}^2\log(x^3 - x^2)$ geeft $h'(x) = \frac{1}{(x^3 - x^2)\ln(2)} \cdot (3x^2 - 2x) = \frac{3x^2 - 2x}{(x^3 - x^2)\ln(2)}$

d $j(x) = \ln(\ln(2x))$ geeft $j'(x) = \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2 = \frac{1}{x\ln(2x)}$

7 **a** $y = e^{3x-1} \xrightarrow{\text{translatie } (2,\ 0)} y = e^{3(x-2)-1}$

Er geldt $e^{3(x-2)-1} = e^{3x-6-1} = e^{3x-1-6} = e^{3x-1} \cdot e^{-6} = \frac{1}{e^6} \cdot e^{3x-1}$.

Dus de vermenigvuldiging ten opzichte van de x-as met $\frac{1}{e^6}$ levert dezelfde beeldfiguur op.

b $y = \ln(x^2) \xrightarrow{\text{verm. } y\text{-as, } 3} y = \ln((\frac{1}{3}x)^2) = \ln(\frac{1}{9}x^2)$

Er geldt $\ln(\frac{1}{9}x^2) = \ln(\frac{1}{9}) + \ln(x^2) = \ln(3^{-2}) + \ln(x^2) = -2\ln(3) + \ln(x^2)$.

Dus de translatie $(0, -2\ln(3))$ levert dezelfde beeldfiguur op.

8 **a** $f(x) = (x-3)^2 e^x = (x^2 - 6x + 9)e^x$ geeft $f'(x) = (2x-6) \cdot e^x + (x^2 - 6x + 9) \cdot e^x = (x^2 - 4x + 3)e^x$

$f'(x) = 0$ geeft $(x^2 - 4x + 3)e^x = 0$

$x^2 - 4x + 3 = 3$

$(x-1)(x-3) = 0$

$x = 1 \vee x = 3$

$f(1) = (1-3)^2 e^1 = 4e$ en $f(3) = (3-3)^2 e^3 = 0$

De toppen zijn (1, 4e) en (3, 0).

b $f'(x) = (x^2 - 4x + 3)e^x$ geeft $f''(x) = (2x-4) \cdot e^x + (x^2 - 4x + 3) \cdot e^x = (x^2 - 2x - 1)e^x$

$f''(x) = 0$ geeft $(x^2 - 2x - 1)e^x = 0$

$x^2 - 2x - 1 = 0$

$D = (-2^2) - 4 \cdot 1 \cdot -1 = 8$, dus $\sqrt{D} = \sqrt{8} = 2\sqrt{2}$

$x = \dfrac{2 + 2\sqrt{2}}{2} \vee x = \dfrac{2 - 2\sqrt{2}}{2}$

$x = 1 + \sqrt{2} \vee x = 1 - \sqrt{2}$

De x-coördinaten van de buigpunten zijn $x = 1 + \sqrt{2}$ en $x = 1 - \sqrt{2}$.

c Stel $x_A = a$, dan is $x_B = a + 3$.

$p = f(x_A) = f(x_B)$ geeft $(a-3)^2 e^a = a^2 e^{a+3}$

Voer in $y_1 = (x-3)^2 e^x$ en $y_2 = x^2 e^{x+3}$.

De optie snijpunt geeft $x = -0{,}861...$ en $y = 6{,}299...$ en $x = 0{,}547...$ en $y = 10{,}398...$

Dus $p \approx 6{,}30$.

9 **a** $f_{-2}(x) = x^{-2} \cdot \ln(x)$ geeft $f_{-2}'(x) = -2x^{-3} \cdot \ln(x) + x^{-2} \cdot \dfrac{1}{x} = \dfrac{-2\ln(x)}{x^3} + \dfrac{1}{x^2} \cdot \dfrac{1}{x} = \dfrac{-2\ln(x)}{x^3} + \dfrac{1}{x^3} = \dfrac{-2\ln(x) + 1}{x^3}$

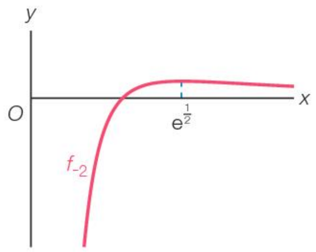
$f_{-2}'(x) = 0$ geeft $\dfrac{-2\ln(x) + 1}{x^3} = 0$

$-2\ln(x) + 1 = 0$

$-2\ln(x) = -1$

$\ln(x) = \frac{1}{2}$

$x = e^{\frac{1}{2}}$



max. is $f_{-2}(e^{\frac{1}{2}}) = (e^{\frac{1}{2}})^{-2} \cdot \frac{1}{2} = e^{-1} \cdot \frac{1}{2} = \dfrac{1}{2e}$

b $f_3(x) = x^3 \cdot \ln(x)$ geeft $f_3'(x) = 3x^2 \cdot \ln(x) + x^3 \cdot \dfrac{1}{x} = 3x^2\ln(x) + x^2$

$f_3'(x) = 3x^2\ln(x) + x^2$ geeft $f_3''(x) = 6x \cdot \ln(x) + 3x^2 \cdot \dfrac{1}{x} + 2x = 6x\ln(x) + 3x + 2x = 6x\ln(x) + 5x$

$f_3''(x) = 0$ geeft $6x\ln(x) + 5x = 0$

$x(6\ln(x) + 5) = 0$

$x = 0 \quad \vee \quad 6\ln(x) + 5 = 0$

vold. niet $6\ln(x) = -5$

$\ln(x) = -\frac{5}{6}$

$x = e^{-\frac{5}{6}}$

De x-coördinaat van het buigpunt van de grafiek van f_3 is $x = e^{-\frac{5}{6}}$.

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c $f_p(x) = x^p \cdot \ln(x)$ geeft $f_p'(x) = px^{p-1} \cdot \ln(x) + x^p \cdot \frac{1}{x} = px^{p-1}\ln(x) + x^p \cdot x^{-1} = px^{p-1}\ln(x) + x^{p-1}$

$f_p'(x) = 0$ geeft $px^{p-1}\ln(x) + x^{p-1} = 0$

$x^{p-1}(p\ln(x) + 1) = 0$

$x = 0 \vee p\ln(x) + 1 = 0$

vold. niet $p\ln(x) = -1$

$\ln(x) = -\frac{1}{p}$

$x = e^{-\frac{1}{p}}$

$f_p(e^{-\frac{1}{p}}) = (e^{-\frac{1}{p}})^p \cdot -\frac{1}{p} = e^{-1} \cdot -\frac{1}{p} = -\frac{1}{pe}$

$y_{top} = \frac{1}{3}e$ geeft $-\frac{1}{pe} = \frac{1}{3}e$

$-\frac{1}{pe} = \frac{e}{3}$

$pe^2 = -3$

$p = \frac{-3}{e^2}$

10 **a** $f(x) = g(x)$ geeft $\ln(4x) = \ln\left(\frac{1}{x}\right)$

$4x = \frac{1}{x}$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \frac{1}{2} \vee x = -\frac{1}{2}$

vold. niet

$f(\frac{1}{2}) = \ln(2)$ geeft $A(\frac{1}{2}, \ln(2))$

$f(x) = \ln(4x) = \ln(4) + \ln(x)$ geeft $f'(x) = \frac{1}{x}$

Stel $y = ax + b$ met $a = f'(\frac{1}{2}) = 2$.

$\left.\begin{array}{l} y = 2x + b \\ \text{door } A(\frac{1}{2}, \ln(2)) \end{array}\right\} \begin{array}{l} 2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) - 1 \end{array}$

De lijn $y = 2x + \ln(2) - 1$ snijdt de y-as in $(0, \ln(2) - 1)$.

$g(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -\ln(x)$ geeft $g'(x) = -\frac{1}{x}$

Stel $y = ax + b$ met $a = g'(\frac{1}{2}) = -2$.

$\left.\begin{array}{l} y = -2x + b \\ \text{door } A(\frac{1}{2}, \ln(2)) \end{array}\right\} \begin{array}{l} -2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) + 1 \end{array}$

De lijn $y = -2x + \ln(2) + 1$ snijdt de y-as in $(0, \ln(2) + 1)$.

De lengte van het gevraagde lijnstuk is $\ln(2) + 1 - (\ln(2) - 1) = 2$.

b Stel $x_P = p$.

$PS = g(p) = \ln\left(\frac{1}{p}\right)$

$\left.\begin{array}{l} PQRS \text{ is een vierkant, dus } PQ = PS = \ln\left(\frac{1}{p}\right) \\ x_P = p \end{array}\right\} x_Q = p + \ln\left(\frac{1}{p}\right)$

$x_Q = p + \ln\left(\frac{1}{p}\right)$ geeft $QR = f\left(p + \ln\left(\frac{1}{p}\right)\right) = \ln\left(4\left(p + \ln\left(\frac{1}{p}\right)\right)\right) = \ln\left(4p + 4\ln\left(\frac{1}{p}\right)\right)$

$PQRS$ is een vierkant, dus $PS = QR$

$\ln\left(\frac{1}{p}\right) = \ln\left(4p + 4\ln\left(\frac{1}{p}\right)\right)$

$\frac{1}{p} = 4p + 4\ln\left(\frac{1}{p}\right)$

Voer in $y_1 = \frac{1}{x}$ en $y_2 = 4x + 4\ln\left(\frac{1}{x}\right)$.

De optie snijpunt geeft $x = 0{,}1065...$

Dus $x_P \approx 0{,}107$.

Bladzijde 228

11 **a** $p = 10\,e^{-\frac{1}{2}t+1}$

$e^{-\frac{1}{2}t+1} = \frac{1}{10}p$

$-\frac{1}{2}t + 1 = \ln(\frac{1}{10}p)$

$-\frac{1}{2}t = -1 + \ln(\frac{1}{10}p)$

$t = 2 - 2\ln(\frac{1}{10}p)$

$t = \ln(e^2) + \ln((\frac{1}{10}p)^{-2})$

$t = \ln(e^2) + \ln(100p^{-2})$

$t = \ln(100e^2 p^{-2})$

b $y = 3\ln\left(\frac{2}{x}\right) + 2\ln(5x)$

$y = 3(\ln(2) - \ln(x)) + 2(\ln(5) + \ln(x))$

$y = 3\ln(2) - 3\ln(x) + 2\ln(5) + 2\ln(x)$

$y = \ln(2^3) + \ln(5^2) - \ln(x)$

$\ln(x) = \ln(2^3 \cdot 5^2) - y$

$\ln(x) = \ln(200) + \ln(e^{-y})$

$\ln(x) = \ln(200\,e^{-y})$

$x = 200\,e^{-y}$

c $y = e^x + e^{x+1} + e^{x+2} + e^{x+3}$

$e^x + e^{x+1} + e^{x+2} + e^{x+3} = y$

$e^x + e^x \cdot e + e^x \cdot e^2 + e^x \cdot e^3 = y$

$e^x(1 + e + e^2 + e^3) = y$

$e^x = \dfrac{y}{1 + e + e^2 + e^3}$

$x = \ln\left(\dfrac{y}{1 + e + e^2 + e^3}\right)$

$\dfrac{e-1}{e^4-1} = \dfrac{e-1}{(e^2+1)(e^2-1)} = \dfrac{e-1}{(e^2+1)(e+1)(e-1)} = \dfrac{1}{(e^2+1)(e+1)} = \dfrac{1}{e^3+e^2+e+1}$

Dus $x = \ln\left(\dfrac{y}{1 + e + e^2 + e^3}\right)$ kan worden geschreven als $x = \ln\left(\dfrac{e-1}{e^4-1}y\right)$.

10 Meetkunde met vectoren

12 **a** Substitutie van $x = -1 + 5t$ en $y = -t$ in $3x + 2y = 10$ geeft $3(-1 + 5t) + 2 \cdot -t = 10$

$-3 + 15t - 2t = 10$

$13t = 13$

$t = 1$

$t = 1$ geeft $x = -1 + 5 = 4$ en $y = -1$, dus $S(4, -1)$.

b $m \perp k$, dus $\vec{n}_m = \vec{r}_k = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

$\left.\begin{array}{l} m\text{: } 5x - y = c \\ \text{door } A(1, 3) \end{array}\right\} c = 5 \cdot 1 - 3 = 2$

Dus m: $5x - y = 2$.

c $\vec{n}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left|\begin{pmatrix} 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}\right|}{\left|\begin{pmatrix} 5 \\ -1 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -3 \end{pmatrix}\right|} = \frac{|10 + 3|}{\sqrt{26} \cdot \sqrt{13}} = \frac{13}{13\sqrt{2}} = \tfrac{1}{2}\sqrt{2}$$

$\angle(k, l) = 45°$

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d P op k, dus $\vec{p} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix}$.

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5t - 2 \\ -t - 3 \end{pmatrix}$

De lijnen AP en l zijn evenwijdig en $\vec{n}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, dus $\begin{pmatrix} 5t - 2 \\ -t - 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 0$

$15t - 6 - 2t - 6 = 0$

$13t = 12$

$t = \frac{12}{13}$

$t = \frac{12}{13}$ geeft $P(-1 + 5 \cdot \frac{12}{13}, -\frac{12}{13}) = P(3\frac{8}{13}, -\frac{12}{13})$

13 **a** $\vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$$\cos(\angle(k, m)) = \frac{\left|\begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}\right|}{\left|\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -5 \end{pmatrix}\right|} = \frac{|-4 - 15|}{\sqrt{13} \cdot \sqrt{29}} = \frac{19}{\sqrt{377}}$$

$\angle(k, m) \approx 11{,}9°$

b $\vec{r}_k = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$\left.\begin{array}{l} k\text{: } 3x + 2y = c \\ \text{door } (1, 2) \end{array}\right\} c = 3 \cdot 1 + 2 \cdot 2 = 7$

Dus k: $3x + 2y = 7$.

$x = 2u - 6$ en $y = -5u + 4$ substitueren in $3x + 2y = 7$ geeft $3(2u - 6) + 2(-5u + 4) = 7$

$6u - 18 - 10u + 8 = 7$

$-4u = 17$

$u = -4\frac{1}{4}$

$u = -4\frac{1}{4}$ geeft $x = 2 \cdot -4\frac{1}{4} - 6 = -14\frac{1}{2}$ en $y = -5 \cdot -4\frac{1}{4} + 4 = 25\frac{1}{4}$

Dus $S(-14\frac{1}{2}, 25\frac{1}{4})$.

c m: $x = 2u - 6 \wedge y = -5u + 4$ geeft m: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix} + u \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$\left.\begin{array}{l} n\text{: } 2x - 5y = c \\ \text{door } O(0, 0) \end{array}\right\} n\text{: } 2x - 5y = 0$

d $p \perp l$, dus $\left.\begin{array}{l} p\text{: } 3x - y = c \\ \text{door } A(6, 8) \end{array}\right\} c = 3 \cdot 6 - 8 = 10$

Dus p: $3x - y = 10$.

$x = 1 - 2t$ en $y = 2 + 3t$ substitueren in $3x - y = 10$ geeft $3(1 - 2t) - (2 + 3t) = 10$

$3 - 6t - 2 - 3t = 10$

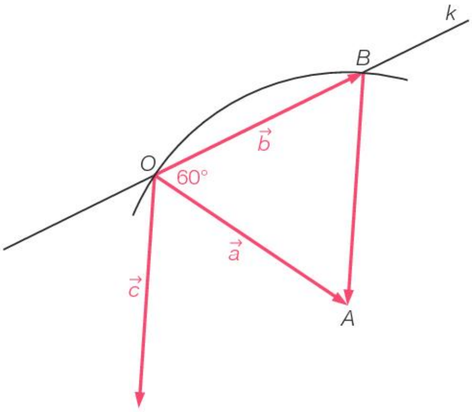
$-9t = 9$

$t = -1$

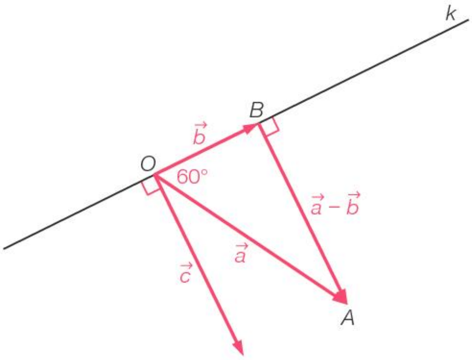
$t = -1$ geeft $x = 1 - 2 \cdot -1 = 3$ en $y = 2 + 3 \cdot -1 = -1$

Dus $T(3, -1)$.

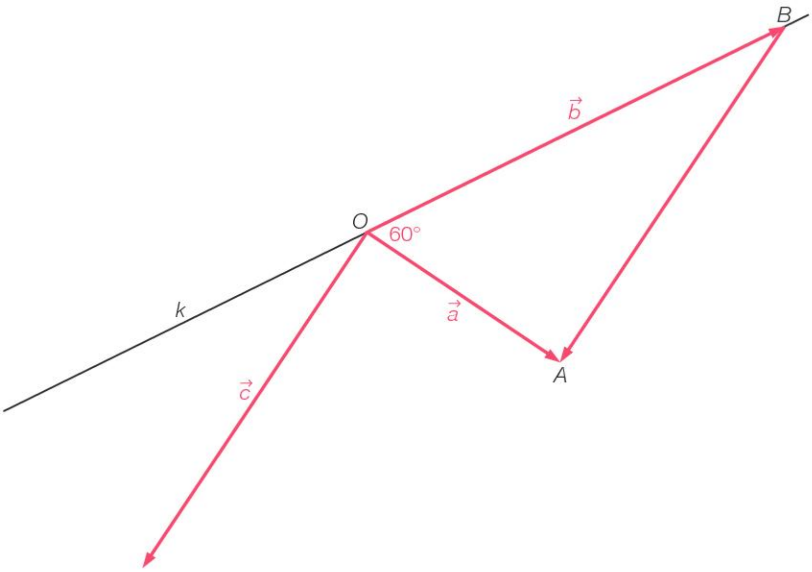
14 **a** A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
Teken een cirkelboog van de cirkel met middelpunt A en straal OA. Deze cirkelboog snijdt k in O en B. Je hebt nu $\vec{b}$ en $\overrightarrow{BA} = \vec{a} - \vec{b}$.
Teken $\vec{c}$ evenwijdig met $\overrightarrow{BA}$ en even lang als $\overrightarrow{BA}$.



b A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
$|\vec{c}| = \sqrt{3} \cdot |\vec{b}|$ betekent dat driehoek OBA een bijzondere rechthoekige driehoek is met zijden waarvan de lengten zich verhouden als $1 : 2 : \sqrt{3}$.
Teken lijnstuk AB loodrecht op k. Je hebt nu $\vec{b}$ en $\overrightarrow{BA} = \vec{a} - \vec{b}$.
Teken $\vec{c}$ evenwijdig met $\overrightarrow{BA}$ en even lang als $\overrightarrow{BA}$.



c Zie de figuur hieronder.
A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2OA$,
$\overrightarrow{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2OA)^2 - 2 \cdot OA \cdot 2OA \cdot \cos(60°)$
$AB^2 = OA^2 + 4OA^2 - 4OA^2 \cdot \frac{1}{2}$
$AB^2 = 3OA^2$
$AB = |\overrightarrow{BA}| = |\vec{c}|$ en $OA = |\vec{a}|$, dus $|\vec{c}| = \sqrt{3} \cdot |\vec{a}|$.

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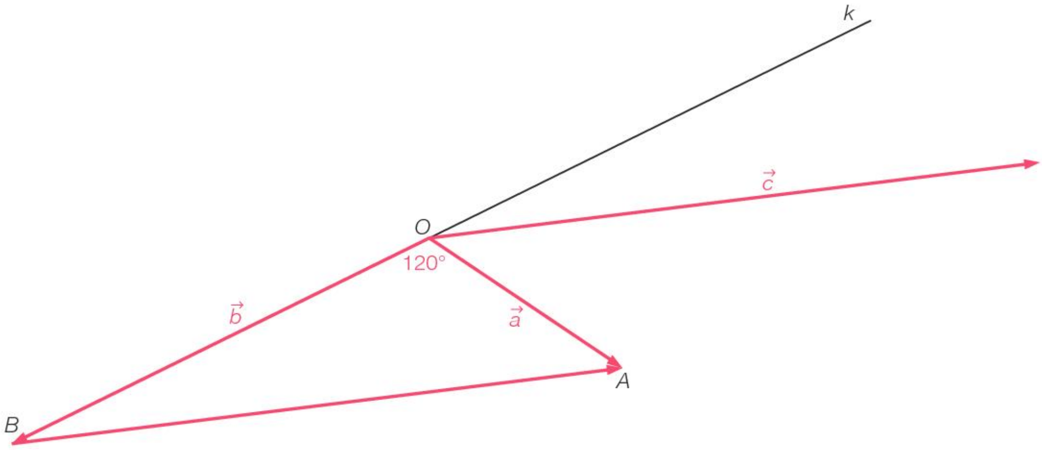
Zie de figuur hieronder.
A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2OA$,
$\overrightarrow{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2OA)^2 - 2 \cdot OA \cdot 2OA \cdot \cos(120°)$

$$AB^2 = OA^2 + 4OA^2 - 4OA^2 \cdot -\tfrac{1}{2}$$
$$AB^2 = 7OA^2$$

$AB = |\overrightarrow{BA}| = |\vec{c}|$ en $OA = |\vec{a}|$, dus $|\vec{c}| = \sqrt{7} \cdot |\vec{a}|$.

Bladzijde 229

15 **a** E is het snijpunt van de diagonalen DH en CI van het vierkant $DCHI$ en is dus het midden van diagonaal DH.

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix}$$

$$\vec{d} = \vec{a} + \overrightarrow{AD} = \vec{a} + \overrightarrow{AB}_L = \begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$$

$$\vec{h} = \vec{b} + 2\overrightarrow{BC} = \vec{b} + 2\overrightarrow{AD} = \begin{pmatrix}3\\0\end{pmatrix} + 2\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}5\\6\end{pmatrix}$$

$$\vec{e} = \tfrac{1}{2}(\vec{d} + \vec{h}) = \tfrac{1}{2}\left(\begin{pmatrix}1\\4\end{pmatrix} + \begin{pmatrix}5\\6\end{pmatrix}\right) = \tfrac{1}{2}\begin{pmatrix}6\\10\end{pmatrix} = \begin{pmatrix}3\\5\end{pmatrix}, \text{ dus } E(3, 5).$$

b $\vec{c} = \vec{b} + \overrightarrow{BC} = \vec{b} + \overrightarrow{AD} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$, dus $C(4, 3)$.

$$\vec{f} = \tfrac{1}{2}(\vec{d} + \vec{e}) = \tfrac{1}{2}\left(\begin{pmatrix}1\\4\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}\right) = \tfrac{1}{2}\begin{pmatrix}4\\9\end{pmatrix} = \begin{pmatrix}2\\4\tfrac{1}{2}\end{pmatrix}, \text{ dus } F(2, 4\tfrac{1}{2}).$$

Stel CF: $y = ax + b$ met $a = \dfrac{3 - 4\frac{1}{2}}{4 - 2} = -\tfrac{3}{4}$.

$$\left.\begin{array}{l} y = -\tfrac{3}{4}x + b \\ \text{door } C(4, 3) \end{array}\right\} \begin{array}{l} -\tfrac{3}{4} \cdot 4 + b = 3 \\ -3 + b = 3 \\ b = 6 \end{array}$$

Dus CF: $y = -\tfrac{3}{4}x + 6$ en $G(0, 6)$.

$$CG = \sqrt{(0-4)^2 + (6-3)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

16 **a** $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix}c\\d\end{pmatrix} - \begin{pmatrix}a\\0\end{pmatrix} = \begin{pmatrix}c - a\\d\end{pmatrix}$

$$\vec{e} = \vec{a} + \overrightarrow{AE} = \vec{a} + \overrightarrow{AC}_L = \begin{pmatrix}a\\0\end{pmatrix} + \begin{pmatrix}-d\\c - a\end{pmatrix} = \begin{pmatrix}a - d\\c - a\end{pmatrix}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix}c\\d\end{pmatrix} - \begin{pmatrix}b\\0\end{pmatrix} = \begin{pmatrix}c - b\\d\end{pmatrix}$$

$$\vec{f} = \vec{b} + \overrightarrow{BF} = \vec{b} + \overrightarrow{BC}_R = \begin{pmatrix}b\\0\end{pmatrix} + \begin{pmatrix}d\\b - c\end{pmatrix} = \begin{pmatrix}b + d\\b - c\end{pmatrix}$$

$$\vec{m} = \tfrac{1}{2}(\vec{e} + \vec{f}) = \tfrac{1}{2}\left(\begin{pmatrix}a - d\\c - a\end{pmatrix} + \begin{pmatrix}b + d\\b - c\end{pmatrix}\right) = \tfrac{1}{2}\begin{pmatrix}a + b\\-a + b\end{pmatrix}, \text{ dus } M(\tfrac{1}{2}a + \tfrac{1}{2}b, -\tfrac{1}{2}a + \tfrac{1}{2}b).$$

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b $\vec{n} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix}$

$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{AE} = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} -d \\ c-a \end{pmatrix} = \begin{pmatrix} c-d \\ -a+c+d \end{pmatrix}$

$\vec{p} = \frac{1}{2}(\vec{a} + \vec{d}) = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} c-d \\ -a+c+d \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a+c-d \\ -a+c+d \end{pmatrix}$

$\overrightarrow{NP} = \vec{p} - \vec{n} = \frac{1}{2}\begin{pmatrix} a+c-d \\ -a+c+d \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -b+c-d \\ -a+c+d \end{pmatrix}$

$\vec{g} = \vec{c} + \overrightarrow{CG} = \vec{c} + \overrightarrow{BF} = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} d \\ b-c \end{pmatrix} = \begin{pmatrix} c+d \\ b-c+d \end{pmatrix}$

$\vec{q} = \frac{1}{2}(\vec{b} + \vec{g}) = \frac{1}{2}\left(\begin{pmatrix} b \\ 0 \end{pmatrix} + \begin{pmatrix} c+d \\ b-c+d \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} b+c+d \\ b-c+d \end{pmatrix}$

$\overrightarrow{NQ} = \vec{q} - \vec{n} = \frac{1}{2}\begin{pmatrix} b+c+d \\ b-c+d \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a+c+d \\ b-c+d \end{pmatrix}$

$\overrightarrow{NQ} = \overrightarrow{NP}_R$, dus $NP = NQ$ en $NP \perp NQ$, dus driehoek PQN is een gelijkbenige rechthoekige driehoek.

17 **a** $\overrightarrow{DA} = \vec{a} - \vec{d} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ d \end{pmatrix} = \begin{pmatrix} a \\ -d \end{pmatrix}$

$\vec{c} = \vec{d} + \overrightarrow{DA}_L = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a+d \end{pmatrix}$

$\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} d \\ a+d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} d-a \\ d+a \end{pmatrix}$

$\vec{p} = \vec{a} + \overrightarrow{AP} = \vec{a} + \overrightarrow{AC}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} a+d \\ a-d \end{pmatrix} = \begin{pmatrix} 2a+d \\ a-d \end{pmatrix}$

$P(10, -1)$ geeft $\begin{cases} 2a + d = 10 \\ a - d = -1 \end{cases}$ +

$3a = 9$

$\left.\begin{array}{l} a = 3 \\ a - d = -1 \end{array}\right\} \begin{array}{l} 3 - d = -1 \\ d = 4 \end{array}$

Dus $a = 3$ en $d = 4$.

b $\left.\begin{array}{l} x + y = 6 \\ P(2a+d, a-d) \end{array}\right\} \begin{array}{l} 2a + d + a - d = 6 \\ 3a = 6 \\ a = 2 \end{array}$

$a = 2$ geeft $\vec{p} = \begin{pmatrix} 4+d \\ 2-d \end{pmatrix}$ en $\overrightarrow{AC} = \begin{pmatrix} d-2 \\ d+2 \end{pmatrix}$.

$\vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{AC} = \begin{pmatrix} 4+d \\ 2-d \end{pmatrix} + \begin{pmatrix} d-2 \\ d+2 \end{pmatrix} = \begin{pmatrix} 2d+2 \\ 4 \end{pmatrix}$

$\left.\begin{array}{l} y = \frac{1}{2}x^2 \\ Q(2d+2, 4) \end{array}\right\} \begin{array}{l} \frac{1}{2}(2d+2)^2 = 4 \\ (2d+2)^2 = 8 \\ 2d + 2 = 2\sqrt{2} \vee 2d + 2 = -2\sqrt{2} \\ d + 1 = \sqrt{2} \vee d + 1 = -\sqrt{2} \\ d = -1 + \sqrt{2} \vee d = -1 - \sqrt{2} \end{array}$

vold. niet

Dus $d = -1 + \sqrt{2}$.

18 **a** $\left.\begin{array}{l} AB: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{a}) \\ \vec{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{array}\right\} AB: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$x = 99$ oftewel $-1 + t = 99$ geeft $t = 100$

$t = 100$ geeft $y = 5 + 100 \cdot -2 = -195$

Dus $D(99, -195)$ op de lijn AB.

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b $BC: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t \cdot (\vec{c} - \vec{b})$

$\vec{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ en $\vec{c} - \vec{b} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\Big\}$ $BC: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$k: x + 3y = c$
door $A(-1, 5)$ $\Big\}$ $c = -1 + 3 \cdot 5 = 14$

Dus $k: x + 3y = 14$.

Substitutie van $x = 2 + t$ en $y = -1 + 3t$ in $x + 3y = 14$ geeft $2 + t + 3(-1 + 3t) = 14$

$2 + t - 3 + 9t = 14$
$10t = 15$
$t = 1\frac{1}{2}$

$t = 1\frac{1}{2}$ geeft $x = 2 + 1\frac{1}{2} = 3\frac{1}{2}$ en $y = -1 + 3 \cdot 1\frac{1}{2} = 3\frac{1}{2}$, dus $S(3\frac{1}{2}, 3\frac{1}{2})$.

c $AP = BP$, dus P ligt op de middelloodlijn m van AB.

$m \perp AB$, dus $\vec{r}_m = \vec{n}_{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

$\vec{s}_m = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$

Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

P op m, dus $\vec{p} = \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix}$.

$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2t + 1\frac{1}{2} \\ t - 3 \end{pmatrix}$

$\overrightarrow{BP} = \vec{p} - \vec{b} = \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2t - 1\frac{1}{2} \\ t + 3 \end{pmatrix}$

$AP \perp BP$, dus $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$

$\begin{pmatrix} 2t + 1\frac{1}{2} \\ t - 3 \end{pmatrix} \cdot \begin{pmatrix} 2t - 1\frac{1}{2} \\ t + 3 \end{pmatrix} = 0$
$4t^2 - 2\frac{1}{4} + t^2 - 9 = 0$
$5t^2 = 11\frac{1}{4}$
$t^2 = 2\frac{1}{4}$
$t = 1\frac{1}{2} \vee t = -1\frac{1}{2}$

$t = 1\frac{1}{2}$ geeft $P(\frac{1}{2} + 2 \cdot 1\frac{1}{2}, 2 + 1\frac{1}{2}) = P(3\frac{1}{2}, 3\frac{1}{2})$

$t = -1\frac{1}{2}$ geeft $P(\frac{1}{2} + 2 \cdot -1\frac{1}{2}, 2 - 1\frac{1}{2}) = P(-2\frac{1}{2}, \frac{1}{2})$

d $BC: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (zie b)
$BC: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ 38 \end{pmatrix} + t \cdot \begin{pmatrix} q \\ 3 \end{pmatrix}$ $\Big\}$ $q = 1$

$(2, -1)$ op BC, dus $38 + 3t = -1$
$3t = -39$
$t = -13$

$q = 1$ en $t = -13$ geeft $p + -13 \cdot 1 = 2$
$p - 13 = 2$
$p = 15$

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19 **a** $\overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$

$\vec{c} = \vec{b} + \overrightarrow{BC} = \vec{b} + \overrightarrow{BA}_R = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 16 \\ 8 \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix}$

$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{b} + \overrightarrow{BA} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 16 \end{pmatrix} = \begin{pmatrix} 16 \\ 24 \end{pmatrix}$

$$\left.\begin{array}{l} CD: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t \cdot (\vec{d} - \vec{c}) \\ \vec{c} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} 16 \\ 24 \end{pmatrix} - \begin{pmatrix} 24 \\ 8 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{array}\right\} CD: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$x_P = 22$ en P op CD geeft $24 + t = 22$, dus $t = -2$.
$t = -2$ geeft $y = 8 + -2 \cdot -2 = 12$ en dit is y_P.
Verder geldt $x_D \leq x_P \leq x_C$ en $y_C \leq y_P \leq y_D$.
Dus P ligt op de zijde CD.

b
$$\left.\begin{array}{l} AP: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{p} - \vec{a}) \\ \vec{a} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} \text{ en } \vec{p} - \vec{a} = \begin{pmatrix} 22 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 16 \end{pmatrix} = \begin{pmatrix} 22 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 11 \\ -2 \end{pmatrix} \end{array}\right\} AP: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} + t \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix}$$

Q op AP, dus $\vec{q} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix}$.

$\overrightarrow{DQ} = \vec{q} - \vec{d} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix} - \begin{pmatrix} 16 \\ 24 \end{pmatrix} = \begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix}$

DQ loodrecht op AP, dus $\begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix} = 0$

$121t - 176 + 4t + 16 = 0$
$125t = 160$
$t = 1\frac{7}{25}$

$t = 1\frac{7}{25}$ geeft $Q(11 \cdot 1\frac{7}{25}, 16 - 2 \cdot 1\frac{7}{25}) = Q(14\frac{2}{25}, 13\frac{11}{25})$

20 **a** $x(t) = t^2 - 4t$ geeft $x'(t) = 2t - 4$
$y(t) = t^4 - 4t^3 + 4t^2$ geeft $y'(t) = 4t^3 - 12t^2 + 8t$
Evenwijdig met de x-as als $y'(t) = 0 \wedge x'(t) \neq 0$
$4t^3 - 12t^2 + 8t = 0 \wedge 2t - 4 \neq 0$
$4t(t^2 - 3t + 2) = 0 \wedge 2t \neq 4$
$4t(t - 1)(t - 2) = 0 \wedge t \neq 2$
$(t = 0 \vee t = 1 \vee t = 2) \wedge t \neq 2$
$t = 0 \vee t = 1$

$t = 0$ geeft het punt $(0, 0)$.
$t = 1$ geeft het punt $(-3, 1)$.
Dus evenwijdig met de x-as in de punten $(0, 0)$ en $(-3, 1)$.
Evenwijdig met de y-as als $x'(t) = 0 \wedge y'(t) \neq 0$
$t = 2 \wedge t \neq 0 \wedge t \neq 1 \wedge t \neq 2$
Er is dus geen waarde van t die voldoet, dus er zijn geen punten waarin de raaklijn evenwijdig is met de y-as.

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b Naar links betekent $x'(t) < 0$

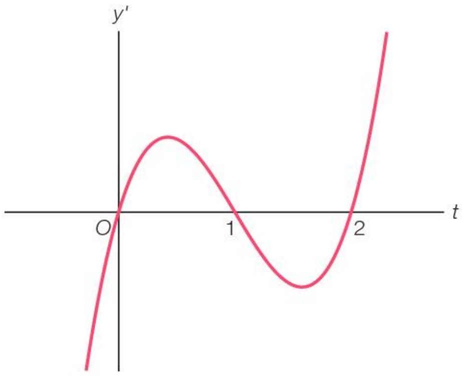
$2t - 4 < 0$

$2t < 4$

$t < 2$

$y'(t) = 0$ geeft $t = 0 \vee t = 1 \vee t = 2$ (zie a)

Omhoog betekent $y'(t) > 0$.



$y'(t) > 0$ geeft $0 < t < 1 \vee t > 2$

$x'(t) < 0 \wedge y'(t) > 0$

$t < 2 \wedge (0 < t < 1 \vee t > 2)$

$0 < t < 1$

Dus naar links en omhoog voor $0 < t < 1$.

c $x(t) = -3$ geeft $t^2 - 4t = -3$

$t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$

$t = 1 \vee t = 3$

$t = 1$ geeft het punt $(-3, 1)$, dus voldoet niet.

$t = 3$ geeft het punt $(-3, 9)$, dus voldoet.

$x'(3) = 2 \cdot 3 - 4 = 2$ en $y'(3) = 4 \cdot 3^3 - 12 \cdot 3^2 + 8 \cdot 3 = 24$

De baansnelheid is $v'(3) = \sqrt{2^2 + 24^2} = \sqrt{580} = 2\sqrt{145}$.

d $v(t) = \sqrt{(2t - 4)^2 + (4t^3 - 12t^2 + 8t)^2}$

Voer in $y_1 = \sqrt{(2x - 4)^2 + (4x^3 - 12x^2 + 8x)^2}$.

De optie minimum geeft $x = 2$ en $y = 0$.

De minimale baansnelheid is 0 voor $t = 2$.

e $x'(t) = 2t - 4$ geeft $x''(t) = 2$

$y'(t) = 4t^3 - 12t^2 + 8t$ geeft $y''(t) = 12t^2 - 24t + 8$

De versnellingsvector is evenwijdig met de lijn $y = x$ als $y''(t) = x''(t)$

$12t^2 - 24t + 8 = 2$

$12t^2 - 24t + 6 = 0$

$2t^2 - 4t + 1 = 0$

$D = (-4)^2 - 4 \cdot 2 \cdot 1 = 8$, dus $\sqrt{D} = \sqrt{8} = 2\sqrt{2}$

$t = \frac{4 + 2\sqrt{2}}{4} \vee t = \frac{4 - 2\sqrt{2}}{4}$

$t = 1 + \frac{1}{2}\sqrt{2} \vee t = 1 - \frac{1}{2}\sqrt{2}$

21 **a** $y(t) = 0$ geeft $t^2 - 1 = 0$

$t^2 = 1$

$t = 1 \vee t = -1$

$t = 1$ geeft het punt $(0, 0)$.

$t = -1$ geeft het punt $(6, 0)$.

Dus de snijpunten met de x-as zijn $(0, 0)$ en $(6, 0)$.

$x(t) = 0$ geeft $-t^3 + 3t^2 - 2t = 0$

$-t(t^2 - 3t + 2) = 0$

$-t(t - 1)(t - 2) = 0$

$t = 0 \vee t = 1 \vee t = 2$

$t = 0$ geeft het punt $(0, -1)$.

$t = 1$ geeft het punt $(0, 0)$.

$t = 2$ geeft het punt $(0, 3)$.

Dus de snijpunten met de y-as zijn $(0, -1)$, $(0, 0)$ en $(0, 3)$.

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b $x(t) = -t^3 + 3t^2 - 2t$ geeft $x'(t) = -3t^2 + 6t - 2$

$y(t) = t^2 - 1$ geeft $y'(t) = 2t$

Evenwijdig met de x-as als $y'(t) = 0 \wedge x'(t) \neq 0$ en evenwijdig met de y-as als $x'(t) = 0 \wedge y'(t) \neq 0$.

$x'(t) = 0$ geeft $-3t^2 + 6t - 2 = 0$

$$D = 6^2 - 4 \cdot -3 \cdot -2 = 12, \text{ dus } \sqrt{D} = \sqrt{12} = 2\sqrt{3}$$

$$t = \frac{-6 + 2\sqrt{3}}{-6} \vee t = \frac{-6 - 2\sqrt{3}}{-6}$$

$$t = 1 - \tfrac{1}{3}\sqrt{3} \vee t = 1 + \tfrac{1}{3}\sqrt{3}$$

$y'(t) = 0$ geeft $2t = 0$

$$t = 0$$

Dus raaklijn evenwijdig met de x-as of met de y-as voor $t = 0 \vee t = 1 - \frac{1}{3}\sqrt{3} \vee t = 1 + \frac{1}{3}\sqrt{3}$.

c Evenwijdig met de lijn $y = 2x + 3$ geeft $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$2 \cdot x'(t) = 1 \cdot y'(t)$$

$$2(-3t^2 + 6t - 2) = 2t$$

$$-6t^2 + 12t - 4 = 2t$$

$$-6t^2 + 10t - 4 = 0$$

$$3t^2 - 5t + 2 = 0$$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$$

$$t = \frac{5 + 1}{6} = 1 \vee t = \frac{5 - 1}{6} = \tfrac{2}{3}$$

$t = 1$ geeft het punt $(0, 0)$.

$t = \frac{2}{3}$ geeft het punt $(-\frac{8}{27}, -\frac{5}{9})$.

Dus de punten zijn $(0, 0)$ en $(-\frac{8}{27}, -\frac{5}{9})$.

d $y(t) = 8$ geeft $t^2 - 1 = 8$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$t = 3$ geeft het punt $(-6, 8)$, dus voldoet.

$t = -3$ geeft het punt $(60, 8)$, dus voldoet niet.

$x'(3) = -3 \cdot 3^2 + 6 \cdot 3 - 2 = -11$ en $y'(3) = 2 \cdot 3 = 6$

De baansnelheid is $v(3) = \sqrt{(-11)^2 + 6^2} = \sqrt{157}$.

e $y(t) = -\frac{3}{4}$ geeft $t^2 - 1 = -\frac{3}{4}$

$$t^2 = \tfrac{1}{4}$$

$$t = \tfrac{1}{2} \vee t = -\tfrac{1}{2}$$

$x(\frac{1}{2}) = -\frac{1}{8} + 3 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2} = -\frac{3}{8}$, dus $t = \frac{1}{2}$ voldoet.

$x(-\frac{1}{2}) = \frac{1}{8} + 3 \cdot \frac{1}{4} - 2 \cdot -\frac{1}{2} = 1\frac{7}{8}$, dus $t = -\frac{1}{2}$ voldoet niet.

$$v(t) = \sqrt{(-3t^2 + 6t - 2)^2 + (2t)^2} = \sqrt{9t^4 - 18t^3 + 6t^2 - 18t^3 + 36t^2 - 12t + 6t^2 - 12t + 4 + 4t^2}$$

$$= \sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}} \cdot (36t^3 - 108t^2 + 104t - 24) = \frac{18t^3 - 54t^2 + 52t - 12}{\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}}$$

$$a(\tfrac{1}{2}) = \frac{18 \cdot \frac{1}{8} - 54 \cdot \frac{1}{4} + 52 \cdot \frac{1}{2} - 12}{\sqrt{9 \cdot \frac{1}{16} - 36 \cdot \frac{1}{8} + 52 \cdot \frac{1}{4} - 24 \cdot \frac{1}{2} + 4}} = \frac{2\frac{3}{4}}{\sqrt{1\frac{1}{16}}} = \tfrac{11}{17}\sqrt{17}$$

Dus de baanversnelling is $\frac{11}{17}\sqrt{17}$.

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Bladzijde 231

22 **a** $F(x) = x + \ln\left(\frac{x-1}{x+1}\right)$ geeft

$$F'(x) = 1 + \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2} = 1 + \frac{x+1}{x-1} \cdot \frac{x+1-x+1}{(x+1)^2} = 1 + \frac{2}{(x-1)(x+1)} = \frac{x^2-1+2}{x^2-1}$$
$$= \frac{x^2+1}{x^2-1}$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

b $F(x) = x\ln\left(\frac{x+1}{x-1}\right) + \ln(x^2-1)$ geeft

$$F'(x) = 1 \cdot \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{1}{\frac{x+1}{x-1}} \cdot \frac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2} + \frac{1}{x^2-1} \cdot 2x$$
$$= \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{x-1}{x+1} \cdot \frac{x-1-x-1}{(x-1)^2} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{x-1}{x+1} \cdot \frac{-2}{(x-1)^2} + \frac{2x}{x^2-1}$$
$$= \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{1}{x+1} \cdot \frac{-2}{x-1} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right) - \frac{2x}{x^2-1} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right)$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

c $F(x) = -x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x)$ geeft

$$F'(x) = -3x^2 \cdot \cos(x) - x^3 \cdot -\sin(x) + 6x \cdot \sin(x) + 3x^2 \cdot \cos(x) + 6 \cdot \cos(x) + 6x \cdot -\sin(x) - 6\cos(x)$$
$$= -3x^2\cos(x) + x^3\sin(x) + 6x\sin(x) + 3x^2\cos(x) + 6\cos(x) - 6x\sin(x) - 6\cos(x)$$
$$= x^3\sin(x)$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

d $F(x) = (6x^2 - 2x - 4)\sqrt{x-1}$ geeft

$$F'(x) = (12x-2) \cdot \sqrt{x-1} + (6x^2-2x-4) \cdot \frac{1}{2\sqrt{x-1}} = (12x-2)\sqrt{x-1} + \frac{3x^2-x-2}{\sqrt{x-1}}$$
$$= \frac{(12x-2)(x-1)}{\sqrt{x-1}} + \frac{3x^2-x-2}{\sqrt{x-1}} = \frac{12x^2-12x-2x+2+3x^2-x-2}{\sqrt{x-1}}$$
$$= \frac{15x^2-15x}{\sqrt{x-1}} = \frac{15x(x-1)}{\sqrt{x-1}} = 15x\sqrt{x-1}$$

Dus $F'(x) = f(x)$ oftewel F is een primitieve van f.

23 **a** $f(x) = 7 \cdot \log(5x)$ geeft $F(x) = 7 \cdot \frac{1}{5} \cdot \frac{1}{\ln(10)} \cdot (5x\ln(5x) - 5x) + c = \frac{7(x\ln(5x) - x)}{\ln(10)} + c$

b $f(x) = e^{\frac{1}{2}x-1}$ geeft $F(x) = 2e^{\frac{1}{2}x-1} + c$

c $f(x) = 10\ln(2x-4)$ geeft
$F(x) = 10 \cdot \frac{1}{2}((2x-4)\ln(2x-4) - (2x-4)) + c = 5(2x-4)\ln(2x-4) - 5(2x-4) + c$

d $f(x) = \frac{x^4 - 6x^2\sqrt{x} + 8x}{x^3} = \frac{x^4}{x^3} - \frac{6x^{2\frac{1}{2}}}{x^3} + \frac{8x}{x^3} = x - 6x^{-\frac{1}{2}} + 8x^{-2}$ geeft

$F(x) = \frac{1}{2}x^2 - \frac{6}{\frac{1}{2}} \cdot x^{\frac{1}{2}} + \frac{8}{-1} \cdot x^{-1} + c = \frac{1}{2}x^2 - 12\sqrt{x} - \frac{8}{x} + c$

e $f(x) = (x^2+3)^2 = x^4 + 6x^2 + 9$ geeft $F(x) = \frac{1}{5}x^5 + 2x^3 + 9x + c$

f $f(x) = 3\sin(4x)$ geeft $F(x) = 3 \cdot -\frac{1}{4}\cos(4x) + c = -\frac{3}{4}\cos(4x) + c$

24 **a** $f(x) = \sqrt{6x+3} = (6x+3)^{\frac{1}{2}}$ geeft $F(x) = \frac{1}{6} \cdot \frac{2}{3}(6x+3)^{1\frac{1}{2}} + c = \frac{1}{9}(6x+3)\sqrt{6x+3} + c$

b $f(x) = \frac{10}{2x-1}$ geeft $F(x) = 10 \cdot \frac{1}{2}\ln|2x-1| + c = 5\ln|2x-1| + c$

c $f(x) = (3x-6)^{-2}$ geeft $F(x) = \frac{1}{3} \cdot -(3x-6)^{-1} + c = -\frac{1}{3}(3x-6)^{-1} + c$

d $f(x) = (2x+5)^{-1} = \frac{1}{2x+5}$ geeft $F(x) = \frac{1}{2}\ln|2x+5| + c$

e $f(x) = 10^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \frac{10^{2x-3}}{\ln(10)} + c = \frac{10^{2x-3}}{2\ln(10)} + c$

f $f(x) = \frac{8^x - 1}{2^x} = \frac{8^x}{2^x} - \frac{1}{2^x} = 4^x - 2^{-x}$ geeft $F(x) = \frac{4^x}{\ln(4)} + \frac{2^{-x}}{\ln(2)} + c$

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25 $F(x) = (ax^2 + bx + c)\,e^x$ geeft
$F'(x) = (2ax + b)\cdot e^x + (ax^2 + bx + c)\cdot e^x = (ax^2 + (2a + b)x + b + c)\,e^x$
Dit moet gelijk zijn aan $f(x) = (x^2 + 1)\,e^x$, dus $a = 1$, $2a + b = 0$ en $b + c = 1$.

$$\left.\begin{array}{l} a = 1 \\ 2a + b = 0 \end{array}\right\} \begin{array}{l} 2 + b = 0 \\ b = -2 \end{array}$$

$$\left.\begin{array}{l} b = -2 \\ b + c = 1 \end{array}\right\} \begin{array}{l} -2 + c = 1 \\ c = 3 \end{array}$$

Dus $a = 1$, $b = -2$ en $c = 3$.

26 $f(x) = px$ geeft $x^3 - 3x = px$

$$\begin{aligned} x^3 &= px + 3x \\ x^3 &= (p + 3)x \\ x = 0 &\vee x^2 = p + 3 \\ x = 0 &\vee x = \sqrt{p+3} \vee x = -\sqrt{p+3} \end{aligned}$$

vold. niet

$$O(V) = \int_0^{\sqrt{p+3}} (px - f(x))\,dx = \int_0^{\sqrt{p+3}} (px - (x^3 - 3x))\,dx = \int_0^{\sqrt{p+3}} (-x^3 + (p + 3)x)\,dx$$

$$= \left[-\tfrac{1}{4}x^4 + (p + 3)\cdot\tfrac{1}{2}x^2\right]_0^{\sqrt{p+3}} = -\tfrac{1}{4}\cdot(p + 3)^2 + (p + 3)\cdot\tfrac{1}{2}\cdot(p + 3) - 0 = \tfrac{1}{4}(p + 3)^2$$

$O(V) = 9$ geeft $\tfrac{1}{4}(p + 3)^2 = 9$

$$\begin{aligned} (p + 3)^2 &= 36 \\ p + 3 = 6 &\vee p + 3 = -6 \\ p = 3 &\vee p = -9 \end{aligned}$$

vold. niet

Dus $p = 3$.

27 $O(ABCD) = 5\cdot(2 - p) = 10 - 5p$
V is het vlakdeel dat wordt ingesloten door de grafiek van f en het lijnstuk CD.

$$O(V) = \int_0^5 (2 - f(x))\,dx = \int_0^5 \left(2 - \tfrac{1}{2}x + 1 - \frac{3}{x+1}\right)dx = \int_0^5 \left(3 - \tfrac{1}{2}x - \frac{3}{x+1}\right)dx = \left[3x - \tfrac{1}{4}x^2 - 3\ln|x + 1|\right]_0^5$$

$$= 15 - 6\tfrac{1}{4} - 3\ln(6) - (0 - 0 - 0)) = 8\tfrac{3}{4} - 3\ln(6)$$

$O(V) = \tfrac{1}{2}O(ABCD)$ geeft $8\tfrac{3}{4} - 3\ln(6) = 5 - 2\tfrac{1}{2}p$

$$\begin{aligned} 2\tfrac{1}{2}p &= 3\ln(6) - 3\tfrac{3}{4} \\ p &= 1\tfrac{1}{5}\ln(6) - 1\tfrac{1}{2} \end{aligned}$$

Bladzijde 232

28 **a** 54 km/uur = 15 m/s
Versnelling van de wielrenner constant a en beginsnelheid 15 geeft snelheid $v_w(t) = at + 15$.
$v_w(t) = at + 15$ geeft $s_w(t) = \tfrac{1}{2}at^2 + 15t + c$

$$\left.\begin{array}{l} s_w(t) = \tfrac{1}{2}at^2 + 15t + c \\ s_w(0) = 0 \end{array}\right\} s_w(t) = \tfrac{1}{2}at^2 + 15t$$

$v_w(t) = 0$ geeft $at + 15 = 0$

$$\begin{aligned} at &= -15 \\ t &= -\frac{15}{a} \end{aligned}$$

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$s_w\left(-\frac{15}{a}\right) = 40$ geeft $\frac{1}{2}a \cdot \left(-\frac{15}{a}\right)^2 + 15 \cdot -\frac{15}{a} = 40$

$$\frac{1}{2}a \cdot \frac{225}{a^2} - \frac{225}{a} = 40$$

$$\frac{112\frac{1}{2}}{a} - \frac{225}{a} = 40$$

$$-\frac{112\frac{1}{2}}{a} = 40$$

$$a = -\frac{112\frac{1}{2}}{40} = -2{,}8125$$

Dus de versnelling van de wielrenner is $-2{,}8125$ m/s^2.

b $a = -2{,}5$, $v_w(0) = 15$ en $s_w(0) = 0$ geeft $v_w(t) = -2{,}5t + 15$ en $s_w(t) = -1{,}25t^2 + 15t$
6 km/uur $= 1\frac{2}{3}$ m/s
Voor de auto geldt $v_a(t) = 1\frac{2}{3}$.

$$\left.\begin{array}{l} v_a(t) = 1\frac{2}{3} \text{ geeft } s_a(t) = 1\frac{2}{3}t + b \\ s_a(0) = 40 \end{array}\right\} s_a(t) = 1\frac{2}{3}t + 40$$

$v_w(t) = v_a(t)$
$-2{,}5t + 15 = 1\frac{2}{3}$
$-2{,}5t = -13\frac{1}{3}$
$t = 5\frac{1}{3}$
$s_w(5\frac{1}{3}) = -1{,}25 \cdot (5\frac{1}{3})^2 + 15 \cdot 5\frac{1}{3} = 44\frac{4}{9}$
$s_a(5\frac{1}{3}) = 1\frac{2}{3} \cdot 5\frac{1}{3} + 40 = 48\frac{8}{9}$
Dus op het moment dat de wielrenner en de auto dezelfde snelheid hebben, rijdt de auto $48\frac{8}{9} - 44\frac{4}{9} = 4\frac{4}{9}$ meter voor de wielrenner.
De wielrenner kan dus op tijd stoppen.

29 **a** $O(V) = \int_0^1 (3^x + 1)\,dx = \left[\frac{3^x}{\ln(3)} + x\right]_0^1 = \frac{3}{\ln(3)} + 1 - \frac{1}{\ln(3)} = \frac{2}{\ln(3)} + 1$

b $I(L) = \pi\int_0^1 (3^x + 1)^2\,dx = \pi\int_0^1 (3^{2x} + 2 \cdot 3^x + 1)\,dx = \pi\left[\frac{1}{2} \cdot \frac{3^{2x}}{\ln(3)} + 2 \cdot \frac{3^x}{\ln(3)} + x\right]_0^1$

$$= \pi\left(\frac{4\frac{1}{2}}{\ln(3)} + \frac{6}{\ln(3)} + 1 - \left(\frac{\frac{1}{2}}{\ln(3)} + \frac{2}{\ln(3)} + 0\right)\right) = \pi\left(\frac{4\frac{1}{2} + 6 - \frac{1}{2} - 2}{\ln(3)} + 1\right) = \frac{8\pi}{\ln(3)} + \pi$$

c $y = 3^x + 1$ geeft $3^x + 1 = y$
$3^x = y - 1$
$x = {}^3\log(y - 1)$
$f(0) = 2$ en $f(1) = 4$

$$I(M) = \pi \cdot 1^2 \cdot 4 - \pi\int_2^4 ({}^3\log(y - 1))^2\,dy \approx 9{,}89$$

30 **a** $I = \pi\int_{\frac{1}{3}r}^{\frac{1}{2}r} y^2\,dx = \pi\int_{\frac{1}{3}r}^{\frac{1}{2}r} (r^2 - x^2)\,dx = \pi\left[r^2x - \frac{1}{3}x^3\right]_{\frac{1}{3}r}^{\frac{1}{2}r} = \pi(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot (\frac{1}{2}r)^3) - \pi(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot (\frac{1}{3}r)^3)$

$$= \pi(\tfrac{1}{2}r^3 - \tfrac{1}{24}r^3) - \pi(\tfrac{1}{3}r^3 - \tfrac{1}{81}r^3) = \tfrac{11}{24}\pi r^3 - \tfrac{26}{81}\pi r^3 = \tfrac{89}{648}\pi r^3$$

b $I_{bol} = \frac{4}{3}\pi r^3$

$$I = \pi\int_{-pr}^{pr} y^2\,dx = \pi\int_{-pr}^{pr} (r^2 - x^2)\,dx = \pi\left[r^2x - \tfrac{1}{3}x^3\right]_{-pr}^{pr} = \pi(r^2 \cdot pr - \tfrac{1}{3}(pr)^3) - \pi(r^2 \cdot -pr - \tfrac{1}{3}(-pr)^3)$$

$$= \pi(pr^3 - \tfrac{1}{3}p^3r^3) + \pi(pr^3 - \tfrac{1}{3}p^3r^3) = \pi(2pr^3 - \tfrac{2}{3}p^3r^3) = (2p - \tfrac{2}{3}p^3)\pi r^3$$

$I = \frac{1}{2}I_{bol}$ geeft $(2p - \frac{2}{3}p^3)\pi r^3 = \frac{2}{3}\pi r^3$
$2p - \frac{2}{3}p^3 = \frac{2}{3}$
Voer in $y_1 = 2x - \frac{2}{3}x^3$ en $y_2 = \frac{2}{3}$.
De optie snijpunt geeft $x = 0{,}347...$
Dus $p \approx 0{,}35$.

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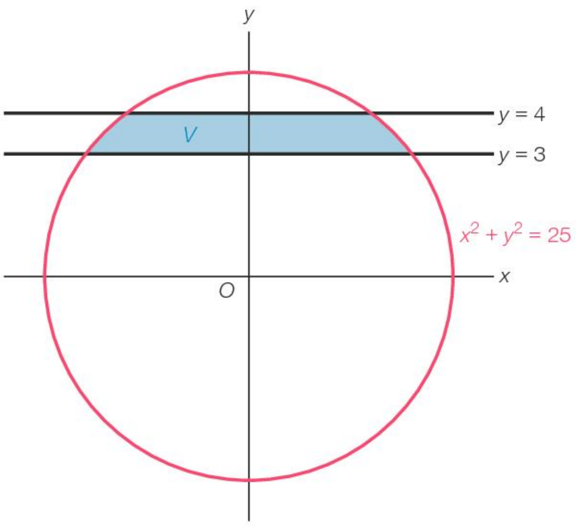
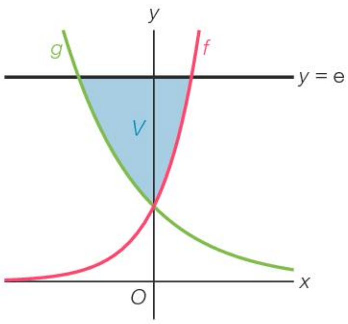


Substitutie van $y = 3$ in $x^2 + y^2 = 25$ geeft $x^2 + 9 = 25$
$x^2 = 16$
$x = 4 \vee x = -4$
Substitutie van $y = 4$ in $x^2 + y^2 = 25$ geeft $x^2 + 16 = 25$
$x^2 = 9$
$x = 3 \vee x = -3$

$$I(L) = \pi\int_{-4}^{-3} y^2\,dx + \pi\int_{-3}^{3} 4^2\,dx + \pi\int_{3}^{4} y^2\,dx - \pi\int_{-4}^{4} 3^2\,dx$$
$$= \pi\int_{-4}^{-3}(25 - x^2)\,dx + \pi\big[16x\big]_{-3}^{3} + \pi\int_{3}^{4}(25 - x^2)\,dx - \pi\big[9x\big]_{-4}^{4}$$
$$= \pi\big[25x - \tfrac{1}{3}x^3\big]_{-4}^{-3} + \pi(48 - -48) + \pi\big[25x - \tfrac{1}{3}x^3\big]_{3}^{4} - \pi(36 - -36)$$
$$= \pi(-75 + 9) - \pi(-100 + \tfrac{64}{3}) + 96\pi + \pi(100 - \tfrac{64}{3}) - \pi(75 - 9) - 72\pi$$
$$= -66\pi + 78\tfrac{2}{3}\pi + 96\pi + 78\tfrac{2}{3}\pi - 66\pi - 72\pi = 49\tfrac{1}{3}\pi$$

Bladzijde 233

32 **a**



$e^{2x} = e$	$e^{-x} = e$	$e^{2x} = e^{-x}$
$2x = 1$	$-x = 1$	$2x = -x$
$x = \frac{1}{2}$	$x = -1$	$x = 0$

e omlaag schuiven geeft

$$I(L) = \pi\int_{-1}^{0}(e^{-x} - e)^2\,dx + \pi\int_{0}^{\frac{1}{2}}(e^{2x} - e)^2\,dx = \pi\int_{-1}^{0}(e^{-2x} - 2\,e^{-x+1} + e^2)\,dx + \pi\int_{0}^{\frac{1}{2}}(e^{4x} - 2\,e^{2x+1} + e^2)\,dx$$
$$= \pi\big[-\tfrac{1}{2}e^{-2x} + 2\,e^{-x+1} + e^2x\big]_{-1}^{0} + \pi\big[\tfrac{1}{4}e^{4x} - e^{2x+1} + e^2x\big]_{0}^{\frac{1}{2}}$$
$$= \pi(-\tfrac{1}{2} + 2e) - \pi(-\tfrac{1}{2}e^2 + 2e^2 - e^2) + \pi(\tfrac{1}{4}e^2 - e^2 + \tfrac{1}{2}e^2) - \pi(\tfrac{1}{4} - e) = (-\tfrac{3}{4}e^2 + 3e - \tfrac{3}{4})\pi$$

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b $\int_{-1}^{0} g(x)\,dx = \int_{-1}^{0} e^{-x}\,dx = [-e^{-x}]_{-1}^{0} = -1 + e$

$\int_{0}^{\frac{1}{2}} f(x)\,dx = \int_{0}^{\frac{1}{2}} e^{2x}\,dx = [\frac{1}{2}e^{2x}]_{0}^{\frac{1}{2}} = \frac{1}{2}e - \frac{1}{2}$

Dus $O(W) = -1 + e + \frac{1}{2}e - \frac{1}{2} = 1\frac{1}{2}e - 1\frac{1}{2}$.

$\int_{-1}^{\ln\left(\frac{4}{e+3}\right)} e^{-x}\,dx = \left[-e^{-x}\right]_{-1}^{\ln\left(\frac{4}{e+3}\right)} = -e^{-\ln\left(\frac{4}{e+3}\right)} + e = -e^{\ln\left(\frac{e+3}{4}\right)} + e = -\frac{e+3}{4} + e = -\frac{1}{4}e - \frac{3}{4} + e = \frac{3}{4}e - \frac{3}{4}$

$\frac{1}{2}O(W) = \frac{1}{2}(1\frac{1}{2}e - 1\frac{1}{2}) = \frac{3}{4}e - \frac{3}{4}$

Dus de lijn $x = \ln\left(\frac{4}{e+3}\right)$ verdeelt W in twee delen met gelijke oppervlakte.

12 Goniometrische formules

33 **a** $\sin(x + \frac{1}{3}\pi) = -\cos(x - \frac{2}{3}\pi)$

$\cos(x - \frac{1}{6}\pi) = \cos(x + \frac{1}{3}\pi)$

$x - \frac{1}{6}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -x - \frac{1}{3}\pi + k \cdot 2\pi$

geen oplossing $\quad 2x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{12}\pi + k \cdot \pi$

b $2\sin(x)\cos(x) = \cos(x + \frac{1}{6}\pi)$

$\sin(2x) = \sin(x + \frac{2}{3}\pi)$

$2x = x + \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = \pi - x - \frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

c $\cos(x + \frac{1}{2}\pi) \cdot \cos(x) = \frac{1}{4}\sqrt{2}$

$\cos(x + \frac{1}{2}\pi) \cdot \sin(x + \frac{1}{2}\pi) = \frac{1}{4}\sqrt{2}$

$\frac{1}{2}\sin(2x + \pi) = \frac{1}{4}\sqrt{2}$

$\sin(2x + \pi) = \frac{1}{2}\sqrt{2}$

$2x + \pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x + \pi = \frac{3}{4}\pi + k \cdot 2\pi$

$2x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{8}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \pi$

d $\sqrt{3} \cdot \sin(2x) + \cos(2x) = 1$

$2\sqrt{3}\sin(x)\cos(x) + 1 - 2\sin^2(x) = 1$

$2\sqrt{3}\sin(x)\cos(x) - 2\sin^2(x) = 0$

$2\sin(x)(\sqrt{3} \cdot \cos(x) - \sin(x)) = 0$

$2\sin(x) = 0 \vee \sqrt{3} \cdot \cos(x) - \sin(x) = 0$

$\sin(x) = 0 \vee \sin(x) = \sqrt{3} \cdot \cos(x)$

$x = k \cdot \pi \vee \frac{\sin(x)}{\cos(x)} = \sqrt{3}$

$x = k \cdot \pi \vee \tan(x) = \sqrt{3}$

$x = k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot \pi$

34 **a** $f(x) = (2 - \sin(x))^2 = 4 - 4\sin(x) + \sin^2(x) = 4 - 4\sin(x) + \frac{1}{2} - \frac{1}{2}\cos(2x)$

$= 4\frac{1}{2} - 4\sin(x) - \frac{1}{2}\cos(2x)$

$F(x) = 4\frac{1}{2}x + 4\cos(x) - \frac{1}{4}\sin(2x) + c$

b $g(x) = \sin(3x)\cos(\frac{1}{2}x) + \cos(3x)\sin(\frac{1}{2}x) = \sin(3\frac{1}{2}x)$

$G(x) = -\frac{2}{7}\cos(3\frac{1}{2}x) + c$

c $h(x) = 4\cos^4(x) - \cos^2(2x) = (2\cos^2(x))^2 - \cos^2(2x) = (1 + \cos(2x))^2 - \cos^2(2x)$

$= 1 + 2\cos(2x) + \cos^2(2x) - \cos^2(2x) = 1 + 2\cos(2x)$

$H(x) = x + \sin(2x) + c$

35 **a** $f(x) = \sin(x) - \cos(x) + \sqrt{2}$ geeft $f'(x) = \cos(x) + \sin(x)$

$f'(x) = 0$ geeft $\cos(x) + \sin(x) = 0$

$\sin(x) = -\cos(x)$

$\tan(x) = -1$

$x = -\frac{1}{4}\pi + k \cdot \pi$

$f(-\frac{1}{4}\pi) = \sin(-\frac{1}{4}\pi) - \cos(-\frac{1}{4}\pi) + \sqrt{2} = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} + \sqrt{2} = 0$

$f(-\frac{1}{4}\pi) = 0$ en $f'(-\frac{1}{4}\pi) = 0$, dus de grafiek van f raakt de x-as.

b $f(\frac{1}{4}\pi - p) + f(\frac{1}{4}\pi + p) = \sin(\frac{1}{4}\pi - p) - \cos(\frac{1}{4}\pi - p) + \sqrt{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + \sqrt{2}$

$= \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) - (\cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p)) + \sqrt{2}$

$+ \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) - (\cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p)) + \sqrt{2}$

$= \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) - \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$

$+ \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) - \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2} = 2\sqrt{2} = 2y_A$

Dus de grafiek van f is puntsymmetrisch in $A(\frac{1}{4}\pi, \sqrt{2})$.

c $f(\frac{3}{4}\pi - p) = \sin(\frac{3}{4}\pi - p) - \cos(\frac{3}{4}\pi - p) + \sqrt{2}$

$= \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) - (\cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p)) + \sqrt{2}$

$= \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$

$= \sqrt{2} \cdot \cos(p) + \sqrt{2}$

$f(\frac{3}{4}\pi + p) = \sin(\frac{3}{4}\pi + p) - \cos(\frac{3}{4}\pi + p) + \sqrt{2}$

$= \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) - (\cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p)) + \sqrt{2}$

$= \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$

$= \sqrt{2} \cdot \cos(p) + \sqrt{2}$

$f(\frac{3}{4}\pi - p) = f(\frac{3}{4}\pi + p)$

Dus de lijn $x = \frac{3}{4}\pi$ is symmetrieas van de grafiek van f.

36 **a** $\vec{v}(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -ab\sin(bt) \\ ab\cos(bt) \end{pmatrix}$

$v(t) = \sqrt{(-ab\sin(bt))^2 + (ab\cos(bt))^2} = \sqrt{a^2b^2\sin^2(bt) + a^2b^2\cos^2(bt)}$

$= \sqrt{a^2b^2(\sin^2(bt) + \cos^2(bt))} = \sqrt{a^2b^2} = ab$

b $\vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} -ab^2\cos(bt) \\ -ab^2\sin(bt) \end{pmatrix}$

$\vec{v}(t) \cdot \vec{a}(t) = \begin{pmatrix} -ab\sin(bt) \\ ab\cos(bt) \end{pmatrix} \cdot \begin{pmatrix} -ab^2\cos(bt) \\ -ab^2\sin(bt) \end{pmatrix} = a^2b^3\sin(bt)\cos(bt) - a^2b^3\cos(bt)\sin(bt) = 0$

Dus $\vec{v}(t) \perp \vec{a}(t)$.

c $|\vec{a}(t)| = \sqrt{(-ab^2\cos(bt))^2 + (-ab^2\sin(bt))^2} = \sqrt{a^2b^4\cos^2(bt) + a^2b^4\sin^2(bt)}$

$= \sqrt{a^2b^4(\sin^2(bt) + \cos^2(bt))} = \sqrt{a^2b^4} = ab^2$

$v = 10$ en $|\vec{a}(t)| = 5$ geeft $ab = 10$ en $ab^2 = 5$

$\frac{ab^2}{ab} = \frac{5}{10}$

$b = \frac{1}{2}$

$b = \frac{1}{2}$ en $ab = 10$ geeft $a = 20$

Dus de straal van de cirkel is 20.

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37 **a** $f(x) = 2\cos^2(x) + \sin(2x) = 2\cos^2(x) - 1 + 1 + \sin(2x) = \cos(2x) + \sin(2x) + 1$

$f(x) = \cos(2x) + \sin(2x) + 1$ geeft $f'(x) = -2\sin(2x) + 2\cos(2x)$

$f'(x) = 0$ geeft $-2\sin(2x) + 2\cos(2x) = 0$

$\sin(2x) = \cos(2x)$

$\tan(2x) = 1$

$2x = \frac{1}{4}\pi + k \cdot \pi$

$x = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$

x in $[0, \pi]$ geeft $x = \frac{1}{8}\pi \vee x = \frac{5}{8}\pi$

max. is $f(\frac{1}{8}\pi) = \cos(\frac{1}{4}\pi) + \sin(\frac{1}{4}\pi) + 1 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} + 1 = 1 + \sqrt{2}$

min. is $f(\frac{5}{8}\pi) = \cos(1\frac{1}{4}\pi) + \sin(1\frac{1}{4}\pi) + 1 = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} + 1 = 1 - \sqrt{2}$

Dus $B_f = [1 - \sqrt{2}, 1 + \sqrt{2}]$.

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b $f(x) = g(x)$ geeft $2\cos^2(x) + \sin(2x) = 2\sin(2x)$

$$2\cos^2(x) = \sin(2x)$$
$$2\cos^2(x) = 2\sin(x)\cos(x)$$
$$2\cos(x) = 0 \vee \cos(x) = \sin(x)$$
$$\cos(x) = 0 \vee \tan(x) = 1$$
$$x = \tfrac{1}{2}\pi + k \cdot \pi \vee x = \tfrac{1}{4}\pi + k \cdot \pi$$

x in $[0, \pi]$ geeft $x = \frac{1}{4}\pi \vee x = \frac{1}{2}\pi$

$$O(V) = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2\sin(2x) - (2\cos^2(x) + \sin(2x)))\,dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2\sin(2x) - (\cos(2x) + \sin(2x) + 1))\,dx$$
$$= \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2\sin(2x) - \cos(2x) - \sin(2x) - 1)\,dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (\sin(2x) - \cos(2x) - 1)\,dx$$
$$= \left[-\tfrac{1}{2}\cos(2x) - \tfrac{1}{2}\sin(2x) - x\right]_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} = -\tfrac{1}{2}\cos(\pi) - \tfrac{1}{2}\sin(\pi) - \tfrac{1}{2}\pi - (-\tfrac{1}{2}\cos(\tfrac{1}{2}\pi) - \tfrac{1}{2}\sin(\tfrac{1}{2}\pi) - \tfrac{1}{4}\pi)$$
$$= \tfrac{1}{2} - 0 - \tfrac{1}{2}\pi + 0 + \tfrac{1}{2} + \tfrac{1}{4}\pi = 1 - \tfrac{1}{4}\pi$$

c C is het midden van AB als $g(p) = \frac{1}{2}f(p)$.

Dit geeft $2\sin(2p) = \cos^2(p) + \frac{1}{2}\sin(2p)$

$$1\tfrac{1}{2}\sin(2p) = \cos^2(p)$$
$$3\sin(p)\cos(p) = \cos^2(p)$$
$$\cos(p) = 0 \vee 3\sin(p) = \cos(p)$$
$$p = \tfrac{1}{2}\pi + k \cdot \pi \vee 3 \cdot \frac{\sin(p)}{\cos(p)} = 1$$

vold. niet $\qquad \tan(p) = \frac{1}{3}$

38 **a** $u_P = 25\sin(8\pi t)$

Per trilling wordt $4 \cdot 25 = 100$ mm afgelegd.

Er zijn $\frac{8\pi}{2\pi} = 4$ trillingen per seconde,

dus P legt per minuut $60 \cdot 4 \cdot 100 = 24\,000$ mm = 24 m af.

$u_Q = 40\sin(6\pi t)$.

Per trilling wordt $4 \cdot 40 = 160$ mm afgelegd.

Er zijn $\frac{6\pi}{2\pi} = 3$ trillingen per seconde,

dus Q legt per minuut $60 \cdot 3 \cdot 160 = 28\,800$ mm = 28,8 m af.

Het punt Q legt de grootste afstand per minuut af. Het verschil is 4,8 m.

b Op $t = 0$ beginnen P en Q beide aan een nieuwe trilling.

P heeft trillingstijd $\frac{1}{4}$ seconde en Q heeft trillingstijd $\frac{1}{3}$ seconde. Dus na 1 seconde heeft P 4 trillingen gemaakt en Q 3 trillingen, dus beginnen P en Q na 1 seconde beide de tweede keer aan een nieuwe trilling.

Dus op $t = 1$.

c De afstand tussen P en Q is $|25\sin(8\pi t) - 40\sin(6\pi t)|$.

Voer in $y_1 = |25\sin(8\pi x) - 40\sin(6\pi x)|$ en $y_2 = 30$.

De optie snijpunt geeft $x = 0{,}2360...$

Dus het eerste tijdstip is $t \approx 0{,}236$.

39 **a** $f(x) = 0$ geeft $2\sin^2(x) + \sin(x) = 0$

$$\sin(x)(2\sin(x) + 1) = 0$$
$$\sin(x) = 0 \vee \sin(x) = -\tfrac{1}{2}$$
$$x = k \cdot \pi \vee x = -\tfrac{1}{6}\pi + k \cdot 2\pi \vee x = 1\tfrac{1}{6}\pi + k \cdot 2\pi$$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi \vee x = 2\pi$

$$O = \int_0^{\pi} f(x)\,dx = \int_0^{\pi} (2\sin^2(x) + \sin(x))\,dx = \int_0^{\pi} (2\sin^2(x) - 1 + 1 + \sin(x))\,dx$$
$$= \int_0^{\pi} (-\cos(2x) + \sin(x) + 1)\,dx = \left[-\tfrac{1}{2}\sin(2x) - \cos(x) + x\right]_0^{\pi}$$
$$= -\tfrac{1}{2}\sin(2\pi) - \cos(\pi) + \pi - (-\tfrac{1}{2}\sin(0) - \cos(0) + 0) = 0 - -1 + \pi + 0 + 1 = 2 + \pi$$

b $f(x) = 2\sin^2(x) + \sin(x)$ geeft $f'(x) = 4\sin(x)\cos(x) + \cos(x)$

$f'(x) = 0$ geeft $4\sin(x)\cos(x) + \cos(x) = 0$

$\cos(x)(4\sin(x) + 1) = 0$

$\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$

$x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee \sin(x) = -\frac{1}{4}$

$f(\frac{1}{2}\pi) = 2\sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) = 2 \cdot 1 + 1 = 3$

$f(1\frac{1}{2}\pi) = 2\sin^2(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) = 2 \cdot 1 - 1 = 1$

$\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} = -\frac{1}{8}$

$f(x) = p$ heeft precies vier oplossingen voor $-\frac{1}{8} < p < 0 \vee 0 < p < 1$.

c $f(\frac{1}{2}\pi - p) = 2\sin^2(\frac{1}{2}\pi - p) + \sin(\frac{1}{2}\pi - p)$

$= 2(\sin(\frac{1}{2}\pi)\cos(p) - \cos(\frac{1}{2}\pi)\sin(p))^2 + \sin(\frac{1}{2}\pi)\cos(p) - \cos(\frac{1}{2}\pi)\sin(p)$

$= 2(1 \cdot \cos(p) - 0)^2 + 1 \cdot \cos(p) - 0$

$= 2\cos^2(p) + \cos(p)$

$f(\frac{1}{2}\pi + p) = 2\sin^2(\frac{1}{2}\pi + p) + \sin(\frac{1}{2}\pi + p)$

$= 2(\sin(\frac{1}{2}\pi)\cos(p) + \cos(\frac{1}{2}\pi)\sin(p))^2 + \sin(\frac{1}{2}\pi)\cos(p) + \cos(\frac{1}{2}\pi)\sin(p)$

$= 2(1 \cdot \cos(p) + 0)^2 + 1 \cdot \cos(p) + 0$

$= 2\cos^2(p) + \cos(p)$

Er geldt $f(\frac{1}{2}\pi - p) = f(\frac{1}{2}\pi + p)$, dus de grafiek van f is symmetrisch op het interval $[0, \pi]$.

$p = f(\frac{1}{2}\pi - \frac{1}{3}\pi) = f(\frac{1}{6}\pi) = 2 \cdot (\frac{1}{2})^2 + \frac{1}{2} = 1$

40 **a** $\begin{cases} x(t) = \sin(t) + \cos(t) \\ y(t) = 1 + 2\sin(2t + \pi) \end{cases}$

$x^2 = (\sin(t) + \cos(t))^2 = \sin^2(t) + 2\sin(t)\cos(t) + \cos^2(t) = 1 + \sin(2t)$

$\left.\begin{array}{l} y = 1 + 2\sin(2t + \pi) = 1 - 2\sin(2t) \\ x^2 = 1 + \sin(2t) \text{ geeft } \sin(2t) = x^2 - 1 \end{array}\right\} y = 1 - 2(x^2 - 1)$

$y = 1 - 2x^2 + 2$

$y = -2x^2 + 3$

Dus de baan van P is een deel van de bergparabool $y = -2x^2 + 3$.

Dus $a = -2$ en $b = 3$.

b $\left.\begin{array}{l} x^2 = 1 + \sin(2t) \\ -1 \leq \sin(2t) \leq 1 \end{array}\right\} 0 \leq x^2 \leq 2$

$-\sqrt{2} \leq x \leq \sqrt{2}$

$x = -\sqrt{2}$ en $y = -2x^2 + 3$ geeft $y = -1$

$x = \sqrt{2}$ en $y = -2x^2 + 3$ geeft $y = -1$

Dus de keerpunten zijn $(-\sqrt{2}, -1)$ en $(\sqrt{2}, -1)$.

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41 **a** De baan van P is de cirkel met middelpunt O en straal 4, de baan van Q is de cirkel met middelpunt O en straal 5.

De minimale afstand tussen P en Q is $5 - 4 = 1$ voor $5t = 3t + k \cdot 2\pi$

$2t = k \cdot 2\pi$

$t = k \cdot \pi$

Dus de minimale afstand wordt bereikt voor $t = 0$, $t = \pi$ en $t = 2\pi$.

De maximale afstand tussen P en Q is $5 + 4 = 9$ voor $5t = 3t + \pi + k \cdot 2\pi$

$2t = \pi + k \cdot 2\pi$

$t = \frac{1}{2}\pi + k \cdot \pi$

Dus de maximale afstand wordt bereikt voor $t = \frac{1}{2}\pi$ en $t = 1\frac{1}{2}\pi$.

b $PQ^2 = (5\cos(3t) - 4\cos(5t))^2 + (5\sin(3t) - 4\sin(5t))^2$

$= 25\cos^2(3t) - 40\cos(3t)\cos(5t) + 16\cos^2(5t) + 25\sin^2(3t) - 40\sin(3t)\sin(5t) + 16\sin^2(5t)$

$= 25(\cos^2(3t) + \sin^2(3t)) + 16(\cos^2(5t) + \sin^2(5t)) - 40(\cos(3t)\cos(5t) + \sin(3t)\sin(5t))$

$= 25 + 16 - 40\cos(3t - 5t) = 41 - 40\cos(-2t) = 41 - 40\cos(2t) = 41 - 40(1 - 2\sin^2(t))$

$= 41 - 40 + 80\sin^2(t) = 1 + 80\sin^2(t)$

Dus $PQ = \sqrt{1 + 80\sin^2(t)}$.

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c $PQ = \sqrt{41}$
$\sqrt{1 + 80\sin^2(t)} = \sqrt{41}$
$1 + 80\sin^2(t) = 41$
$80\sin^2(t) = 40$
$\sin^2(t) = \frac{1}{2}$
$\sin(t) = \frac{1}{2}\sqrt{2} \vee \sin(t) = -\frac{1}{2}\sqrt{2}$
$t = \frac{1}{4}\pi + k \cdot 2\pi \vee t = \frac{3}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{4}\pi + k \cdot 2\pi \vee t = 1\frac{1}{4}\pi + k \cdot 2\pi$
t in $[0, 2\pi]$ geeft $t = \frac{1}{4}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{1}{4}\pi \vee t = 1\frac{3}{4}\pi$

d Evenwijdig met de y-as, dus $x_P = x_Q$
$4\cos(5t) = 5\cos(3t)$
Voer in $y_1 = 4\cos(5x)$ en $y_2 = 5\cos(3x)$.
De optie snijpunt geeft $x = 0{,}756...$
Dus voor $t \approx 0{,}76$.

42 **a** $y = 0$ geeft $\cos(3t) = 0$
$3t = \frac{1}{2}\pi + k \cdot \pi$
$t = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$
$t = \frac{1}{6}\pi$ geeft het punt $(\frac{1}{2}, 0)$.
$t = \frac{1}{2}\pi$ geeft het punt $(-1, 0)$.
$t = \frac{5}{6}\pi$ geeft het punt $(\frac{1}{2}, 0)$.
$\begin{cases} x(t) = \cos(2t) \\ y(t) = \cos(3t) \end{cases}$ geeft $\begin{cases} x'(t) = -2\sin(2t) \\ y'(t) = -3\sin(3t) \end{cases}$ en dus $\vec{v}(t) = \begin{pmatrix} -2\sin(2t) \\ -3\sin(3t) \end{pmatrix}$.
$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} -2\sin(\frac{1}{3}\pi) \\ -3\sin(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -2 \cdot \frac{1}{2}\sqrt{3} \\ -3 \cdot 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$
$\vec{v}(\frac{5}{6}\pi) = \begin{pmatrix} -2\sin(1\frac{2}{3}\pi) \\ -3\sin(2\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -2 \cdot -\frac{1}{2}\sqrt{3} \\ -3 \cdot 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$
$$\cos(\varphi) = \frac{\left|\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}\right|} = \frac{|1 - 3|}{\sqrt{4} \cdot \sqrt{4}} = \frac{2}{4} = \frac{1}{2}$$
Dus $\varphi = 60°$.

b $x = 0$ geeft $\cos(2t) = 0$
$2t = \frac{1}{2}\pi + k \cdot \pi$
$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
$t = \frac{1}{4}\pi$ geeft het punt $(0, -\frac{1}{2}\sqrt{2})$.
$t = \frac{3}{4}\pi$ geeft het punt $(0, \frac{1}{2}\sqrt{2})$.
$\vec{v}(\frac{3}{4}\pi) = \begin{pmatrix} -2 \cdot -1 \\ -3 \cdot \frac{1}{2}\sqrt{2} \end{pmatrix} = \begin{pmatrix} 2 \\ -1\frac{1}{2}\sqrt{2} \end{pmatrix}$
Stel k: $y = ax + b$ met $a = \dfrac{-1\frac{1}{2}\sqrt{2}}{2} = -\frac{3}{4}\sqrt{2}$ en k door $(0, \frac{1}{2}\sqrt{2})$.
Dus k: $y = -\frac{3}{4}\sqrt{2} \cdot x + \frac{1}{2}\sqrt{2}$.

c De baansnelheid is $v(\frac{1}{2}\pi) = \sqrt{(-2\sin(\pi))^2 + (-3\sin(1\frac{1}{2}\pi))^2} = \sqrt{0 + 9} = 3$.

d $d(A, B) = 1$, dus A op de lijn $y = \frac{1}{2}$, omdat de baan symmetrisch is in de x-as.
$\cos(3t) = \frac{1}{2}$
$3t = \frac{1}{3}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{3}\pi + k \cdot 2\pi$
$t = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee t = -\frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$
t in $[0, \pi]$ geeft $t = \frac{1}{9}\pi \vee t = \frac{7}{9}\pi \vee t = \frac{5}{9}\pi$
$x(\frac{1}{9}\pi) = \cos(\frac{2}{9}\pi) = 0{,}766...$
$x(\frac{7}{9}\pi) = \cos(\frac{14}{9}\pi) = 0{,}173...$
$x(\frac{5}{9}\pi) = \cos(\frac{10}{9}\pi) = -0{,}939...$
Dus $p \approx -0{,}94 \vee p \approx 0{,}17 \vee p \approx 0{,}77$.

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43 **a** $y = 0$ geeft $2\cos(2t) = 0$

$\cos(2t) = 0$

$2t = \frac{1}{2}\pi + k \cdot \pi$

$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$t = -\frac{1}{4}\pi$ geeft het punt $(1\frac{1}{2}\sqrt{2}, 0)$.

$\begin{cases} x(t) = 3\cos(t + \frac{1}{2}\pi) \\ y(t) = 2\cos(2t) \end{cases}$ geeft $\begin{cases} x'(t) = -3\sin(t + \frac{1}{2}\pi) \\ y'(t) = -4\sin(2t) \end{cases}$

Dus $\vec{v}(t) = \begin{pmatrix} -3\sin(t + \frac{1}{2}\pi) \\ -4\sin(2t) \end{pmatrix}$.

$\vec{v}(-\frac{1}{4}\pi) = \begin{pmatrix} -3 \cdot \frac{1}{2}\sqrt{2} \\ -4 \cdot -1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}\sqrt{2} \\ 4 \end{pmatrix}$

$\tan(\varphi) = \dfrac{4}{-1\frac{1}{2}\sqrt{2}}$ geeft $\varphi = -62{,}061...°$

Dus de hoek waaronder de baan de positieve x-as snijdt is ongeveer 62,1°.

b De baansnelheid is $v(-\frac{1}{4}\pi) = \sqrt{(-1\frac{1}{2}\sqrt{2})^2 + 4^2} = \sqrt{4\frac{1}{2} + 16} = \sqrt{\frac{41}{2}} = \frac{1}{2}\sqrt{82}$

c $x = 0$ geeft $3\cos(t + \frac{1}{2}\pi) = 0$

$\cos(t + \frac{1}{2}\pi) = 0$

$t + \frac{1}{2}\pi = \frac{1}{2}\pi + k \cdot \pi$

$t = k \cdot \pi$

$t = 0$ geeft $y = 2$, dus de baan snijdt de y-as in het punt $(0, 2)$.

Dus de vergelijking van de parabool is van de vorm $y = ax^2 + 2$.

$\left.\begin{array}{l} y = ax^2 + 2 \\ t = \frac{1}{2}\pi \text{ geeft het punt } (-3, -2) \end{array}\right\}$ $a \cdot (-3)^2 + 2 = -2$

$9a = -4$

$a = -\frac{4}{9}$

Dus $y = -\frac{4}{9}x^2 + 2$.

Substitutie van $x = 3\cos(t + \frac{1}{2}\pi)$ en $y = 2\cos(2t)$ in $y = -\frac{4}{9}x^2 + 2$ geeft

$2\cos(2t) = -\frac{4}{9} \cdot (3\cos(t + \frac{1}{2}\pi))^2 + 2$

$2\cos(2t) = -\frac{4}{9} \cdot 9(\cos(t + \frac{1}{2}\pi))^2 + 2$

$2\cos(2t) = -4(\sin(t + \pi))^2 + 2$

$\cos(2t) = -2(-\sin(t))^2 + 1$

$\cos(2t) = 1 - 2\sin^2(t)$

Dit klopt voor elke t, dus de baan van P is een deel van de parabool $y = -\frac{4}{9}x^2 + 2$.

d $\vec{p} = \begin{pmatrix} 3\cos(t + \frac{1}{2}\pi) \\ 2\cos(2t) \end{pmatrix}$

$\overrightarrow{PQ} = \vec{p}_L = \begin{pmatrix} -2\cos(2t) \\ 3\cos(t + \frac{1}{2}\pi) \end{pmatrix}$

$\vec{q} = \vec{p} + \vec{p}_L = \begin{pmatrix} 3\cos(t + \frac{1}{2}\pi) \\ 2\cos(2t) \end{pmatrix} + \begin{pmatrix} -2\cos(2t) \\ 3\cos(t + \frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 3\cos(t + \frac{1}{2}\pi) - 2\cos(2t) \\ 2\cos(2t) + 3\cos(t + \frac{1}{2}\pi) \end{pmatrix}$

Dus de bewegingsvergelijkingen van Q zijn $\begin{cases} x_Q = 3\cos(t + \frac{1}{2}\pi) - 2\cos(2t) \\ y_Q = 2\cos(2t) + 3\cos(t + \frac{1}{2}\pi) \end{cases}$

e Q op de y-as geeft $x_Q = 0$

$3\cos(t + \frac{1}{2}\pi) - 2\cos(2t) = 0$

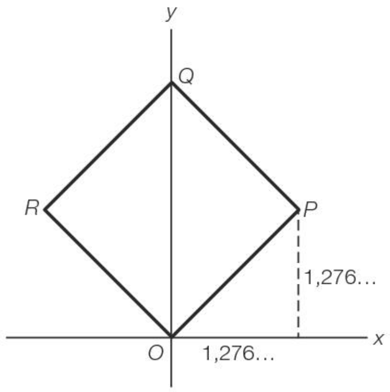
$3\cos(t + \frac{1}{2}\pi) = 2\cos(2t)$

Voer in $y_1 = 3\cos(x + \frac{1}{2}\pi)$ en $y_2 = 2\cos(2x)$.

De optie snijpunt geeft $x = -0{,}439...$

Dus $t = -0{,}439...$ en dit geeft $P(1{,}276...; 1{,}276...)$.

$O(OPQR) = OP^2 = 1{,}276...^2 + 1{,}276...^2 \approx 3{,}26$.



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Bladzijde 236

44 **a** $f(x) = \dfrac{x+2}{x^2-4x+8} = \dfrac{x-2+4}{x^2-4x+8} = \dfrac{\frac{1}{2}(2x-4)+4}{x^2-4x+8} = \dfrac{\frac{1}{2}(2x-4)}{x^2-4x+8} + \dfrac{4}{x^2-4x+8}$

$\int \dfrac{\frac{1}{2}(2x-4)}{x^2-4x+8}\,dx = \int \frac{1}{2} \cdot \dfrac{1}{x^2-4x+8}(2x-4)\,dx = \int \frac{1}{2} \cdot \dfrac{1}{x^2-4x+8}\,d(x^2-4x+8) = \frac{1}{2}\ln|x^2-4x+8| + c$

$\int \dfrac{4}{x^2-4x+8}\,dx = \int \dfrac{4}{(x-2)^2-4+8}\,dx = \int \dfrac{1}{\frac{1}{4}(x-2)^2+1}\,dx = \int \dfrac{1}{(\frac{1}{2}(x-2))^2+1}\,dx$

$= \int \dfrac{1}{(\frac{1}{2}x-1)^2+1}\,dx = 2\arctan(\frac{1}{2}x-1) + c$

Dus $F(x) = \frac{1}{2}\ln|x^2-4x+8| + 2\arctan(\frac{1}{2}x-1) + c$.

b $F(x) = \int 3x^2\sin(x^3)\,dx = \int \sin(x^3)\,dx^3 = -\cos(x^3) + c$

c $F(x) = \int \cos(x) \cdot \sqrt[3]{1+\sin(x)}\,dx = \int \sqrt[3]{1+\sin(x)}\,d\sin(x) = \int \sqrt[3]{1+u}\,du = \int (1+u)^{\frac{1}{3}}\,du$

$= \frac{3}{4}(1+u)^{1\frac{1}{3}} + c = \frac{3}{4}(1+u) \cdot \sqrt[3]{1+u} + c = \frac{3}{4}(1+\sin(x)) \cdot \sqrt[3]{1+\sin(x)} + c$

d $F(x) = \int (2x+4)\cos(2x)\,dx = \int (2x+4)\,d\frac{1}{2}\sin(2x) = (2x+4) \cdot \frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)\,d(2x+4)$

$= (x+2)\sin(2x) - \int \sin(2x)\,dx = (x+2)\sin(2x) + \frac{1}{2}\cos(2x) + c$

e $F(x) = \int \dfrac{2x+1}{\sqrt{16-x^2}}\,dx = \int \left(\dfrac{2x}{\sqrt{16-x^2}} + \dfrac{1}{\sqrt{16-x^2}}\right)dx$

$= \int -\dfrac{1}{\sqrt{16-x^2}}\,d(16-x^2) + \int \dfrac{\frac{1}{4}}{\sqrt{1-(\frac{1}{4}x)^2}}\,dx$

$= \int -\dfrac{1}{\sqrt{u}}\,du + \arcsin(\frac{1}{4}x) = -2\sqrt{u} + \arcsin(\frac{1}{4}x) + c = -2\sqrt{16-x^2} + \arcsin(\frac{1}{4}x) + c$

f $F(x) = \int \dfrac{2x+4}{\sqrt{-x^2-4x-3}}\,dx = \int \dfrac{-1}{\sqrt{-x^2-4x-3}}\,d(-x^2-4x-3) = \int \dfrac{-1}{\sqrt{u}}\,du = -2\sqrt{u} + c$

$= -2\sqrt{-x^2-4x-3} + c$

g $f(x) = \dfrac{2x+7}{x^2+6x+8} = \dfrac{2x+7}{(x+2)(x+4)} = \dfrac{a}{x+2} + \dfrac{b}{x+4} = \dfrac{a(x+4)+b(x+2)}{(x+2)(x+4)} = \dfrac{(a+b)x+4a+2b}{(x+2)(x+4)}$

$\begin{cases} a+b=2 \\ 4a+2b=7 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 2a+2b=4 \\ 4a+2b=7 \end{cases}$ −

$-2a = -3$

$\left.\begin{aligned} a &= 1\tfrac{1}{2} \\ a+b &= 2 \end{aligned}\right\}$ $1\frac{1}{2} + b = 2$

$b = \frac{1}{2}$

$f(x) = \dfrac{1\frac{1}{2}}{x+2} + \dfrac{\frac{1}{2}}{x+4}$ geeft $F(x) = 1\frac{1}{2}\ln|x+2| + \frac{1}{2}\ln|x+4| + c$

45 **a** $f(x) = 4\sin^2(x)\cos(x)$ geeft

$f'(x) = 8\sin(x)\cos(x) \cdot \cos(x) + 4\sin^2(x) \cdot -\sin(x) = 8\sin(x)\cos^2(x) - 4\sin^3(x)$

$f(x) = 0$ geeft $4\sin^2(x) \cdot \cos(x) = 0$

$\sin(x) = 0 \vee \cos(x) = 0$

$x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$

$f'(0) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

$f'(\pi) = 8 \cdot 0 \cdot (-1)^2 - 4 \cdot 0 = 0$

$f'(2\pi) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

Dus de grafiek raakt de x-as in $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.

b Stel $k: y = ax + b$ met $a = f'(\frac{1}{2}\pi) = 8 \cdot 1 \cdot 0 - 4 \cdot 1 = -4$.

$\left.\begin{aligned} &y = -4x + b \\ &f(\tfrac{1}{2}\pi) = 0, \text{ dus } A(\tfrac{1}{2}\pi, 0) \end{aligned}\right\}$ $-4 \cdot \frac{1}{2}\pi + b = 0$

$b = 2\pi$

Dus $k: y = -4x + 2\pi$.

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c $O(V) = \int_0^{\frac{1}{2}\pi} 4\sin^2(x)\cos(x)\,dx = \int_{x=0}^{x=\frac{1}{2}\pi} 4\sin^2(x)\,d\sin(x) = \left[\frac{4}{3}\sin^3(x)\right]_0^{\frac{1}{2}\pi} = \frac{4}{3}\cdot 1 - \frac{4}{3}\cdot 0 = 1\frac{1}{3}$

46 **a** $f(x) = \frac{2}{3}$ geeft $\frac{2x}{x^2-4} = \frac{2}{3}$

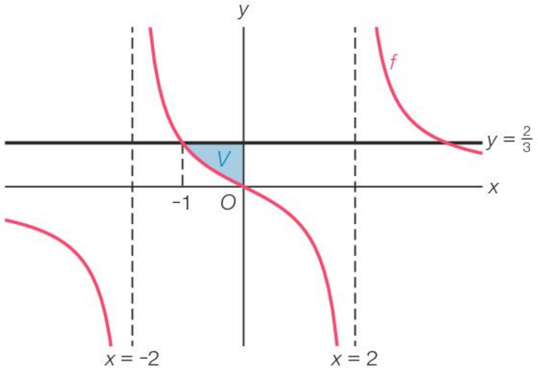
$2x^2 - 8 = 6x$

$2x^2 - 6x - 8 = 0$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x = -1 \vee x = 4$



$O(V) = \int_{-1}^{0} \left(\frac{2}{3} - \frac{2x}{x^2-4}\right)dx$

$\int \frac{2x}{x^2-4}\,dx = \int \frac{1}{x^2-4}\,2x\,dx = \int \frac{1}{x^2-4}\,d(x^2-4) = \ln|x^2-4| + c$

Dus $O(V) = \left[\frac{2}{3}x - \ln|x^2-4|\right]_{-1}^{0} = 0 - \ln(4) - (-\frac{2}{3} - \ln(3)) = -\ln(4) + \frac{2}{3} + \ln(3) = \frac{2}{3} + \ln(\frac{3}{4})$.

b $\int_p^{p+4} f(x)\,dx = \int_p^{p+4} \frac{2x}{x^2-4}\,dx = \left[\ln(x^2-4)\right]_p^{p+4} = \ln((p+4)^2-4) - \ln(p^2-4)$

$= \ln(p^2+8p+16-4) - \ln(p^2-4) = \ln\left(\frac{p^2+8p+12}{p^2-4}\right)$

Omdat $p^2 - 4 > 0$ als $p > 2$, kan de modulus wegblijven.

$\int_p^{p+4} f(x)\,dx = \ln(2)$

$\ln\left(\frac{p^2+8p+12}{p^2-4}\right) = \ln(2)$

$\frac{p^2+8p+12}{p^2-4} = 2$

$p^2 + 8p + 12 = 2p^2 - 8$

$p^2 - 8p - 20 = 0$

$(p+2)(p-10) = 0$

$p = -2 \quad \vee \quad p = 10$

vold. niet

Dus voor $p = 10$.

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47 **a** $f(x) = \dfrac{x^3 + 10x}{x^2 + 1}$ geeft

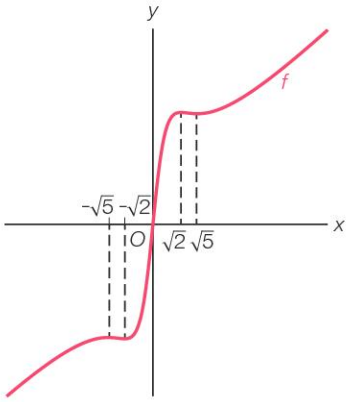
$$f'(x) = \frac{(x^2+1)\cdot(3x^2+10) - (x^3+10x)\cdot 2x}{(x^2+1)^2} = \frac{3x^4 + 10x^2 + 3x^2 + 10 - 2x^4 - 20x^2}{(x^2+1)^2} = \frac{x^4 - 7x^2 + 10}{(x^2+1)^2}$$

$f'(x) = 0$ geeft $x^4 - 7x^2 + 10 = 0$

$(x^2 - 2)(x^2 - 5) = 0$

$x^2 = 2 \vee x^2 = 5$

$x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{5} \vee x = -\sqrt{5}$



$$f(\sqrt{2}) = \frac{(\sqrt{2})^3 + 10\cdot\sqrt{2}}{(\sqrt{2})^2 + 1} = \frac{2\sqrt{2} + 10\sqrt{2}}{2+1} = \frac{12\sqrt{2}}{3} = 4\sqrt{2}$$

$$f(-\sqrt{2}) = \frac{(-\sqrt{2})^3 + 10\cdot -\sqrt{2}}{(-\sqrt{2})^2 + 1} = \frac{-2\sqrt{2} - 10\sqrt{2}}{2+1} = \frac{-12\sqrt{2}}{3} = -4\sqrt{2}$$

$$f(\sqrt{5}) = \frac{(\sqrt{5})^3 + 10\cdot\sqrt{5}}{(\sqrt{5})^2 + 1} = \frac{5\sqrt{5} + 10\sqrt{5}}{5+1} = \frac{15\sqrt{5}}{6} = 2\tfrac{1}{2}\sqrt{5}$$

$$f(-\sqrt{5}) = \frac{(-\sqrt{5})^3 + 10\cdot -\sqrt{5}}{(-\sqrt{5})^2 + 1} = \frac{-5\sqrt{5} - 10\sqrt{5}}{5+1} = \frac{-15\sqrt{5}}{6} = -2\tfrac{1}{2}\sqrt{5}$$

De toppen zijn $(-\sqrt{5}, -2\tfrac{1}{2}\sqrt{5})$, $(-\sqrt{2}, -4\sqrt{2})$, $(\sqrt{2}, 4\sqrt{2})$ en $(\sqrt{5}, 2\tfrac{1}{2}\sqrt{5})$.

b Stel $k{:}\, y = ax + b$ met $a = f'(1) = \dfrac{1 - 7 + 10}{(1+1)^2} = 1$.

$\left.\begin{array}{l} y = x + b \\ f(1) = 5\tfrac{1}{2}, \text{ dus } A(1, 5\tfrac{1}{2}) \end{array}\right\}$ $1 + b = 5\tfrac{1}{2}$

$b = 4\tfrac{1}{2}$

Dus $k{:}\, y = x + 4\tfrac{1}{2}$.

c $f(x) = p$ heeft precies één oplossing voor $p < -4\sqrt{2} \vee -2\tfrac{1}{2}\sqrt{5} < p < 2\tfrac{1}{2}\sqrt{5} \vee p > 4\sqrt{2}$.

d $x^2 + 1 \,/\, x^3 + 10x \,\backslash\, x$

$\underline{x^3 + x}$ −

$9x$

$f(x) = x + \dfrac{9x}{x^2 + 1}$

$$\int \frac{9x}{x^2+1}\,dx = \int 4\tfrac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x\,dx = \int 4\tfrac{1}{2}\cdot\frac{1}{x^2+1}\,d(x^2+1) = 4\tfrac{1}{2}\ln\left|x^2+1\right| + c$$

$$O(V) = \int_0^3 f(x)\,dx = \int_0^3 \left(x + \frac{9x}{x^2+1}\right)dx = \left[\tfrac{1}{2}x^2 + 4\tfrac{1}{2}\ln(x^2+1)\right]_0^3$$

$$= 4\tfrac{1}{2} + 4\tfrac{1}{2}\ln(10) - 0 = 4\tfrac{1}{2} + 4\tfrac{1}{2}\ln(10)$$

Bladzijde 237

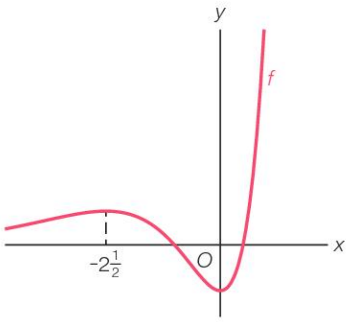
48 **a** $f(x) = (2x^2 + x - 1)\,e^x$ geeft $f'(x) = (4x + 1) \cdot e^x + (2x^2 + x - 1) \cdot e^x = (2x^2 + 5x)\,e^x$

$f'(x) = 0$ geeft $(2x^2 + 5x)\,e^x = 0$

$2x^2 + 5x = 0$

$x(2x + 5) = 0$

$x = 0 \vee x = -2\frac{1}{2}$



$f(0) = -1$ en $f(-2\frac{1}{2}) = 9\,e^{-2\frac{1}{2}} = \dfrac{9}{e^{2\frac{1}{2}}} = \dfrac{9}{e^2 \cdot \sqrt{e}}$

Dus de toppen zijn $(0, -1)$ en $\left(-2\frac{1}{2}, \dfrac{9}{e^2 \cdot \sqrt{e}}\right)$.

b $f(x) = 0$ geeft $(2x^2 + x - 1)\,e^x = 0$

$2x^2 + x - 1 = 0$

$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$

$x = \dfrac{-1 + 3}{4} = \frac{1}{2} \vee x = \dfrac{-1 - 3}{4} = -1$

$$\int(2x^2 + x - 1)\,e^x\,dx = \int(2x^2 + x - 1)\,d\,e^x = (2x^2 + x - 1)\,e^x - \int e^x d(2x^2 + x - 1)$$

$$= (2x^2 + x - 1)\,e^x - \int(4x + 1)\,e^x dx = (2x^2 + x - 1)\,e^x - \int(4x + 1)\,d\,e^x$$

$$= (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + \int e^x d(4x + 1) = (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + \int 4\,e^x\,dx$$

$$= (2x^2 + x - 1)\,e^x - (4x + 1)\,e^x + 4\,e^x = (2x^2 - 3x + 2)\,e^x$$

$$O(V) = -\int_{-1}^{\frac{1}{2}} f(x)\,dx = -\left[(2x^2 - 3x + 2)\,e^x\right]_{-1}^{\frac{1}{2}} = -(\tfrac{1}{2} - 1\tfrac{1}{2} + 2)e^{\frac{1}{2}} + (2 + 3 + 2)e^{-1} = -e^{\frac{1}{2}} + 7e^{-1} = \frac{7}{e} - \sqrt{e}$$

c $-\displaystyle\int_{-1}^{p} f(x)\,dx = \frac{1}{2}\left(\frac{7}{e} - \sqrt{e}\right)$ geeft $-\left[(2x^2 - 3x + 2)\,e^x\right]_{-1}^{p} = \dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$-(2p^2 - 3p + 2)\,e^p + \dfrac{7}{e} = \dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$-(2p^2 - 3p + 2)\,e^p = -\dfrac{7}{2e} - \frac{1}{2}\sqrt{e}$

$(2p^2 - 3p + 2)\,e^p = \dfrac{7}{2e} + \frac{1}{2}\sqrt{e}$

Voer in $y_1 = (2x^2 - 3x + 2)\,e^x$ en $y_2 = \dfrac{7}{2e} + \frac{1}{2}\sqrt{e}$.

De optie snijpunt geeft $x = -0{,}1130...$

Dus $p \approx -0{,}113$.

49 **a** $F(x) = \int x^3 \ln(x)\,dx = \int \ln(x)\,d(\frac{1}{4}x^4) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4\,d\ln(x) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \dfrac{1}{x}dx$

$= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3\,dx = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + c$

b $F(x) = \int x \ln(x^3)\,dx = \int 3x \ln(x)\,dx = \int 3\ln(x)\,d\frac{1}{2}x^2 = 3\ln(x) \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 d\,3\ln(x)$

$= 1\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \dfrac{3}{x}dx = 1\frac{1}{2}x^2 \ln(x) - \int 1\frac{1}{2}x\,dx = 1\frac{1}{2}x^2 \ln(x) - \frac{3}{4}x^2 + c$

c $F_n(x) = \int x^n \ln(x)\,dx = \int \ln(x)\,d\dfrac{1}{n+1}x^{n+1} = \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^{n+1}\,d\ln(x)$

$= \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^{n+1} \cdot \dfrac{1}{x}dx = \dfrac{1}{n+1}x^{n+1}\ln(x) - \int \dfrac{1}{n+1}x^n\,dx$

$= \dfrac{1}{n+1}x^{n+1}\ln(x) - \dfrac{1}{(n+1)^2}x^{n+1} + c$

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d Er geldt $x > 0$, omdat anders x^m niet bestaat.

Uit c volgt $\int x \ln(x)\,dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + c$.

$F_m(x) = \int x \ln(x^m)\,dx = \int mx \ln(x)\,dx = \frac{1}{2}mx^2 \ln(x) - \frac{1}{4}mx^2 + c$

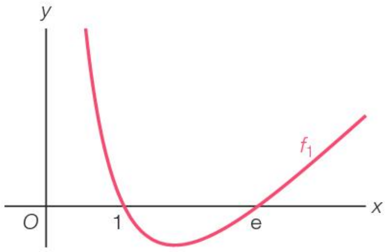
50 **a** $f_1(x) = 2\ln^2(x) - 2\ln(x)$

$f_1(x) = 0$ geeft $2\ln^2(x) - 2\ln(x) = 0$

$2\ln(x)(\ln(x) - 1) = 0$

$\ln(x) = 0 \vee \ln(x) = 1$

$x = 1 \vee x = \mathrm{e}$



$$\int 2\ln^2(x)\,dx = 2x\ln^2(x) - \int 2x\,d\ln^2(x) = 2x\ln^2(x) - \int 2x \cdot 2\ln(x) \cdot \frac{1}{x}dx$$
$$= 2x\ln^2(x) - \int 4\ln(x)\,dx$$
$$= 2x\ln^2(x) - 4(x\ln(x) - x) + c = 2x\ln^2(x) - 4x\ln(x) + 4x + c$$
$$\int 2\ln(x)\,dx = 2(x\ln(x) - x) + c = 2x\ln(x) - 2x + c$$
$$\int_1^{\mathrm{e}} f_1(x)\,dx = \int_1^{\mathrm{e}} (2\ln^2(x) - 2\ln(x))\,dx = \left[2x\ln^2(x) - 4x\ln(x) + 4x - (2x\ln(x) - 2x)\right]_1^{\mathrm{e}}$$
$$= \left[2x\ln^2(x) - 6x\ln(x) + 6x\right]_1^{\mathrm{e}} = 2\mathrm{e} - 6\mathrm{e} + 6\mathrm{e} - (0 - 0 + 6) = 2\mathrm{e} - 6$$

De oppervlakte is $-(2\mathrm{e} - 6) = 6 - 2\mathrm{e}$.

b $f_p(x) = 0$ geeft $2\ln^2(x) - 2p\ln(x) = 0$

$2\ln(x)(\ln(x) - p) = 0$

$\ln(x) = 0 \vee \ln(x) = p$

$x = 1 \vee x = \mathrm{e}^p$

Uit a volgt $F_p(x) = 2x\ln^2(x) - 4x\ln(x) + 4x - 2px\ln(x) + 2px + c$.

$$\int_1^{\mathrm{e}^p} f_p(x)\,dx = \left[2x\ln^2(x) - 4x\ln(x) + 4x - 2px\ln(x) + 2px\right]_1^{\mathrm{e}^p}$$
$$= 2\mathrm{e}^p \cdot p^2 - 4\mathrm{e}^p \cdot p + 4\mathrm{e}^p - 2p\mathrm{e}^p \cdot p + 2p\mathrm{e}^p - (0 - 0 + 4 - 0 + 2p)$$
$$= 2p^2\mathrm{e}^p - 4p\mathrm{e}^p + 4\mathrm{e}^p - 2p^2\mathrm{e}^p + 2p\mathrm{e}^p - 4 - 2p$$
$$= -2p\mathrm{e}^p + 4\mathrm{e}^p - 4 - 2p = (4 - 2p)\mathrm{e}^p - 4 - 2p$$

$f_p(x) \leq 0$ voor $1 \leq x \leq \mathrm{e}^p$, dus opp $= (2p - 4)\mathrm{e}^p + 2p + 4$.

opp = 8 geeft $(2p - 4)\mathrm{e}^p + 2p + 4 = 8$

$(2p - 4)\mathrm{e}^p + 2p - 4 = 0$

$(2p - 4)(\mathrm{e}^p + 1) = 0$

$2p - 4 = 0 \vee \mathrm{e}^p + 1 = 0$

$2p = 4$ geen opl.

$p = 2$

Dus voor $p = 2$.

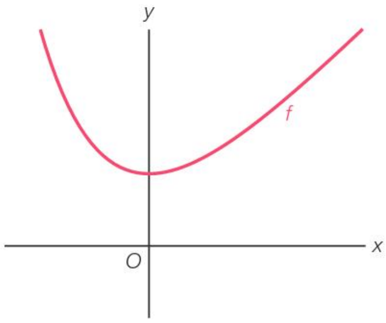
51 **a** $f(x) = x + e^{-x}$ geeft $f'(x) = 1 - e^{-x}$

$f'(x) = 0$ geeft $1 - e^{-x} = 0$

$e^{-x} = 1$

$-x = 0$

$x = 0$



min. is $f(0) = 1$

$B_f = [1, \rightarrow\rangle$

b $p > 0$

$O(V_p) = 6$ geeft $\int_{-p}^{p} (x + e^{-x})\,dx = 6$

$\left[\frac{1}{2}x^2 - e^{-x}\right]_{-p}^{p} = 6$

$\frac{1}{2}p^2 - e^{-p} - (\frac{1}{2}p^2 - e^{p}) = 6$

$\frac{1}{2}p^2 - e^{-p} - \frac{1}{2}p^2 + e^{p} = 6$

$e^{p} - 6 - e^{-p} = 0$

$(e^{p})^2 - 6e^{p} - 1 = 0$

Stel $e^{p} = u$.

$u^2 - 6u - 1 = 0$

$D = (-6)^2 - 4 \cdot 1 \cdot -1 = 40$

$u = \frac{6 + 2\sqrt{10}}{2} \vee u = \frac{6 - 2\sqrt{10}}{2}$

$u = 3 + \sqrt{10} \quad \vee \quad u = 3 - \sqrt{10}$

$e^{p} = 3 + \sqrt{10} \vee e^{p} = 3 - \sqrt{10}$

$p = \ln(3 + \sqrt{10})$ geen opl.

$p < 0$

$O(V_p) = 6$ geeft $\int_{p}^{-p} (x + e^{-x})\,dx = 6$

$\left[\frac{1}{2}x^2 - e^{-x}\right]_{p}^{-p} = 6$

$\frac{1}{2}p^2 - e^{p} - (\frac{1}{2}p^2 - e^{-p}) = 6$

$\frac{1}{2}p^2 - e^{p} - \frac{1}{2}p^2 + e^{-p} = 6$

$-e^{p} - 6 + e^{-p} = 0$

$(e^{p})^2 + 6e^{p} - 1 = 0$

Stel $e^{p} = u$.

$u^2 + 6u - 1 = 0$

$D = 6^2 - 4 \cdot 1 \cdot -1 = 40$

$u = \frac{-6 + 2\sqrt{10}}{2} \vee u = \frac{-6 - 2\sqrt{10}}{2}$

$u = -3 + \sqrt{10} \quad \vee \quad u = -3 - \sqrt{10}$

$e^{p} = -3 + \sqrt{10} \vee e^{p} = -3 - \sqrt{10}$

$p = \ln(-3 + \sqrt{10})$ geen opl.

Dus voor $p = \ln(3 + \sqrt{10})$ en voor $p = \ln(-3 + \sqrt{10})$.

c $I = \pi\int_{-1}^{0} (f(x))^2\,dx = \pi\int_{-1}^{0} (x + e^{-x})^2\,dx = \pi\int_{-1}^{0} (x^2 + 2x\,e^{-x} + e^{-2x})\,dx$

$\int 2x\,e^{-x}\,dx = \int 2x\,d{-e^{-x}} = -2x\,e^{-x} + \int e^{-x}\,d\,2x = -2x\,e^{-x} + \int 2e^{-x}\,dx = -2x\,e^{-x} - 2\,e^{-x} + c$

$I = \pi\left[\frac{1}{3}x^3 - 2x\,e^{-x} - 2\,e^{-x} - \frac{1}{2}e^{-2x}\right]_{-1}^{0}$

$= \pi(0 - 0 - 2 - \frac{1}{2}) - \pi(-\frac{1}{3} + 2e - 2e - \frac{1}{2}e^2)$

$= \pi(-2\frac{1}{2} + \frac{1}{3} + \frac{1}{2}e^2) = (\frac{1}{2}e^2 - 2\frac{1}{6})\pi$

52 **a** $F(x) = \int \frac{1}{e^x + 1}\,dx = \int \frac{e^{-x}}{1 + e^{-x}}\,dx = \int \frac{-1}{1 + e^{-x}}\,d(1 + e^{-x}) = \int -\frac{1}{u}\,du = -\ln|u| + c = -\ln(1 + e^{-x}) + c$

b $F(x) = \int \frac{\ln(\sin(x))}{\cos^2(x)}\,dx = \int \ln(\sin(x))\,d\tan(x) = \tan(x) \cdot \ln(\sin(x)) - \int \tan(x)\,d\ln(\sin(x))$

$= \tan(x) \cdot \ln(\sin(x)) - \int \tan(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x)\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - \int \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\sin(x)} \cdot \cos(x)\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - \int 1\,dx$

$= \tan(x) \cdot \ln(\sin(x)) - x + c$

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c $\sqrt{2x+1} = u$

$2x + 1 = u^2$

$2x = u^2 - 1$

$x = \frac{1}{2}u^2 - \frac{1}{2}$

$F(x) = \int x\sqrt{2x+1}\,dx = \int (\frac{1}{2}u^2 - \frac{1}{2}) \cdot u\,d(\frac{1}{2}u^2 - \frac{1}{2}) = \int (\frac{1}{2}u^2 - \frac{1}{2}) \cdot u \cdot u\,du = \int (\frac{1}{2}u^4 - \frac{1}{2}u^2)\,du$

$= \frac{1}{10}u^5 - \frac{1}{6}u^3 + c = \frac{1}{10}(2x+1)^2 \cdot \sqrt{2x+1} - \frac{1}{6}(2x+1)\sqrt{2x+1} + c$

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53 **a** $x = 11$ geeft $t^2 - 4 = 11$

$t^2 = 15$

$t = \sqrt{15} \vee t = -\sqrt{15}$

$y(\sqrt{15}) = \sqrt{15}\ln(4 + \sqrt{15})$

$y(-\sqrt{15}) = -\sqrt{15}\ln(4 - \sqrt{15}) = \sqrt{15}\ln((4 - \sqrt{15})^{-1}) = \sqrt{15}\ln\left(\frac{1}{4 - \sqrt{15}}\right)$

$= \sqrt{15}\ln\left(\frac{1}{4 - \sqrt{15}} \cdot \frac{4 + \sqrt{15}}{4 + \sqrt{15}}\right) = \sqrt{15}\ln\left(\frac{4 + \sqrt{15}}{16 - 15}\right) = \sqrt{15}\ln(4 + \sqrt{15})$

Dus $y(\sqrt{15}) = y(-\sqrt{15})$ en dus snijdt K zichzelf in het punt $(11, \sqrt{15}\ln(4 + \sqrt{15}))$.
Dit snijpunt ligt op de lijn $x = 11$, dus K snijdt zichzelf op de lijn $x = 11$.

b $\int y\,dx = \int t\ln(t+4)\,d(t^2 - 4) = \int t\ln(t+4) \cdot 2t\,dt = \int 2t^2\ln(t+4)\,dt$

De GR geeft $\int_{-\sqrt{15}}^{\sqrt{15}} 2x^2\ln(x+4)\,dx = 67{,}636...$

Dus oppervlakte $\approx 67{,}64$.

54 **a** $x = 0$ geeft $4\sin(t) = 0$

$t = k \cdot \pi$

$t = \frac{1}{2}\pi$ geeft het punt $(4, 4)$ en $t = \frac{1}{4}\pi$ geeft het punt $(2\sqrt{2}, 2\sqrt{2} - 2)$.
Dus V wordt in positieve richting omlopen.

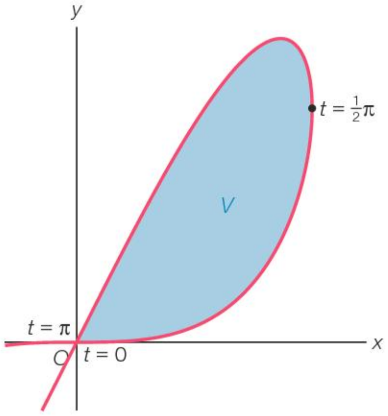
$O(V) = \int_{t=\pi}^{t=0} y\,dx = \int_{t=\pi}^{t=0} (4\sin(t) - 2\sin(2t))\,d\,4\sin(t) = \int_{\pi}^{0} (4\sin(t) - 4\sin(t)\cos(t)) \cdot 4\cos(t)\,dt$

$= \int_{\pi}^{0} (16\sin(t)\cos(t) - 16\sin(t)\cos^2(t))\,dt = \int_{\pi}^{0} 16\sin(t)\cos(t)\,dt - \int_{\pi}^{0} 16\sin(t)\cos^2(t)\,dt$

$= \int_{\pi}^{0} 8\sin(2t)\,dt + \int_{t=\pi}^{t=0} 16\cos^2(t)\,d\cos(t) = \left[-4\cos(2t)\right]_{\pi}^{0} + \left[5\frac{1}{3}\cos^3(t)\right]_{\pi}^{0}$

$= -4\cos(0) + 4\cos(2\pi) + 5\frac{1}{3}\cos^3(0) - 5\frac{1}{3}\cos^3(\pi) = -4 + 4 + 5\frac{1}{3} \cdot 1^3 - 5\frac{1}{3} \cdot (-1)^3 = 10\frac{2}{3}$

b x is maximaal 4 voor $t = \frac{1}{2}\pi$.



$$I(L) = \pi\int_{t=\pi}^{t=\frac{1}{2}\pi} y^2\,dx - \pi\int_{t=0}^{t=\frac{1}{2}\pi} y^2\,dx = \pi\int_{t=\pi}^{t=\frac{1}{2}\pi} y^2\,dx + \pi\int_{t=\frac{1}{2}\pi}^{t=0} y^2\,dx = \pi\int_{t=\pi}^{t=0} y^2\,dx$$

$$= \pi\int_{t=\pi}^{t=0} (4\sin(t) - 2\sin(2t))^2\,d\,4\sin(t) = \pi\int_{\pi}^{0}(4\sin(t) - 2\sin(2t))^2 \cdot 4\cos(t)\,dt$$

De GR geeft $\pi\int_{\pi}^{0}(4\sin(x) - 2\sin(2x))^2 \cdot 4\cos(x)\,dx = 157{,}913...$

Dus $I(L) \approx 157{,}91$.

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